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Multiplicative Connectivity Indices of Nanostructures

V.R. KULLI

Department of Mathematics Gulbarga University, Gulbarga 585 106 Karnataka, India

E-Mail of Corresponding Author : vrkulli@gmail.com

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Abstract

A single number that can be used to characterize some property of the graph of a molecular is called a topological index for that graph. In this paper, we compute several topological indices of linear [*n*]-anthracene, *V*-anthracene nanotube and nanotori: multiplicative Zagreb, multiplicative hyper-Zagreb, multiplicative sum connectivity, multiplicative product connectivity, general multiplicative Zagreb, multiplicative *ABC* and multiplicative *GA* indices.

Key words: Multiplicative indices, molecular graphs, linear [*n*] anthracene, *V*-anthracene nanotube, *V*-antracene nanotori.

Mathematics Subject Classification: 05*C*05, 05*C*07, 05*C*35.

1. Introduction

In this paper, we consider only finite, connected undirected without loops and multiple edges. A molecular graph is a simple graph such that its vertices correspond to the atoms and the edges to the bonds. We note that hydrogen atoms are often omitted. Recently nanostructures involving carbon have been the focus of an intense research activity. A topological index is a real number that is derived from molecular graphs of chemical compounds. In organic chemistry, topological indices have been found to be useful in chemical documentation, isomer discrimination structure property relationships, structure activity relationships and pharmaceutical drug design. There has been considerable interest in the general problem of computing topological indices. In this paper, we determine some topological indices for a family of linear [*n*] anthracene, lattice of *V*-antracene nanotube and nanotori.

Let G be a graph with a vertex set $V(G)$ and an edge set $E(G)$. The degree $d_G(v)$ of a vertex v is the number of vertices adjacent to v. For other undefined notations, readers may refer to [1].

In[2], two fairly new indices with higher prediction ability are respectively defined as

$$II_1(G)=\prod_{u\in V(G)} d_G(u)^2, \quad II_2(G)=\prod_{uv\in E(G)} d_G(u)\,d_G(v).$$

These two graph invariants are called first and second multiplicative Zagreb indices by Gutman in[3].

Recently, in[4], Eliasi *et al*. introduced a multiplicative version of the first Zagreb index as

$$II_1^*(G)=\prod_{uv\in E(G)}\left(d_G(u)+d_G(v)\right).$$

In[5], Kulli proposed the first and second multiplicative hypes-Zagreb indices as

$$HII_1(G)=\prod_{uv\in E(G)}\left(d_G(u)+d_G(v)\right)^2,\qquad HII_2(G)=\prod_{uv\in E(G)}\left(d_G(u)d_G(v)\right)^2.$$

Motivated by the definition of the first and second multiplicative hyper-Zagreb indices, the general first and second multiplicative Zagreb indices were very recently introduced in[6]. These indices are respectively defined as

$$MZ_1^a(G)=\prod_{uv\in E(G)}\left(d_G(u)+d_G(v)\right)^a,\qquad MZ_2^a(G)=\prod_{uv\in E(G)}\left(d_G(u)d_G(v)\right)^a.$$

One of the best known and widely used topological index is the product connectivity index or Randiæ index, introduced by Randiæ in[7]. The product connectivity index is defined as

$$\chi(G)=\sum_{uv\in E(G)}\frac{1}{\sqrt{d_G(u)d_G(v)}}.$$

Inspired by work on Rendić index, Kulli[8] introduced the multiplicative product connectivity index, multiplicative sum connectivity index, multiplicative atom bond connectivity index and multiplicative geometric- arithmetic index.

The multiplicative sum connectivity index of a graph G is defined as

$$XII(G)=\prod_{uv\in E(G)}\frac{1}{\sqrt{d_G(u)+d_G(v)}}.$$

The multiplicative product connectivity index of a graph G is defined as

$$\chi(G)=\prod_{uv\in E(G)}\frac{1}{\sqrt{d_G(u)d_G(v)}}.$$

The multiplicative atom bond connectivity index of a graph G is defined as

$$ABCII(G)=\prod_{uv\in E(G)}\sqrt{\frac{d_G(u)+d_G(v)-2}{d_G(u)d_G(v)}}.$$

The multiplicative geometric-arithmetic index of a graph G is defined as follows:

$$GAII(G)=\prod_{uv\in E(G)}\frac{2\sqrt{d_G(u)d_G(v)}}{d_G(u)+d_G(v)}.$$

Recently many other multiplicative indices were studied, for example, in [9-17].

In this paper, we compute the multiplicative Zagreb, multiplicative hyper-Zagreb, multiplicative sum connectivity, multiplicative product connectivity, general multiplicative Zagreb, multiplicative *ABC* and multiplicative *GA* indices for linear [*n*] anthracene, *V*-anthracene nanotube and nanotori.

2. Results for linear [n] Anthracenes

We compute index for anthracene graph. Anthracene is a solid polycylic aromatic hydrocarbon of formula $C_{14}H_{10}$ consisting of three fused benzene rings. It is a component of coal tar. Antracene is used in the production of the red dye alizarin and other dyes.

Theorem 1. Let T be a linear $[n]$ anthracene. Then

1) $II_1^*(T) = \left(\frac{4}{15}\right)^4 5^{12n}6^{6n}$. 2) $II_2(T) = \left(\frac{4}{27}\right)^4 6^{12n}9^{6n}$.

3) $HII_1(T) = \left(\frac{4}{15}\right)^8 5^{24n}6^{12n}$. 4) $HII_2(T) = \left(\frac{4}{27}\right)^8 6^{24n}9^{12n}$.

5) $XII(T) = \left(\frac{15}{4}\right)^2 5^{-6n}6^{-3n}$. 6) $\chi II(T) = \left(\frac{27}{4}\right)^2 6^{-6n}3^{-6n}$.

7) $MZ_1^a(T) = \left(\frac{4}{15}\right)^{4a} 5^{12na}6^{6na}$. 8) $MZ_2^a(T) = \left(\frac{4}{27}\right)^{4a} 6^{12na}9^{6na}$.

9) $ABCII(T) = \frac{81}{32}3^{-6n}$. 10) $GAII(T) = \left(\frac{2\sqrt{6}}{5}\right)^{12n-4}$.

Proof: Let T be a linear $[n]$-anthracene, see Figure 1. By algebraic method, we get $|V(T)| = 14n$ vertices and $|E(T)| = 18n - 2$ edges. We have three partitions of the edge set $E(T)$ as given in Table 1.

$d_T(u), d_T(v) \setminus uv \in E(T)$	$E_1 = (2, 2)$	$E_2 = (2, 3)$	$E_3 = (3, 3)$
Number of edges	6	$12n - 4$	$6n - 4$

Table 1. Computing the number of edges for a linear $[n]$ anthracene

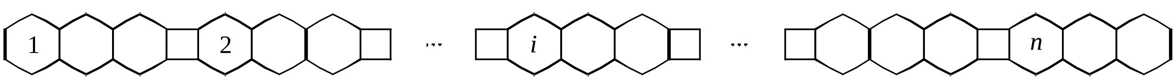

Figure 1

1) $$II_1^*(T) = \prod_{uv\in E(T)} \left[d_T(u) + d_T(v)\right] = \prod_{uv\in E_1} 4 \times \prod_{uv\in E_2} 5 \times \prod_{uv\in E_3} 6$$

$$= 4^6 \times 5^{12n-4} \times 6^{6n-4} = \left(\frac{4}{15}\right)^4 \times 5^{12n} \times 6^{6n}.$$

2) $$II_2(T) = \prod_{uv\in E(T)} \left[d_T(u) d_T(v)\right] = \prod_{uv\in E_1} 4 \times \prod_{uv\in E_2} 6 \times \prod_{uv\in E_3} 9$$

$$= 4^6 \times 6^{12n-4} \times 9^{6n-4} = \left(\frac{4}{27}\right)^4 \times 6^{12n} \times 9^{6n}.$$

3) $$HII_1(T) = \prod_{uv\in E(G)} \left[d_T(u) + d_T(v)\right]^2 = \prod_{uv\in E_1} 4^2 \times \prod_{uv\in E_2} 5^2 \times \prod_{uv\in E_3} 6^2$$

$$= 4^{12} \times 5^{2(12n-4)} \times 6^{2(6n-4)} = \left(\frac{4}{15}\right)^8 \times 5^{24n} \times 6^{12n}.$$

4) $$HII_2(T) = \prod_{uv\in E(G)} \left[d_T(u) d_T(v)\right]^2 = \prod_{uv\in E_1} 4^2 \times \prod_{uv\in E_2} 6^2 \times \prod_{uv\in E_3} 9^2$$

$$= 4^{12} \times 6^{2(12n-4)} \times 9^{2(6n-4)} \quad = \left(\frac{4}{27}\right)^{8} \times 6^{24n} \times 9^{12n}$$

5) $$XII(T) = \prod_{uv \in E(T)} \frac{1}{\sqrt{d_T(u) + d_T(v)}} = \prod_{uv \in E_1} \frac{1}{\sqrt{4}} \times \prod_{uv \in E_2} \frac{1}{\sqrt{5}} \times \prod_{uv \in E_3} \frac{1}{\sqrt{6}}$$

$$= \left(\frac{1}{2}\right)^{6} \times \left(\frac{1}{\sqrt{5}}\right)^{12n-4} \times \left(\frac{1}{\sqrt{6}}\right)^{6n-4} = \left(\frac{15}{4}\right)^{2} \times 5^{-6n} \times 6^{-3n}.$$

6) $$\chi II(T) = \prod_{uv \in E(T)} \frac{1}{\sqrt{d_T(u) d_T(v)}} = \prod_{uv \in E_1} \frac{1}{\sqrt{4}} \times \prod_{uv \in E_2} \frac{1}{\sqrt{6}} \times \prod_{uv \in E_3} \frac{1}{\sqrt{9}}$$

$$= \left(\frac{1}{2}\right)^{6} \times \left(\frac{1}{\sqrt{6}}\right)^{12n-4} \times \left(\frac{1}{3}\right)^{6n-4} = \left(\frac{27}{4}\right)^{2} \times 6^{-6n} \times 3^{-6n}.$$

7) $$MZ_1^a(T) = \prod_{uv \in E(T)} \left[d_T(u) + d_T(v)\right]^a = \prod_{uv \in E_1} 4^a \times \prod_{uv \in E_2} 5^a \times \prod_{uv \in E_3} 6^a$$

$$= 4^{6a} \times 6^{(12n-4)a} \times 6^{(6n-4)a} = \left(\frac{4}{15}\right)^{4a} \times 5^{12na} \times 6^{6na}.$$

8) $$MZ_2^a(T) = \prod_{uv \in E(T)} \left[d_T(u) d_T(v)\right]^a = \prod_{uv \in E_1} 4^a \times \prod_{uv \in E_2} 6^a \times \prod_{uv \in E_3} 9^a$$

$$= 4^{6a} \times 6^{(12n-4)a} \times 9^{(6n-4)a} = \left(\frac{4}{27}\right)^{4a} \times 6^{12na} \times 9^{6na}.$$

9) $$ABCII(T) = \prod_{uv \in E(T)} \sqrt{\frac{d_T(u) + d_T(v) - 2}{d_T(u) d_T(v)}}$$

$$= \prod_{uv \in E_1} \sqrt{\frac{2+2-2}{2 \times 2}} \times \prod_{uv \in E_2} \sqrt{\frac{3+2-2}{3 \times 2}} \times \prod_{uv \in E_3} \sqrt{\frac{3+3-2}{3 \times 3}}$$

$$= \left(\sqrt{\frac{1}{2}}\right)^{6} \times \left(\sqrt{\frac{1}{2}}\right)^{12n-4} \times \left(\frac{2}{3}\right)^{6n-4} = \frac{81}{32} 3^{-6n}.$$

10) $$GAII(T) = \prod_{uv \in E(T)} \frac{2\sqrt{d_T(u) d_T(v)}}{d_T(u) + d_T(v)} = \prod_{uv \in E_1} \frac{2\sqrt{2 \times 2}}{2+2} \times \prod_{uv \in E_2} \frac{2\sqrt{3 \times 2}}{3+2} \times \prod_{uv \in E_3} \frac{2\sqrt{3 \times 3}}{3+3}$$

$$= (1)^{6} \times \left(\frac{2\sqrt{6}}{5}\right)^{12n-4} \times (1)^{6n-4} = \left(\frac{2\sqrt{6}}{5}\right)^{12n-4}.$$

3. Results for V-Antracene Nanotubes

Theorem 2. Let G be a 2-dimensional lattice of V-anthracene.Then

1) $II_1^*(G)=6^{21pq}\times\left(\frac{5^4}{6^5}\right)^{3p}\times\left(\frac{5}{18}\right)^{4q}\times\left(\frac{24}{25}\right)^{4}$. 2) $II_2(G)=9^{21pq}\times\left(\frac{6^4}{9^5}\right)^{3p}\times\left(\frac{4}{27}\right)^{4q}$.

3) $HII_1(G)=6^{42pq}\times\left(\frac{5^4}{6^5}\right)^{6p}\times\left(\frac{5}{18}\right)^{8q}\times\left(\frac{24}{25}\right)^{8}$. 4) $HII_2(G)=9^{42pq}\times\left(\frac{6^4}{9^5}\right)^{6p}\times\left(\frac{4}{27}\right)^{8q}$.

5) $XII(G)=6^{\frac{21}{2}pq}\times\left(\frac{5^4}{6^5}\right)^{\frac{3}{2}p}\times\left(\frac{5}{18}\right)^{2q}\times\left(\frac{24}{25}\right)^{2}$. 6) $\chi II(G)=3^{21pq}\times\left(\frac{6^2}{3^5}\right)^{3p}\times\left(\frac{4}{27}\right)^{2q}$.

7) $MZ_1^a(G)=6^{21pqa}\times\left(\frac{5^4}{6^5}\right)^{3pa}\times\left(\frac{5}{18}\right)^{4qa}\times\left(\frac{24}{25}\right)^{4a}$. 8) $MZ_2^a(G)=9^{21pqa}\times\left(\frac{6^4}{9^5}\right)^{3pa}\times\left(\frac{4}{27}\right)^{4qa}$.

9) $ABCII(G)=\left(\frac{2}{3}\right)^{21pq}\times\frac{3^{15p+8q}}{3^{21p+11q}}\times\frac{64}{81}$. 10) $GAII(G)=\left(\frac{2\sqrt{6}}{5}\right)^{12p+4q-8}$.

Proof: Let G be a 2-dimensional lattice of V-anthracene, see Figure 2. By algebraic method, we get $|V(G)|=14pq$ vertices and $|E(G)| = 21pq - 3p - 2q$ edges.

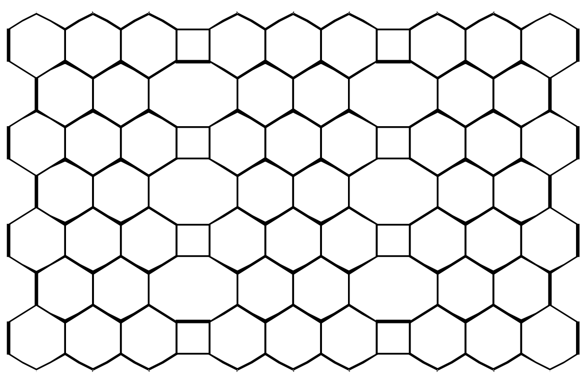

Figure 2

We have three partitions of the edge set $E(G)$ as given in table 2.

$d_G(u), d_G(v) \setminus uv \in E(G)$	$E_1 = (2, 2)$	$E_2 = (2, 3)$	$E_3 = (3, 3)$
Number of edges	$2q + 4$	$12p + 4q - 8$	$21pq - 15q - 8q + 4$

Table 2.Computing the number of edges for molecular graph G

1) $$II_1^*(G)=\prod_{uv\in E(G)}\left[d_G(u)+d_G(v)\right]$$

$$=4^{2q+4}\times 5^{12p+4q-8}\times 6^{21pq-15p-8q+4}=6^{21pq}\times\left(\frac{5^4}{6^5}\right)^{3p}\times\left(\frac{5}{18}\right)^{4q}\times\left(\frac{24}{25}\right)^{4}.$$

2) $$II_2(G)=\prod_{uv\in E(G)}\left[d_G(u)d_G(v)\right]=\prod_{uv\in E_1}4\times\prod_{uv\in E_2}6\times\prod_{uv\in E_3}9$$

$$=4^{2q+4}\times 6^{12p+4q-8}\times 9^{21pq-15p-8q+4}=9^{21pq}\times\left(\frac{6^4}{9^5}\right)^{3p}\times\left(\frac{4}{27}\right)^{4q}.$$

3) $$HII_1(G)=\prod_{uv\in E(G)}\left[d_G(u)+d_G(v)\right]^2=\prod_{uv\in E_1}4^2\times\prod_{uv\in E_2}5^2\times\prod_{uv\in E_3}6^2$$

$$=6^{42pq}\times\left(\frac{5^4}{6^5}\right)^{6p}\times\left(\frac{5}{18}\right)^{8q}\times\left(\frac{24}{25}\right)^{8}.$$

4) $$HII_2(G)=\prod_{uv\in E(G)}\left[d_G(u)d_G(v)\right]^2$$

$$=\prod_{uv\in E_1}4^2\times\prod_{uv\in E_2}6^2\times\prod_{uv\in E_3}9^2=9^{42pq}\times\left(\frac{6^4}{9^5}\right)^{6p}\times\left(\frac{4}{27}\right)^{8q}$$

5) $$XII(G)=\prod_{uv\in E(G)}\frac{1}{\sqrt{d_G(u)+d_G(v)}}=\prod_{uv\in E_1}\frac{1}{\sqrt{4}}\times\prod_{uv\in E_2}\frac{1}{\sqrt{5}}\times\prod_{uv\in E_3}\frac{1}{\sqrt{6}}$$

$$=6^{\frac{21}{2}pq}\times\left(\frac{5^4}{6^5}\right)^{\frac{3}{2}p}\times\left(\frac{5}{18}\right)^{2q}\times\left(\frac{24}{25}\right)^{2}.$$

6) $$\chi II(G)=\prod_{uv\in E(G)}\frac{1}{\sqrt{d_G(u)d_G(v)}}=\prod_{uv\in E_1}\frac{1}{\sqrt{4}}\times\prod_{uv\in E_2}\frac{1}{\sqrt{6}}\times\prod_{uv\in E_3}\frac{1}{\sqrt{9}}$$

$$=3^{21pq}\times\left(\frac{6^2}{3^5}\right)^{3p}\times\left(\frac{4}{27}\right)^{2q}.$$

7) $$MZ_1^a(G)=\prod_{uv\in E(G)}\left[d_G(u)+d_G(v)\right]^a=\prod_{uv\in E_1}4^a\times\prod_{uv\in E_2}5^a\times\prod_{uv\in E_3}6^a$$

$$=6^{21pqa}\times\left(\frac{5^4}{6^5}\right)^{3pa}\times\left(\frac{5}{18}\right)^{4qa}\times\left(\frac{24}{25}\right)^{4a}.$$

8) $$MZ_2^a(G)=\prod_{uv\in E(G)}\left[d_G(u)d_G(v)\right]^a=\prod_{uv\in E_1}4^a\times\prod_{uv\in E_2}6^a\times\prod_{uv\in E_3}9^a$$

$$=9^{21pqa}\times\left(\frac{6^4}{9^5}\right)^{3pa}\times\left(\frac{4}{27}\right)^{4qa}$$

9) $$ABCII(G)=\prod_{uv\in E(G)}\sqrt{\frac{d_G(u)+d_G(v)-2}{d_G(u)d_G(v)}}$$

$$=\prod_{uv\in E_1}\sqrt{\frac{2+2-2}{2\times 2}}\times\prod_{uv\in E_2}\sqrt{\frac{3+2-2}{3\times 2}}\times\prod_{uv\in E_3}\sqrt{\frac{3+3-2}{3\times 3}}$$

$$=\left(\sqrt{\frac{1}{2}}\right)^{2q+4}\times\left(\sqrt{\frac{1}{2}}\right)^{12p+4q-8}\times\left(\frac{2}{3}\right)^{21pq-15p-8q+4}=\left(\frac{2}{3}\right)^{21pq}\times\frac{3^{15p+8q}}{2^{21p+11q}}\times\frac{64}{81}.$$

10) $$GAII(G)=\prod_{uv\in E(G)}\frac{2\sqrt{d_G(u)d_G(v)}}{d_G(u)+d_G(v)}=\prod_{uv\in E_1}\frac{2\sqrt{2\times 2}}{2+2}\times\prod_{uv\in E_2}\frac{2\sqrt{3\times 2}}{3+2}\times\prod_{uv\in E_3}\frac{2\sqrt{3\times 3}}{3+3}$$

$$=(1)^{2q+4}\times\left(\frac{2\sqrt{6}}{5}\right)^{12p+4q-8}\times(1)^{21pq-15p-8q+4}=\left(\frac{2\sqrt{6}}{5}\right)^{12p+4q-8}.$$

Theorem 3. Let K be a lattice of V-anthracene nanotube. Then

1) $II_1^*(K)=5^{12p}\times 6^{21pq-15p}$.

2) $II_2(K)=2^{12p}\times 3^{42pq-18p}$.

3) $HII_1(K)=5^{24p}\times 6^{42pq-30p}$.

4) $HII_2(K)=2^{24p}\times 3^{84pq-36p}$.

5) $XII(K)=\left(\frac{1}{5}\right)^{6p}\times\left(\frac{1}{\sqrt{6}}\right)^{21pq-15p}$.

6) $\chi II(K)=\left(\frac{1}{6}\right)^{6p}\times\left(\frac{1}{3}\right)^{21pq-15p}$.

7) $MZ_1^a(K)=5^{12pa}\times 6^{(21pq-15p)a}$.

8) $MZ_2^a(K)=2^{12pa}\times 3^{(42pq-18p)a}$.

9) $ABCII(K)=\left(\frac{1}{2}\right)^{6p}\times\left(\frac{2}{3}\right)^{21pq-15p}$.

10) $GAII(K)=\left(\frac{2\sqrt{6}}{5}\right)^{12p}$.

Proof: Let K be a lattice of V-anthracene, see Figure 3.

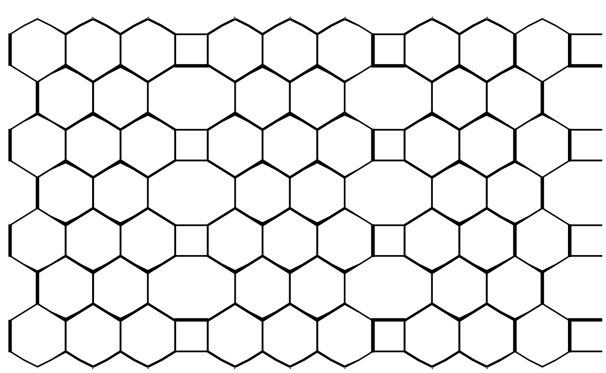

Figure 3

By algebraic method, we get $|V(K)|= 14pq$ vertices and $|E(K)|= 21pq -3p$ edges. We have two partitions of the edge set $E(K)$ as given in Table 3.

$d_K(u), d_K(u) \setminus uv \in E(K)$	$E_1 = (2, 3)$	$E_2 = (3, 3)$
Number of edges	$12p$	$21pq - 15p$

Table 3. Computing the number of edges for molecular graph K

1) $$II_1^*(K) = \prod_{uv\in E(K)} \left[d_K(u)+d_K(v)\right] = \prod_{uv\in E_1} 5 \times \prod_{uv\in E_2} 6 = 5^{12p} \times 6^{21pq-15p}.$$

2) $$II_2(K) = \prod_{uv\in E(K)} \left[d_K(u)d_K(v)\right] = \prod_{uv\in E_1} 6 \times \prod_{uv\in E_2} 9 = 6^{12q} \times 9^{21pq-15p}$$

$$= 2^{12p} \times 3^{42pq-18p}.$$

3) $$HII_1(K) = \prod_{uv\in E(K)} \left[d_K(u)+d_K(v)\right]^2 = \prod_{uv\in E_1} 5^2 \times \prod_{uv\in E_2} 6^2 = 5^{24p} \times 6^{42pq-30p}.$$

4) $$HII_2(K) = \prod_{uv\in E(K)} \left[d_K(u)d_K(v)\right]^2 = \prod_{uv\in E_1} 6^2 \times \prod_{uv\in E_2} 9^2 = \left(6^2\right)^{12p} \times \left(9^2\right)^{21pq-15p}$$

$$= 2^{24p} \times 3^{84pq-36p}.$$

5) $$XII(K) = \prod_{uv\in E(K)} \frac{1}{\sqrt{d_K(u)+d_K(v)}} = \prod_{uv\in E_1} \frac{1}{\sqrt{2+3}} \times \prod_{uv\in E_2} \frac{1}{\sqrt{3+3}}$$

$$= \left(\frac{1}{\sqrt{5}}\right)^{12p} \times \left(\frac{1}{\sqrt{6}}\right)^{21pq-15p}. = \left(\frac{1}{5}\right)^{6p} \times \left(\frac{1}{\sqrt{6}}\right)^{21pq-15p}.$$

6) $$\chi II(K) = \prod_{uv\in E(K)} \frac{1}{\sqrt{d_K(u)d_K(v)}} = \prod_{uv\in E_1} \frac{1}{\sqrt{2\times 3}} \times \prod_{uv\in E_2} \frac{1}{\sqrt{3\times 3}}$$

$$= \left(\frac{1}{\sqrt{6}}\right)^{12p} \times \left(\frac{1}{\sqrt{9}}\right)^{21pq-15p} = \left(\frac{1}{6}\right)^{6p} \times \left(\frac{1}{3}\right)^{21pq-15p}.$$

7) $$MZ_1^a(K) = \prod_{uv\in E(K)} \left[d_K(u)+d_K(v)\right]^a = \prod_{uv\in E_1} 5^a \times \prod_{uv\in E_2} 6^a = 5^{12a} \times 6^{(21pq-15p)a}.$$

8) $$MZ_2^a(K) = \prod_{uv\in E(K)} \left[d_K(u)d_K(v)\right]^a = \prod_{uv\in E_1} (2\times 3)^a \times \prod_{uv\in E_2} (3\times 3)^a = 2^{12pa} \times 3^{(42pq-18p)a}$$

9) $$ABCII(K) = \prod_{uv\in E(K)} \sqrt{\frac{d_K(u)+d_K(v)-2}{d_K(u)d_K(v)}} = \prod_{uv\in E_1} \sqrt{\frac{3+2-2}{3\times 2}} \times \prod_{uv\in E_2} \sqrt{\frac{3+3-2}{3\times 3}}$$

$$= \left(\frac{1}{2}\right)^{6p} \times \left(\frac{2}{3}\right)^{21pq-15p}.$$

10) $$GAII(K)=\prod_{uv\in E(K)}\frac{2\sqrt{d_K(u)d_K(v)}}{d_K(u)+d_K(v)}=\prod_{uv\in E_1}\frac{2\sqrt{3\times 2}}{3+2}\times\prod_{uv\in E_2}\frac{2\sqrt{3\times 3}}{3+3}$$

$$=\left(\frac{2\sqrt{6}}{5}\right)^{12p}\times(1)^{21pq-15p}=\left(\frac{2\sqrt{6}}{5}\right)^{12p}.$$

4. Results for V-Anthracene nanotori

Theorem 4: Let L be a lattice of V-anthracene nanotori. Then

1) $II_1^*(L)=6^{21pq}$.
2) $II_2(L)=9^{21pq}$.
3) $HII_1(L)=6^{42pq}$.
4) $HII_2(L)=9^{42pq}$.
5) $XII(L)=\left(\frac{1}{\sqrt{6}}\right)^{21pq}$.
6) $\chi II(L)=\left(\frac{1}{3}\right)^{21pq}$.
7) $MZ_1^a(L)=6^{21pqa}$.
8) $MZ_2^aII(L)=9^{21pqa}$.
9) $ABCII(L)=\left(\frac{2}{3}\right)^{21pq}$.
10) $GAII(L)=1$.

Proof : Let L be a lattice of V-Anthracene nanotori, see Figure 4

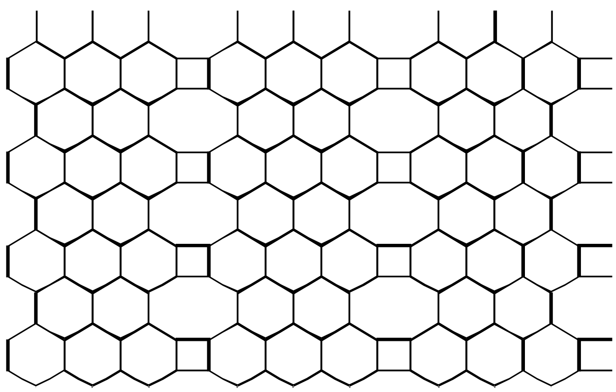

Figure 4

By algebraic method, we get $|V(L)|= 14pq$ vertices and $|E(L)|= 21pq$ edges. We have

$$E_1=\{uv\in E(L):d_L(u)=d_L(v)=3\},|E_1|=21pq.$$

Now

1) $$II_1^*(L)=\prod_{uv\in E(L)}[d_L(u)+d_L(v)]=\prod_{uv\in E_1}(3+3)=6^{21pq}.$$

2) $$II_2(L)=\prod_{uv\in E(L)}[d_L(u)d_L(v)]=\prod_{uv\in E_1}(3\times 3)=9^{21pq}.$$

3) $$HII_1(L)=\prod_{uv\in E(L)}[d_L(u)+d_L(v)]^2=\prod_{uv\in E_1}(3+3)^2=6^{42pq}.$$

4) $$HII_2(L)=\prod_{uv\in E(L)}\left[d_L(u)d_L(v)\right]^2=\prod_{uv\in E_1}(3\times 3)^2=9^{42pq}.$$

5) $$XII(L)=\prod_{uv\in E(L)}\frac{1}{\sqrt{d_L(u)+d_L(v)}}=\prod_{uv\in E_1}\frac{1}{\sqrt{3+3}}=\left(\frac{1}{\sqrt{6}}\right)^{21pq}.$$

6) $$\chi II(L)=\prod_{uv\in E(L)}\frac{1}{\sqrt{d_L(u)d_L(v)}}=\prod_{uv\in E_1}\frac{1}{\sqrt{3\times 3}}=\left(\frac{1}{3}\right)^{21pq}.$$

7) $$MZ_1^a(L)=\prod_{uv\in E(L)}\left[d_L(u)+d_L(v)\right]^a=\prod_{uv\in E_1}(3+3)^a=6^{21pqa}.$$

8) $$MZ_2^a(L)=\prod_{uv\in E(L)}\left[d_L(u)\times d_L(v)\right]^a=\prod_{uv\in E_1}(3\times 3)^a=9^{21pqa}.$$

9) $$ABCII(L)=\prod_{uv\in E(L)}\sqrt{\frac{d_L(u)+d_L(v)-2}{d_L(u)d_L(v)}}=\prod_{uv\in E_1}\sqrt{\frac{3+3-2}{3\times 3}}=\left(\frac{2}{3}\right)^{21pq}.$$

10) $$GAII(L)=\prod_{uv\in E(L)}\frac{2\sqrt{d_L(u)d_L(v)}}{d_L(u)+d_L(v)}=\prod_{uv\in E_1}\frac{2\sqrt{3\times 3}}{3+3}=(1)^{21pq}=1.$$

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