JOURNAL OF ULTRA SCIENTIST OF PHYSICAL SCIENCES
An International Open Free Access Peer Reviewed Research Journal of Mathematics
website:- www.ultrascientist.org

Estd. 1989

Effects of Ohmic dissipation and chemical reaction on MHD free convection flow through porous medium with thermal radiation

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Abstract

The heat and mass transfer impacts on a steady 2–dimensional magnetohydrodynamic (MHD) natural convection boundary layer flow of viscous fluid surrounded by a porous vertical surface with heat source, Ohmic dissipation, chemical reaction and radiation effects are studied. The governing partial differential equations (PDE) are converted into a set of ordinary differential equations (ODE) employing non-dimensional quantities then we solved the ODE employing perturbation method. Most of the studies so far have presented the numerical and semi analytical solutions of flow and heat transfer because of porous vertical surface. Present analysis on analytical solution for the flow velocity and the temperature in the form of a series solution. It was found that increasing the permeability parameter and radiation parameter, the corresponding value of velocity increases and a reverse trend has seen in magnetic parameter.

Key words: Boundary layer; Chemical reaction; MHD; Heat source; Porous medium.
MSC Code: 34K10, 34K28, 76W05, 76S05, 74F05

Nomenclature

B_0	magnetic flux density	C^*	fluid Concentration
C	non-dimensional concentration parameter	C_p	specific molecular diffusivity

C_w stretching sheet concentration C_∞ species concentration at infinity

D coefficient of the mass diffusivity E Eckert number

g acceleration due to gravity Gm Grashof number for mass transfer

Gr Grashof number for heat transfer K material parameter

K_c chemical reaction parameter M magnetic parameter

Nu Nusselt number Pr Prandtl number

Sc Schmidt number Sh Sherwood number

T fluid temperature T_w stretching sheet temperature

T_∞ fluid temperature at infinity x, y coordinate

u, v velocity components along x- and y-direction

Greek symbols

α thermal diffusivity β_T thermal expansion coefficient

β_C concentration expansion coefficient σ Electrical conductivity

λ mixed convection parameter ν kinematic viscosity

θ dimensionless temperature μ dynamic viscosity

κ thermal conductivity ρ density of the fluid

1. Introduction

Convection flow in porous media has yielded remarkable observations in the recent years because of their significance in the field of thermal shelters, geothermal systems, oil withdrawal, solid matrix heat exchangers and reserve of nuclear waste substances. These emerging fields are applied to ground water hydrology, lower level coal gasification, energy-efficient drying processes, wall-cooled catalytic reactors and earth's crust. In general, the resultant rate of heat transfer takes places because of both temperature differences and concentration differences in the flow phenomena. In various transport operations that survive in the industries in which simultaneous occurrence of heat and mass transfer is considered to be an outcome of consolidated buoyancy impact of chemical categories such as thermal diffusion and diffusion thermo. Such a type of situation often exists in chemical processing industries like polymer production and food processing[1]. Natural convective flows in different geometries are noticed in a number of industrial uses such as geothermal systems, fiber and granular insulation.

These days MHD has attracted the attention of researchers because of its demands in applications in geophysics and engineering sciences. According to Raju *et al.*[2], MHD free convection, dissipative boundary layer flow over vertical surface in the presence of chemical reaction, constant suction and thermal radiation. More researchers, Raju and varma[3], Magyari *et al.*[4], Ravi kumar *et al.*[5,6], Chamkha[7], Hayat *et al.*[8], Makinde and Mhone[9] etc. are the few to make a mention who contributed in this area.

When large temperatures acquired in technological tools like gas are ionized and so becomes field a very good electrical conductor. The ionized gas or plasma collaborates across the magnetic and alters heat transfer and friction. On account of this a few fluids also release and occupy thermal radiation. Hence it is necessary to review the result of magnetic field on the temperature allocation and heat transfer whereas the fluid

is both a conductor of electricity also useful in releasing and overwhelming thermal radiation. The goal of heat transfer by thermal radiation is becoming very popular, whereas we are concerned about higher operating temperatures and space applications.

The impact of heat source on magnetohydrodynamic flow in a micropolar fluid has been demonstrated by Mishra et al.[10]. A chemical reaction can be registered either a homogeneous or heterogeneous task, depending on how the reaction comes: on an incorporate chemical reaction or as a single-phase volume reaction. A reaction is a fully developed order one in case its rate is in direct proportion to the concentration. Tripathy et al.[11] have analyzed the impact of chemical reaction on magnetohydrodynamic natural convective sheet over a moving vertical plate with permeable medium. The impact of chemical reaction and radiation on assorted convection flow in the heat and mass transfer due to vertical plate muddy porous medium has been studied by Srinivasacharya and Reddy [12].

Rashidi et al.[13] has investigate the natural convective heat and mass transfer for MHD fluid flow with buoyancy and radiation effects past a permeable vertical stretching sheet. Dessie and Kishan[14] have studied the MHD effects on heat transfer past stretching sheet immersed in a porous medium with variable viscosity, heat source and viscous dissipation.

The impact on thermal radiative natural convection flow of a gas over a semi-infinite plate was studied by Soundalgekar and Takhar[15]. The radiation impact on the magnetohydrodynamic natural convection flow over a semi-infinite vertical plate has been analyzed by Takhar et al.[16].

Subsequently, the impact of radiation on natural convection through a porous vertical plate has been studied by Hossain et al.[17]. Heat and mass transfer impact on movable vertical plate with thermal radiation has been studied by Muthucumarswamy and Kumar[18]. They found that the velocity lessen with enhancing value of radiation parameter. This indicates that the wall shear stress enhances with reducing radiation parameter. It has been noticed that the skin-friction increases with increasing values of the Schmidt number. Interestingly the skin-friction decreases with increasing values of the time. The magnetohydrodynamic flow over an unforcedly commenced infinite vertical plate with heat radiation has been investigated by Mazumdar & Deka[19]. More attention to mass transfer and radiation outcomes on a natural convection flow with a permeable medium in a vertical surface has been examined by[20].

The increasing demand for chemical reactions in the hydro-metallurgical industries prerequisite the illustrate of heat and mass transfer in the presence of chemical reaction. The existence of a distant masses in a fluid origin some kind of chemical reaction. It can be dispensed either by itself or as concoction along a fluid. In various chemical engineering applications, a chemical reaction arises linking a distant mass and the fluid in which the plate is movable. These operations considered in many industrial requisitions, like, food processing, production of ceramics or glassware and polymer production. Following that, the several solution procedures on heat and mass transfer due to chemical reaction parameter in a flow effected with ambition fluid have been studied by Takhar et al. [21], Muthucumarswamy & Ganesan[22] Chamkha[23], Kandasamy et al.[24], Kumar[25], Abdulla[26], Singh et al.[27], Raju et al.[28] and Ravi Kumar et al.[29]. Recently, Manivannan et al.[30] have studied radiation and chemical reaction impact on isothermal vertical sheet in the presence of unstable mass transmission. Consequence of radiation and chemical reaction on unsteady magnetohydrodynamic natural convection flow and mass transfer along viscous incompressible fluid over a hot vertical plate saturated in a permeable medium in the occupancy of heat source was analyzed by Sharma et al.[31]. While a chemical reaction is on, the process generates heat and the use of porous media is common to ensure insulation of the hot shape to keep its porous media are believed to be efficacious inside reducing the flow of temperature. For achieving success inside the hot insulation of a surface, the study of a flow past a permeable medium and to assess the impact on heat and

mass transfer because of chemical reaction parameter.

Hence our effort to enquire into this occurrence. Consequences of chemical reaction on natural convection flow between a permeable medium enclosed by a vertical surface have been studied by Mahapatra *et al.*[32]. Cogley *et al.*[33] have analyzed the differential estimation for radiative transfer in a non-gray gas nearby equilibrium.

The importance of studying the heat and mass transfer impacts on a steady flow of viscous fluid surrounded by a permeable vertical surface with heat source, Ohmic dissipation and radiation. Most of the studies cited above present the numerical and semi analytical solutions of flow and heat transfer because of permeable vertical surface. Our aim is to acquire an analytical solution for the flow velocity and the temperature in the form of a series solution.

This research paper is managed as follows: Section 2 contains the formulation of the problem, and Section 3 describes analysis of the problem. In Section 4 graphical results have been sketched for various physical parameters whereas Section 5 includes the conclusion of the present analysis.

2. Formulation of the problem :

This section dealt with a steady flow of viscous, electrically conducting and radiating fluid with heat source while it occupies a semi-infinite zone of the space surrounded in a vertical porous plate. The x^*-axis is carried across the surface in an upward direction, the y^*-axis being normal to it. A uniform magnetic field B_0 being taken as applied in a direction $\perp$ to the surface. The fluid flow under the above considerations along an extremely porous medium, the governing boundary layer equations for this problem gives as:

$$\frac{\partial v^*}{\partial y^*} = 0 \tag{1}$$

$$v^* \frac{\partial u^*}{\partial y^*} = \vartheta \frac{\partial^2 u^*}{\partial y^{*2}} + g\beta_T (T^* - T_\infty^*) + g\beta_C (C^* - C_\infty^*) - \frac{\sigma B_0^2 u^*}{\rho} - \frac{\vartheta}{k_p} u^* \tag{2}$$

$$v^* \frac{\partial T^*}{\partial y^*} = \frac{\kappa}{\rho C_p} \frac{\partial^2 T^*}{\partial y^{*2}} + \frac{\vartheta}{C_p} \left(\frac{\partial u^*}{\partial y^*} \right)^2 - \frac{1}{\rho C_p} \frac{\partial q_r}{\partial y^*} + \frac{Q_0^* (T^* - T_\infty^*)}{\rho Cp} + \frac{\sigma B_0^2 u^{*2}}{\rho C_p} \tag{3}$$

$$v^* \frac{\partial C^*}{\partial y^*} = D \frac{\partial^2 C^*}{\partial y^{*2}} - k_c C^* \tag{4}$$

The boundary conditions are as follows

$$\begin{aligned} at \quad & y^* = 0: \quad & u^* = 0, \quad & T^* = T_w, \quad & C^* = C_w \\ at \quad & y^* \to 0: \quad & u^* \to 0, \quad & T^* \to T_\infty, \quad & C^* \to C_\infty \end{aligned} \tag{5}$$

From Eq. (1), we have

$$v^* = -v_0 = \text{ constant.} \tag{6}$$

In an attempt, Cogley *et al.*[33] proved that the optically thick boundary for a non-gray gas nearby symmetry as presented below:

$$\frac{\partial q_r}{\partial y^*} = 4\left(T^* - T_\infty^*\right) \int_0^\infty K_{\lambda w} \left(\frac{de_{b\lambda}}{dT^*} \right) d\lambda = 4I\left(T^* - T_\infty^*\right) \tag{7}$$

Now we consider the non-dimensional parameters are:

$$u = \frac{u^*}{v_0}, y = \frac{v_0 y^*}{\vartheta}, \; M = \frac{\sigma B_0^2 \vartheta}{\rho v_0^2}, \; \beta = \frac{Q_0^* \vartheta^2}{\kappa v_0^2}, F = \frac{4I\vartheta}{\kappa v_0^2},$$

$$pr = \frac{\mu C_p}{\kappa}, Sc = \frac{\vartheta}{D}, k_0 = \frac{\vartheta k_c}{v_0^2}, k = \frac{v_0^2 k_p}{\vartheta^2}, E = \frac{v_0^2}{C_p \left(T_w - T_\infty^*\right)}$$

$$Gr = \frac{\vartheta g \beta_T \left(T_w - T_\infty^*\right)}{v_0^3}, Gm = \frac{\vartheta g \beta_C \left(C_w - C_\infty^*\right)}{v_0^3},$$

$$\theta = \frac{\left(T^* - T_\infty^*\right)}{\left(T_w - T^*\right)}, C = \frac{\left(C^* - C_\infty^*\right)}{\left(C_w - C_\infty^*\right)}, N1 = M + \frac{1}{k} \tag{8}$$

The non-dimensional structure of the eqs. (2)- (4) reduce to

$$u'' + u' - N1u = -Gr\theta - GmC \tag{9}$$

$$\theta'' + \Pr\theta' - F\theta = -\Pr Eu'^2 - M \, prEu^2 \tag{10}$$

$$C'' + ScC' - k_0 \, ScC = 0 \tag{11}$$

The boundary conditions are:

$$y = 0: \qquad u = 0, \quad \theta = 1, \quad C = 1$$
$$y \to \infty: \qquad u \to 0, \qquad \theta \to 0, \quad C \to 0 \tag{12}$$

3. The analysis :

To solve coupled nonlinear system of eqs. (9) to (11) with the boundary conditions (12), the following regular perturbation is used. The (9) - (11) are expanded in powers of Eckert number E (<<1)

$$u = u_0 + E.u_1 + O(E^2) \tag{13}$$

$$\theta = \theta_0 + E.\theta_1 + O(E^2) \tag{14}$$

$$C = C_0 + E.C_1 + O(E^2) \tag{15}$$

Substituting Eqs. (13) - (15) in to (9) - (11) and perform the coefficients in terms of the powers of Eckert number, and not considering the terms of higher order, we have
order equations are:

$$u_0'' + u_0' + N1u_0 = -Gr\theta_0 - GmC_0 \tag{16}$$

$$\theta_0'' + \Pr \theta_0' - F\theta_0 = 0 \tag{17}$$

$$C_0'' + Sc \, C_0' - k_0 Sc \, C_0 = 0 \tag{18}$$

the first order equations are:

$$u_1'' + u_1' + N1 \, u_1 = -Gr \, \theta_1 - GmC_1 \tag{19}$$

$$\theta_1'' + \Pr \theta_1' - F \, \theta_1 = - \Pr u_0'^2 - M \, pr \, u_0^2 \tag{20}$$

$$C_1'' + Sc\ C_1' - k_0\ Sc\ C_1 = 0 \tag{21}$$

The modified boundary conditions, obtained from Eq. (12) using eqs. (13) – (15) are

$$
\begin{aligned}
&y = 0: \quad u_0 = 0, \quad u_1 = 0, \quad \theta_0 = 1, \quad \theta_1 = 0, \quad C_0 = 1, \quad\quad C_1 = 0 \\
&y \to \infty: \quad u_0 \to 0, \quad u_1 \to 0, \quad \theta_0 \to 0, \theta_1 \to 0, \quad C_0 \to 0, \quad C_1 \to 0
\end{aligned}
\tag{22}
$$

Solving (16)-(21) with using (22) and substitute the obtained results into eqs. (13) – (15), we get

$$
\begin{aligned}
u(y) &= m_9 e^{m_5 y} + m_7 e^{m_3 y} + m_8 e^{m_1 y} + E. \\
&\begin{bmatrix} C_8 e^{m_5 y} + C_1 e^{m_3 y} + C_2 e^{m_{14} y} + C_3 e^{m_{16} y} \\ + C_4 e^{m_{18} y} + C_5 e^{m_{20} y} + C_6 e^{m_{22} y} + C_7 e^{m_{24} y} \end{bmatrix}
\end{aligned}
\tag{23}
$$

$$
\theta(y) = e^{m_3 y} + E. \begin{bmatrix} a_7 e^{m_3 y} + a_1 e^{m_{14} y} + a_2 e^{m_{16} y} + a_3 e^{m_{18} y} \\ + a_4 e^{m_{20} y} + a_5 e^{m_{22} y} + a_6 e^{m_{24} y} \end{bmatrix}
\tag{24}
$$

$$C(y) = e^{m_1 y} \tag{25}$$

The rate of heat transfer is known as Nusselt number defined by

$$
\begin{aligned}
Nu &= -\left(\frac{\partial \theta}{\partial y}\right)_{y=0} = -m_3 - E. \\
&\left[\begin{bmatrix} a_7 m_3 + a_1 m_{14} + a_2 m_{16} + a_3 m_{18} \\ + a_4 m_{20} + a_5 m_{22} + a_6 m_{24} \end{bmatrix}\right]
\end{aligned}
\tag{26}
$$

The skin friction coefficient τ at the surface is defined by

$$
\begin{aligned}
\tau &= \left(\frac{\partial u}{\partial y}\right)_{y=0} = (m_5 m_9 + m_3 m_7 + m_1 m_8) + E. \\
&(m_5 C_8 + m_3 C_1 + m_{14} C_2 + m_{16} C_3 \\
&\qquad\qquad + m_{18} C_4 + m_{20} C_5 + m_{22} C_6 + m_{24} C_7)
\end{aligned}
\tag{27}
$$

and the another physical quantity, the Sherwood number Sh, defined as

$$Sh = -\left(\frac{\partial C}{\partial y}\right)_{y=0} = -m_1 \tag{28}$$

4. Numerical results :

The numerical outcomes are presented in flow fields for dissimilar values of diverse physical parameters drawn in figures. The fluids chosen as Prandlt number (Pr) (i.e. Pr=0.7 for air and Pr=7 for water at 20^0C). The velocity, temperature and concentration profiles, the skin friction, Nusselt number and Sherwood number are acquired at $Sc = 0.22$ (Hydrogen) for various values of the physical parameters.

Fig.1 depicts the impact of chemical reaction parameter, the corresponding value of velocity increases. Fig. 2 shows that on increasing the permeability of porous medium, the corresponding value of velocity increases. From Fig. 3 it has been noticed that increasing the value of radiation parameter, the corresponding value of velocity reduces. From Fig. 4 it has been noticed that the impact of magnetic parameter M on the velocity, showed that velocity decreases, in both the cases of air and water, as the value of M is increased. From Fig. 5, we can see that on enhancing the value of Grashof number, the corresponding value of skin friction increases. Fig.6 exhibits that temperature reduces along with the enhance in radiation parameter F . In Fig.7 the impact of magnetic parameter M in the case of water and air is observed and it is found that temperature reduced with an enhanced in magnetic parameter M. A change in the transfer rate of heat, demonstrated in terms of the Nusselt number is presented in Fig.8. It has been observed that Nusselt number Nu enhances with an enhance in chemical reaction parameter. From Fig. 9, it has been noticed that on increasing the value of radiation parameter the corresponding value of Nusselt number increases. From Fig. 10 it is found that on increasing the value of chemical reaction parameter, the corresponding value of concentration lessens. From Fig.11, we see that on increasing the values of Schmidt number Sc the corresponding value of Sherwood number Sh increases.
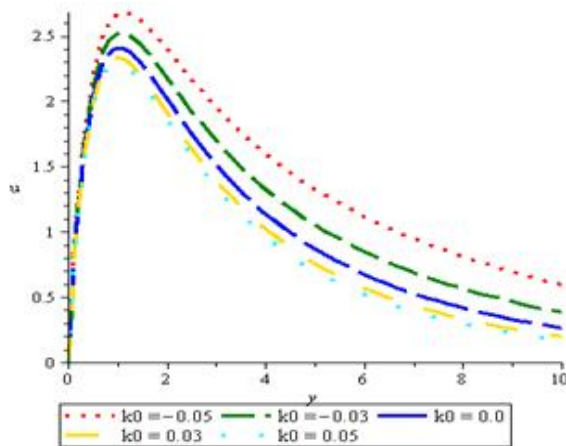

Fig. 1 Impact of k_0 on the velocity profiles

profiles for $pr = 0.71, k = 1, Gr = 10,$

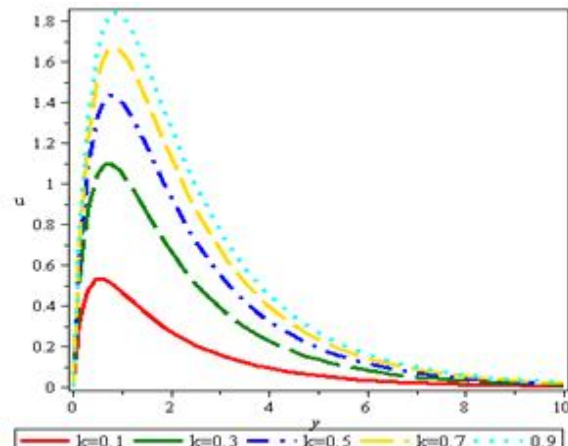$Gm = 10, M = 0.5, F = 0.06 \; and \; Sc = 0.22.$

Fig. 2 Effect of k on the velocity

for $pr = 0.71, k_0 = 0.5, Gr = 10,$

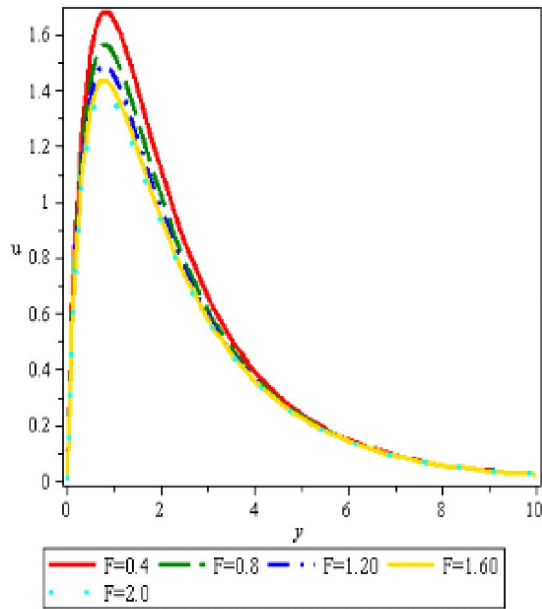$E = 0.01, Gm = 10, M = 0.5, F = 0.05 \; and \; Sc = 0.22.$

Fig. 3 Impact of F on the velocity profiles

for $Pr = 0.71, k_0 = 0.5, Gr = 10,$

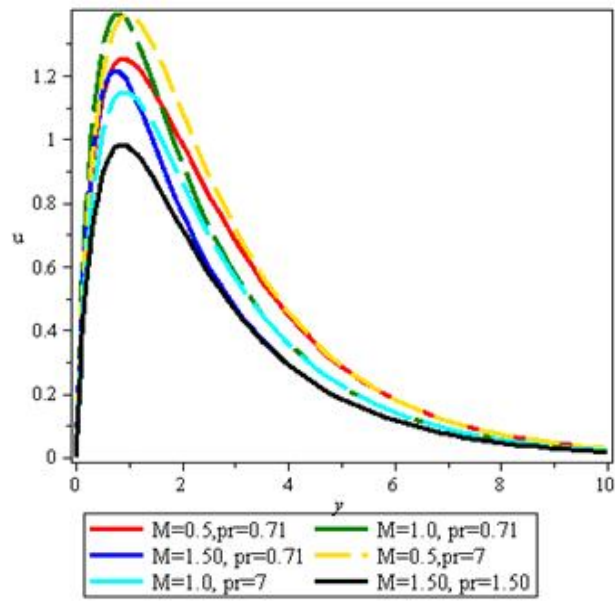$E = 0.01, Gm = 10, M = 1.5, k = 0.5 \ and \ Sc = 0.22.$

Fig. 4 Impact of M on the velocity profiles for

$pr = 0.71, k_0 = 0.5, Gr = 5,$

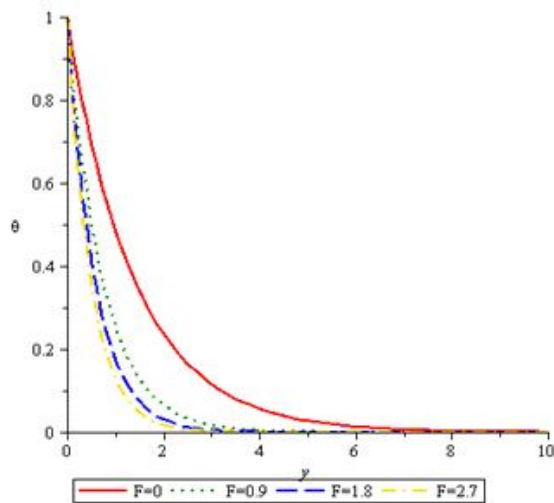$E = 0.01, Gm = 5, k = 1 \ and \ Sc = 0.22.$

Fig.5 Impact of Gr on the skin friction profiles

for $pr = 7, k_0 = 0.5,$

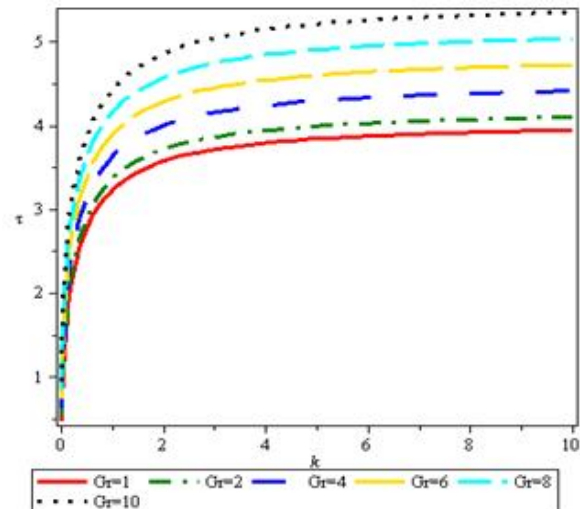$E = 0.01, Gm = 5, k = 1, F = 2.0, \ and \ Sc = 0.22.$

Fig. 6 Impact of F on the Temperature profiles

for $pr = 0.71, k_0 = 0.5, Gr = 5,$

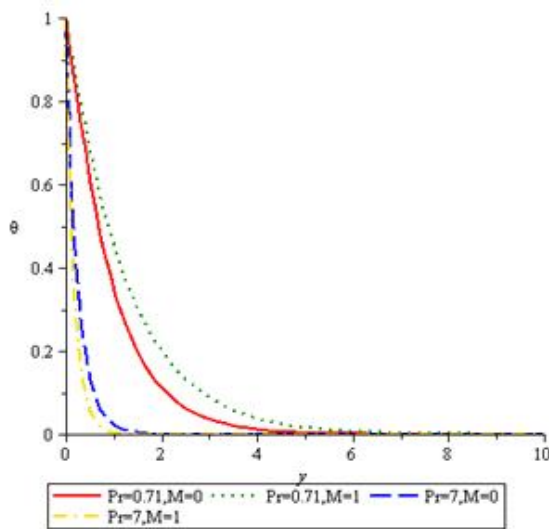$E = 0.01, Gm = 5, M = 0.5, k = 1.5 \ and \ Sc = 0.22.$

Fig. 7 Impacts of pr and M on the temperature

profiles for $F = 0.6, k_0 = 0.8, Gr = 5,$
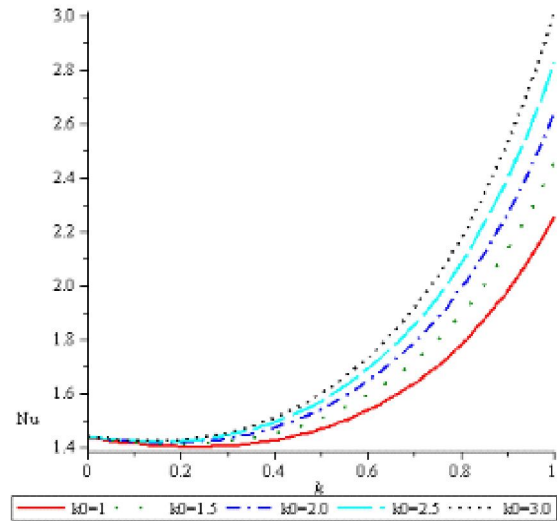
$E = 0.05, Gm = 5, k = 1.2 \ and \ Sc = 0.22.$

Fig. 8 Effects of k0 on Nusselt number

for $pr = 0.71$, $F = 1, k_0 = 1.5, Gr = 5,$

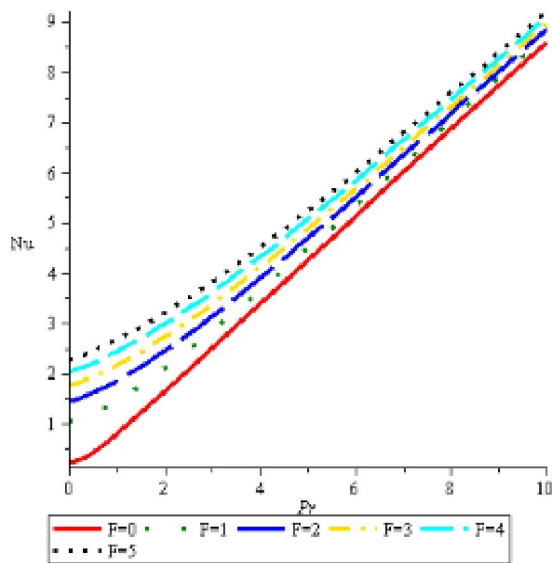$E = 0.05, Gm = 5, k = 1 \ and \ Sc = 0.22.$

Fig. 9 Impact of F on Nusselt number for

$k_0 = 1, Gr = 10,$

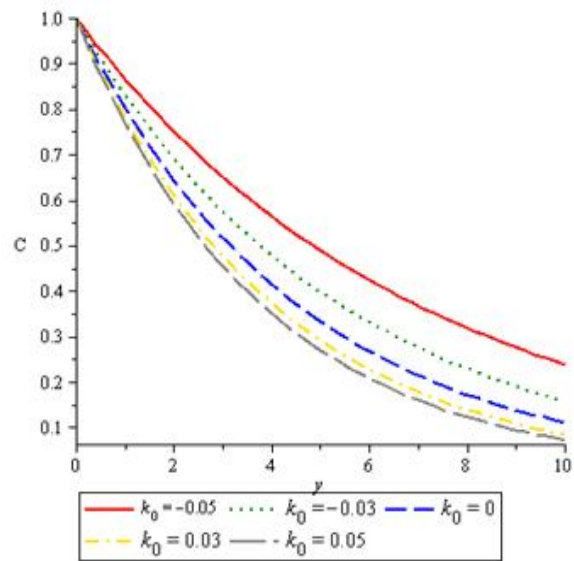$E = 0.05, Gm = 10, M = 1, k = 1 \ and \ Sc = 0.22.$

Fig. 10 Impact of k_0 on concentration
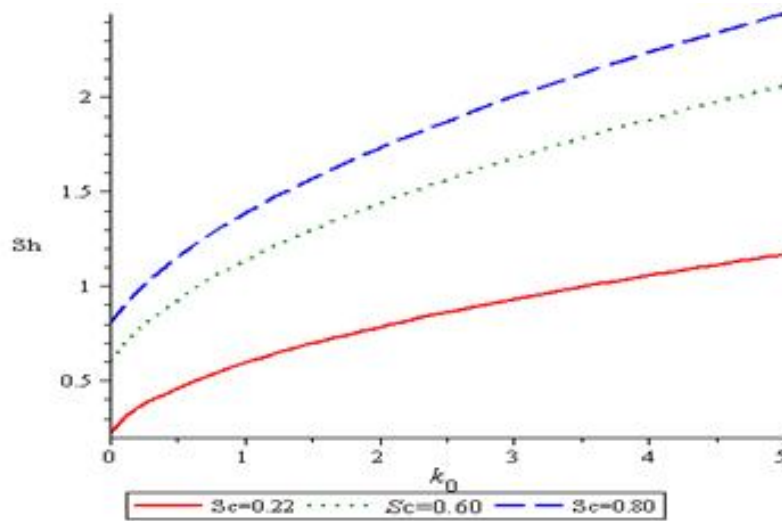
profiles for $Sc = 0.22.$

Fig. 11   Effect of Sc on Sherwood number.

5. Conclusions

In this article, we have analyzed the effects of heat source, chemical reaction and radiation, on MHD natural convection flow through a porous medium bounded by a vertical surface. The numerical computation is conducted by using Maple. The effect of various parameters on velocity, temperature and concentration, Skin friction coefficient, Nusselt number and Sherwood number for different values of various parameters are shown graphically.

The following conclusions are drawn from the present study.

- The velocity profile enhances as the permeability of porous medium and value of radiation parameter increase and reverse trend has seen in magnetic parameter.
- The temperature reduces along with the enhance in radiation parameter and magnetic parameter.
- The enhancing the value of chemical reaction parameter, the corresponding value of concentration decreases.

Acknowledgements

References

1. E. L. Cussler, Diffusion mass transfer in fluid systems. London: Cambridge University Press (1988).
2. M. C. Raju, N. Anand Reddy and S. V. K. Varma, Analytical study of MHD frees Convective, dissipative boundary layer flow past a porous vertical surface in the presence of thermal radiation, chemical reaction and constant suction. *Ain Shams Engineering Journal 5(4),* 1361-1369 (2014).
3. M. C. Raju and S. V. K. Varma, Unsteady MHD free convection oscillatory Couette flow through a porous medium with periodic wall temperature. *J Future Eng Technol 6(4),* 7–12 (2011).
4. E. Magyari, I. Pop and B. Keller, Analytical solutions for unsteady free convection flow through a porous media. *Journal of Engineering Mathematics 48,* 93–104 (2004).

5. V. Ravi Kumar, M.C. Raju and G.S.S. Raju, Combined effects of heat absorption and MHD on convective Rivlin-Erickson flow past a semi-infinite vertical porous plate with variable temperature and suction. *Ain Shams Engineering Journal 5,* 867-875 (2014).

6. V. Ravi Kumar, M.C. Raju and GSS. Raju, Heat and mass transfer effects on MHD flow of viscous fluid through non-homogeneous porous medium in presence of temperature dependent heat source. *Int. J Contemp. Math Sci. 7(32),* 1597–1604 (2012).

7. A. J. Chamkha, Hydro magnetic combined heat and mass transfer by natural convection from a permeable surface embedded in a fluid saturated porous medium. *Int. J.Numer. Methods for Heat & Fluid Flow 10(5),* 455–476 (2012).

8. T. Hayat, M. Mustafa and I. Pop, Heat and mass transfer for Soret and Dufour's effect on mixed convection boundary layer flow over a stretching vertical surface in a porous medium filled with a visco-elastic fluid. Commun. *Nonlinear Sci.Numer.Simul 5 (5),* 183–196 (2010).

9. D. O. Makindeand Y. P. Mhone, Heat transfer to MHD oscillatory flow in a channel filled with porous medium. Roman. *J. Phys. 50,* 931–938 (2005).

10. S. R. Mishra, P. K. Pattnaik and G. C. Das, Effect of heat source and double stratification on MHD free convection in a micropolar fluid. Alexandria Engineering Journal *54,* 681-689 (2015).

11. R. S. Tripathy, G. C. Das, S. R. Mishra and S. Baag, Chemical reaction effect on MHD free convectivesurface over a moving vertical plate through porous medium. *Alexandria Engineering Journal 54,* 673-679 (2015).

12. D. Srinivasacharya and G. Swamy Reddy, Chemical reaction and radiation effects on mixed convection heat and mass transfer over a vertical plate in power-law fluid saturated porous medium. *Journal of the Egyptian Mathematical Society 24,* 108-115 (2016).

13. M. M. Rashidi, B. Rostami, N. Freidoonimehr and S.Abbasbandy, Free convective heat and mass transfer for MHD fluid flow over a permeable vertical stretching sheet in the presence of the radiation and buoyancy effects. *Ain Shams Engineering Journal 5,* 901-912 (2014).

14. H. Dessie and N. Kishan, MHD effects on heat transfer over stretching sheet embedded in porous medium with variable viscosity, viscous dissipation and heat source/sink. Ain Shams Engineering Journal *5,* 967-977 (2014).

15. V. M. Soundalgekar and H.S. Takhar, Radiative convective flow past a semi-infinite vertical plate. Model Meas Control *51,* 31–40 (1992).

16. H.S. Takhar, S.R. Gorla and V.M. Soundalgekar, Radiation effects on MHD free convection flow of a radiating gas past a semi-infinite vertical plate. *Int. J. Numer. Methods for Heat & Fluid Flow 6,* 77–83 (1996).

17. A.M. Hossain, M.A. Alim and DAS. Rees, Effect of radiation on free convection from a porous a vertical plate International *Journal of Heat and Mass Transfer 42,* 181–191 (1999).

18. R. Muthucumarswamy and G.S. Kumar, Heat and mass transfer effects on moving vertical plate in the presence of thermal radiation. Theor. Appl. Mach. *31(1),* 35–46 (2004).

19. M.K. Mazumdar and R.K. Deka, MHD flow past an impulsively started infinite vertical plate in presence of thermal radiation. Romanian *Journal of Physics 52(5-7),* 565-573 (2007).

20. M.C. Raju, SVK Varma and N. Ananda Reddy, Radiation and mass transfer effects on a free convection flow through a porous medium bounded by a vertical surface. *J Future Eng Technol. 7(2),* 7–12 (2012).

21. H.S. Takhar, A.J. Chamkha and G. Nath, Flow and mass transfer on a stretching sheet with a magnetic field and chemically reactive species, International *Journal of Engineering Science 38(12),* 1303–1314 (2000).

22. R. Muthucumarswamy and R.P. Ganesan, Effect of chemical reaction and injection on flow characteristics in an unsteady upward motion of an isothermal plate. *Journal of Applied Mechanics and Technical Physics 42(4),* 665–71 (2001).

23. A.J. Chamkha, MHD flow of a uniformly stretched vertical permeable surface in the presence of heat generation/absorption and a chemical reaction. International Communications in Heat and Mass Transfer *30(3),* 413-422 (2003).

24. R. Kandasamy, K. Periasamy and K.K. Sivagnana Prabhu, Effects of chemical reaction, heat and mass transfer along a wedge with heat source and concentration in the presence of suction or injection. *International Journal of Heat and Mass Transfer 48(7),* 1388-1394 (2005).

25. H. Kumar, Radiative heat transfer with hydro magnetic flow and viscous dissipation over a stretching surface in the presence of variable heat flux. Thermal Science *13(2),* 163–169 (2009).

26. I.A. Abdulla, Analytic solution of heat and mass transfer over a permeable stretching plate affected by chemical reaction, internal heating, Dufour-Soret effect and Hall Effect. Thermal Science *13(2),* 183–197 (2009).

27. P. Singh, N.S.Tomer, S. Kumar and D Sinha, Effect of radiation and porosity parameter on MHD flow due to stretching sheet in porous media. Thermal Science *15(2),* 517-524 (2011).

28. M.C. Raju, SVK. Varma and RRK. Rao, Unsteady MHD free convection and chemically reactive flow past an infinite vertical porous plate. i-Manager's *Journal on Future Engineering and Technology 8(3),* 35–40 (2013).

29. V. Ravi Kumar, M.C. Raju, GSS. Raju and A.J. Chamkha, MHD double diffusive and chemically reactive flow through porous medium bounded by two vertical plates. i-Manager's Journal on Future Engineering and Technology *5(4),* 1–8 (2013).

30. K. Manivannan, R. Muthucumarswamy and V. Thangaraj, Radiation and chemical reaction effects on isothermal vertical oscillating plate with variable mass diffusion. Thermal Science *13(2),* 155–162 (2009).

31. P.R. Sharma, N. Kumar and P. Sharma, Influence of chemical reaction and radiation on unsteady MHD free convection flow and mass transfer through viscous incompressible fluid past a heated vertical plate immersed in porous medium in the presence of heat source. Applied Mathematical Sciences *5(46),* 2249–2260 (2011).

32. N. Mahapatra, G.C. Dash, S. Panda and M. Acharya, Effects of chemical reaction on free convection flow through a porous medium bounded by a vertical surface. *Journal of Engineering Physics and Thermophysics 83(1),* 130-140 (2010).

33. C. L. Cogley, W. G. Vincenti and E. S. Gilles, Differential Approximation for Radiative Heat Transfer in a Non Grey Gas near Equilibrium, American Institute of Aeronautics and Astronautics, *6(3),* 551-553 (1968).