The Soft Separation Axioms Semi- D_2 in Soft Topological Spaces and in Soft Bi-Topological Spaces

ARIF MEHMOOD KHATTAK[1], FAHAD JAMAL[2] and ISHFAQ AHMED[3]

[1]Riphah International University Islamabad Math department (Pakistan)
[2]Qurtuba University of Science and Information Technology Peshawar Math Department (Pakistan)
[3]Qurtuba University of Science and Information Technology Peshawar Math Department (Pakistan)
Email Address for corresponding author: [1]mehdaniyal@gmail.com

Abstract

In this article the concept of Soft semi W-D_2 structure in soft topological spaces is introduced in different ways. There are many topological structures in soft topology but soft semi W-D_2 topological structure is interesting and more practical because it is obtained from the mixture of two structures.

Key words: Soft Topology Soft set, soft semi open set, soft semi closed set. Soft semi W-D_2 space.

1. Introduction

Soft set theory is one of the young topics achieving importance in finding balanced and reasonable way out in day to day life activities problems which involves uncertainty and ambiguity**.** In 1999, Molodtsov[10] originated a new concept of soft set theory, which is absolutely a new method for modelling vagueness and uncertainty. In 1963, Kelly[4] first commenced the notion of bi topological space. He defined a bi topological space (X,τ_1,τ_2) to be a set X full with two topologies τ_1 and τ_2 on X and initiated the systematic study of bitopological space. Also he left no stone un-turn to studied separation properties of bi topological space in 2011, Shabir and Naz[11] defined soft topological spaces and studied separation. Basavaraj M. Ittanagi[3] studied soft bi topological structure and also showed the prettiness of soft separation axioms in soft bi topological spaces with respect to pair wise approach with full depth. In section II of this article, preliminary definitions

concerning soft sets, soft topological spaces and soft bi topological spaces are given. In section 3 of this article, the notion of Soft semi-D_2 space in soft bi topological spaces is familiarized in different ways and its beauty is discussed with full make up.

2. Preliminary :

Throughout this paper, $\overrightarrow{X}$ denotes the master set and $\breve{E}$ denotes the set of parameters for the master set $\overrightarrow{X}$.

Definition 1: [8] Let $\overrightarrow{X}$ be the master and $\breve{E}$ be a set of parameters. Let $P(\overrightarrow{X})$ denotes the power set of $\overrightarrow{X}$ and $\breve{A}$ be a nonempty subset of $\breve{E}$. A pair $(F, \breve{A})$ denoted by $F_{\breve{A}}$ is named a soft set over $\overrightarrow{X}$, where F is a mapping given by $F: \breve{A} \rightarrow P(\overrightarrow{X})$. In other words, a soft set over $\overrightarrow{X}$ is a parameterized family of subsets of the master X. For a specific $e \in \breve{A}$, $F(e)$, may be considered the set of e-approximate elements of the soft set $(F, \breve{A})$ and if $e \notin \breve{A}$, then $F(e) = \phi$

$i.e. F_A = \{F(e): e \in \breve{A} \subseteq \breve{E} ; F: \breve{A} \rightarrow P(\overrightarrow{X})\}$.

The family of all these soft sets over X with respect to the restriction set $\breve{E}$ is signified by $SS(X)_{\breve{E}}$.

Definition 2: [8] Let $F_{\breve{A}}, G_{\breve{B}} \in SS(X)_{\breve{E}}$. Then $F_{\breve{A}}$ is soft sub set of $G_{\breve{B}}$, denoted by $F_{\breve{A}} \widetilde{\subseteq} G_{\breve{B}}$, if

(1) $\breve{A} \widetilde{\subseteq} \breve{B}.$, and

(2) $F(e) \widetilde{\subseteq} G(e), \forall e \in \breve{A}$.

In this case, $F_{\breve{A}}$ is supposed to be a soft subset of $G_{\breve{B}}$ and $G_{\breve{B}}$ is said to be a soft super set of $F_{\breve{A}}$, $G_{\breve{B}} \widetilde{\supseteq} F_{\breve{A}}$

Definition 3: [8] Two soft subsets F_A and G_B over a common universe X are said to be soft equal if F_A is a soft subset of G_B and G_B is a soft subset of F_A.

Definition 4: [8] The complement of a soft set (F, A) denoted by $(F, A)'$ is defined by $(F, A)' = (F', A)$, $F': A \rightarrow P(X)$ is a mapping given by $F'(e) = X - F(e); \forall e \in A$ and F' is called the soft complement function of F. Clearly (F') is the same as F and $((F, A)')' = (F, A)$.

Definition 5: [8] A soft set (F, A) over X is said to be a Null soft set denoted by $\widetilde{\phi}$ or ϕ_A if for all $e \in A, F(e) = \phi$ *(vacuious set)*.

Definition 6: [8] A soft set (F, A) over X is said to be an absolute soft set denoted by $\tilde{A}$ or X_A if for all $e \in A, F(e) = X$. obviously we have $X'_A = \phi_A$ *and* $\phi'_A = X_A$.

Definition 7: [8] The union of two soft sets (F, A) and (G, B) over the common universe X is the soft set (H, C), where $C = A \cup B$ & $\forall e \in C$,

$$H(e) = \begin{cases} F(e), e \in A - B \\ G(e), e \in B - A \\ F(e) \,\breve{\cap}\, G(e), e \in A \,\breve{\cap}\, B \end{cases}$$

Definition 8: [8] The intersection of two soft sets (F, A) and (G, B) over the common universe X is the soft set (H, C), where $C = A \,\breve{\cap}\, B$ and for all $e \in C, H(e) = F(e) \,\breve{\cap}\, G(e)$.

Definition 9: [8] Let $\tilde{\tau}$ be the collection of soft sets over $\overrightarrow{X}$, then $\tilde{\tau}$ is said to be a soft topology on $\overrightarrow{X}$, if

(1) $\phi, \overrightarrow{X} \in \tilde{\tau}$

(2) Union of any number of soft sets in $\tilde{\tau}$ belongs to $\tilde{\tau}$

(3) Intersection of any two soft sets in $\tilde{\tau}$ belongs to $\tilde{\tau}$

Definition 10: [14]. A soft set (F, A) in a soft topological space (X, τ, E) will be termed soft semi open (written S.S.O) if and only if there exists a soft open set (O,E) such that $(O, E) \sqsubseteq (A, E) \subseteq Cl(O, E)$. The set of all semi *open soft* set is denoted by $SSO(X, \tau, E)$ or $SSO(X)$ and the set of all semi *closed soft* set is denoted by $SCS(X, \tau, E)$ or $SCS(X)$

Definition 11: [6]. Let $(\overrightarrow{X}, \tilde{\tau}, E)$ be a soft topological space, $(F, E) \in SS(X)_E$ and $\tilde{Y}$ be a non-vacuous subset of $\overrightarrow{X}$. Then the soft subset of (F, E) over $\tilde{Y}$ signified by $(F_{\tilde{Y}}, E)$, is defined as follows:
$F_Y(e) = \tilde{Y} \,\breve{\cap}\, F(e), \forall e \in E$
In other words, $(F_{\tilde{Y}}, E) = \tilde{Y}_E \,\breve{\cap}\, (F, E)$.

Definition 12: [1]. Let $(X, \tilde{\tau}, E)$ be a soft topological space and $\tilde{Y}$ be a non-vacuous subset of X. Then
$\tilde{\tau}_{\tilde{Y}} = \{(F_{\tilde{Y}}, E) : (F, E) \in \tilde{\tau}\}$
is said to be the relative soft topology on $\tilde{Y}$ and $(Y, \tilde{\tau}_Y, E)$ is called a soft subspace of $(\overrightarrow{X}, \tilde{\tau}, E)$.

Definition 13: [6]. Let $F_A \in SS(X)_E$ & $G_B \in SS(Y)_K$. The Cartesian product $F_A \circledast G_B$ is defined by $(F_A \circledast G_B)(e, k) = F_A(e) \times G_B(k), \forall (e, k) \in A \times B$. According to this definition $F_A \circledast G_B$ is a soft set over $X \times Y$ and its parameter set is $E \times K$.

Definition 14: [2]. Let $(\overrightarrow{X}, \tilde{\tau}_X, E)$ and $(\tilde{Y}, \tilde{\tau}_{\tilde{Y}}, K)$ be two soft topological spaces. The soft product topology $\tilde{\tau}_X \circledast \tau_Y$ over $X \times Y$ with respect to $E \times K$ is the soft topology having the collection $\{F_E \circledast G_K / F_E \in \tilde{\tau}_{\overrightarrow{X}}, G_K \in \tilde{\tau}_{\tilde{Y}}\}$ as the basis.

Definition 15: [3]. Let $\left(\overrightarrow{X}, \tilde{\tau}_{1\overrightarrow{X}}, E\right)$ and $\left(\overrightarrow{X}, \tilde{\tau}_{2\overrightarrow{X}}, E\right)$ be two not the same soft topological spaces on $\overrightarrow{X}$. Then $(X, \quad \tilde{\tau}_{1\overrightarrow{X}}, \tilde{\tau}_{2\overrightarrow{X}}, E)$ is called a Soft bi topological space if the two soft topologies $\tilde{\tau}_{1\overrightarrow{X}}$ and $\tilde{\tau}_{2\overrightarrow{X}}$ individually gratify the axioms of soft topology. The participants of $\tilde{\tau}_{1\overrightarrow{X}}$ are called $\tilde{\tau}_{1\overrightarrow{X}}$ soft open sets and the complements of $\tilde{\tau}_{1\overrightarrow{X}}$ soft open sets are named $\tilde{\tau}_{1\overrightarrow{X}}$ soft closed sets. Similarly, The participants of $\tilde{\tau}_{2\overrightarrow{X}}$ are called $\tilde{\tau}_{2\overrightarrow{X}}$ soft open sets and the complements of $\tilde{\tau}_{2\overrightarrow{X}}$ soft open sets are named $\tilde{\tau}_{2\overrightarrow{X}}$ soft closed sets.

Definition 16: [3]. Let $(X, \tilde{\tau}_{1\widehat{X}}, \tilde{\tau}_{2\widehat{X}}, E)$ be a soft bi topological space over X and Y be a non-empty subset of X. Then $\tau_{1\widehat{Y}} = \{(F_Y, E) : (F, E) \in \tilde{\tau}_{1\widehat{X}}\}$ and $\tilde{\tau}_{2\widehat{Y}} = \{(G_{\widehat{Y}}, E) : (G, E) \in \tilde{\tau}_{2\widehat{X}}\}$ are said to be the relative topologies on Y and $\{Y, \tilde{\tau}_{1\widehat{Y}}, \tilde{\tau}_{2\widehat{Y}}, E\}$ is named as soft sub space of $(X, \tilde{\tau}_{1\widehat{X}}, \tilde{\tau}_{2\widehat{X}}, E)$.

3. Main Results

3.1 In this portion some new results are discussed in Soft Topological Spaces :

In this portion we discussed soft topological structures with the application of soft semi difference set with respect to ordinary points.

Definition 17: Let $(\overset{\leftrightarrow}{X}, \tilde{\tau}, E)$ be a soft topological space and a soft set (S, E) of soft topological space is called soft semi-difference set (in short hand sD-soft set) if there exists two soft semi open sets $(O_1, E), (O_2, E)$ such that $(O_1, E) \neq (\overset{\leftrightarrow}{X}, \tilde{\tau}, E)$ and set $(S, E) = (O_1, E) \backslash (O_2, E)$.

Definition 18 : A soft topological space $(\overset{\leftrightarrow}{X}, \tilde{\tau}, E)$ is said to be Soft semi W-D_2 space of type 1 signified by $semi(DW - H)_1$ if for every $e_1, e_2 \in E, e_1 \neq e_2$ there exists soft sD-soft open sets $(S_1, \breve{A})$, $(S_2, \breve{B})$ such that $S_{1\breve{A}}(e_1) = \overset{\leftrightarrow}{X}, S_{2\breve{B}}(e_2) = \overset{\leftrightarrow}{X}$ and $(S_1, \breve{A}) \cap (S_2, \breve{B}) = \tilde{\phi}$.

Definition 19: Let $(\overset{\leftrightarrow}{X}, \tilde{\tau}, E)$ be a soft topological space and $H \subseteq E$. Then $(X, \tilde{\tau}_H, H)$ is called soft p-subspace of $(X, \tilde{\tau}, E)$ relative to the parameter set H where $\tilde{\tau}_H = \{(S_{\breve{A}}) / H : H \subseteq \breve{A} \subseteq E, sD$ $soft\ open\ set\ S_{\breve{A}} \in \tilde{\tau}\}$ and $(S_{\breve{A}}) / H$ is the restriction map on H.

Theorem19
Proof
(1) Soft subspace of a semi $(DW - H)_1$ space is soft semi $(DW - H)_1$.
(2) Soft p-subspace of a semi $(DW - H)_1$ space is soft semi $(DW - H)_1$.
(3) Product of two soft semi $(DW - H)_1$ space is soft semi $(DW - H)_1$.

(1) Proof

Let $(\overset{\leftrightarrow}{X}, \tilde{\tau}_1, \tilde{\tau}_2)$ be soft semi $(DW - H)_1$ space. Let Y be a non-vacuous sub set of $\overset{\leftrightarrow}{X}$. Let $(\overset{\leftrightarrow}{Y}, \tilde{\tau}_Y, E)$ be a soft sub space of $(\overset{\leftrightarrow}{X}, \tau, E)$ where $\tilde{\tau}_Y = \{(\tilde{F}_Y, E) : (S, E) \in \tilde{\tau}\}$ is the relative soft topology on $\overset{\leftrightarrow}{Y}$. Consider $e_1, e_2 \in E, e_1 \neq e_2$ there exists sD soft open sets $(S_1, A), (S_2, B)$ such that $S_{1\breve{A}}(e_1) = X, S_{2\breve{B}}(e_2) = X$ and $(S_1, \breve{A}) \cap (S_2, \breve{B}) = \tilde{\phi}$. Therefore $((S_{1\breve{A}})_Y, E), ((S_{2\breve{B}})_Y, E) \in \tilde{\tau}_Y$. Also $(S_{1\breve{A}})_Y(e_1) = Y \cap S_{1\breve{A}}(e_1) = Y \cap X = Y$. $(S_{1B})_Y(e_2) = Y \cap S_{2B}(e_2) = Y \cap X = Y$. $((S_{1A})Y \cap S_{2B})\overset{\leftrightarrow}{Y})(e) = ((S_{1A} \cap S_{2B})\overset{\leftrightarrow}{Y})(e)$
$= \overset{\leftrightarrow}{Y} \cap (S_{1A} \cap S_{2B})(e)$
$= \overset{\leftrightarrow}{Y} \cap \tilde{\phi}(e)$

$$= \tilde{Y} \cap \phi$$
$$= \phi$$

$(S_{1_A})_Y \cap (S_{2_{\breve{B}}})_Y = \phi.$ Hence $(\tilde{\tilde{Y}}, \tilde{\tau}_Y, E)$ is semi $(DW - H)_1$.

(2) Proof: Let (X, τ, E) be a semi $(DW - H)_1$ space. Let $H \subseteq E$. Let $(\tilde{\tilde{X}}, \tilde{\tau}_H, H)$ be a soft p-subspace of $(\tilde{\tilde{X}}, \tau, E)$ relative to the parameter set H. $\tilde{\tau}_H = \{(S_{\tilde{A}})/H : H \subseteq A \subseteq E, S_A \in \tau$. Suppose $\tilde{h}_1, \tilde{h}_2 \in H, h_1 \neq h_2$. Then $\tilde{h}_1, \tilde{h}_2 \in E$. Therefore, there happens soft sD soft open sets $(S_1, A), (S_2, B)$ such that $S_{1_{\tilde{A}}}(h_1) = X, S_{2_{\breve{B}}}(h_2) = \tilde{\tilde{X}}$ & $(S_1, \breve{A}) \cap (S_2, \breve{B}) = \tilde{\phi}$. Therefore $(S_{1_{\tilde{A}}})/H (S_{2_{\breve{B}}})/H \in \tilde{\tau}_H$. Also

$$((S_{1_{\tilde{A}}})/H)(h_1) = S_{1_{\tilde{A}}}(h_1) = \tilde{\tilde{X}}$$

$$((S_{2_B})/H)(h_2) = S_{2_B}(h_2) = \tilde{\tilde{X}} \&$$

$$((S_{1_{\tilde{A}}})/H) \cap ((S_{2_{\breve{B}}})/H) = (S_{1_{\tilde{A}}} \cap S_{2_{\breve{B}}})/H$$

$$= \tilde{\phi}/H$$

$$= \tilde{\phi}$$

Hence $(\tilde{\tilde{X}}, \tilde{\tau}_H, H)$ is semi $(DW - H)_1$.

(3) Proof: Let $(\tilde{\tilde{X}}, \tilde{\tau}_x, E)$ and $\{\tilde{Y}, \tilde{\tau}_Y, K\}$ be two $(SDW - H)_1$ spaces. Consider two distinct points $(e_1, k_1), (e_2, k_2 \in E \times K$ either $e_1 \neq e_2$ or

$k_1 \neq k_2$. Suppose $e_1 \neq e_2$. Since $(\tilde{\tilde{X}}, \tilde{\tau}_x, E)$ is $(SDW - H)_1$, there exist sD soft open sets $(S_1, A), (S_2, B)$ such that $S_{1_{\tilde{A}}}(h_1) = X, S_{2_{\breve{B}}}(h_2) = \tilde{\tilde{X}}$ & $(S_1, \breve{A}) \cap (S_2, \breve{B}) = \tilde{\phi}$.

Therefore $S_{1_A} \circledast Y_k \in \tilde{\tau}_{1X} \circledast \tilde{\tau}_{1Y}, S_{2_{\breve{B}}} \circledast Y_K \in \tilde{\tau}_X \circledast \tilde{\tau}_Y$

$$(S_{1_{\tilde{A}}} \circledast \tilde{Y}_K)(e_1, k_1) = S_{1_{\tilde{A}}}(e_1) \times \tilde{Y}_K(k_1) = \tilde{\tilde{X}} \times \tilde{Y}$$

$$(S_{2_{\breve{B}}} \circledast Y_K)(e_2, k_2) = S_{2_{\breve{B}}}(e_2) \times Y_K(k_2) = \tilde{\tilde{X}} \times \tilde{Y}$$

If for any $(e, k) \in (E \times K), (S_{1_A} \circledast \tilde{Y}_K)(e, k) \neq \phi \Rightarrow F_A(e) \times \tilde{Y}_K(k) \neq \phi$

$$\Rightarrow S_{1_{\tilde{A}}}(e) \times \tilde{Y} \neq \phi \Rightarrow S_{1_{\tilde{A}}}(e) \neq \phi \Rightarrow S_{2_{\breve{B}}}(e) = \phi$$

Since $(S_{1_A} \cap S_{2_{\breve{B}}} = \phi \Rightarrow S_{1_A}(e) \cap S_{2_{\breve{B}}}(e) = \phi)$

$$\Rightarrow S_{2_{\breve{B}}}(e) \times \tilde{Y}_K(k) = \phi \Rightarrow (S_{2_B} \otimes \tilde{Y}_K)(e, k) = \phi$$

$$\Rightarrow (S_{1_{\tilde{A}}} \circledast \tilde{Y}_K) \cap (S_{2_{\breve{B}}} \circledast \tilde{Y}_K) = \tilde{\phi}$$

Assume $k_1 \neq k_2$. Since $\{Y, \tilde{\tau}_{1Y}, \tilde{\tau}_{2Y}, K\}$ is semi $(DW - H)_1$ there exist sD soft open sets $(S_1, A), (S_2, B)$ $S_{1_{\tilde{A}}} \in \tilde{\tau}_Y, S_{2_B} \in \tilde{\tau}_Y$ such that $S_{1_{\tilde{A}}}(k_1) =, \tilde{Y} S_{2_{\breve{B}}}(k_2) = \tilde{Y}$ & $S_{1_{\tilde{A}}} \cap S_{2_{\breve{B}}} = \tilde{\phi}$.

Therefore $X_E \circledast S_{1_A} \in \tilde{\tau}_Y$, $\breve{X}_E \circledast S_{2_{\breve{B}}} \in \tilde{\tau}_Y$

$(\breve{X}_E \circledast S_{1_{\breve{A}}})(e_1, k_1) = \breve{X}_E(e_1) \times S_{1_{\breve{A}}}(k_1)$

$= \breve{X} \times \tilde{Y}$

$(X_E \circledast S_{2_{\breve{B}}})(e_2, k_2) = \breve{X}_E(e_2) \times S_{1_{\breve{A}}}(k_1)$

$= \breve{X} \times \tilde{Y}$

If for any $(e, k) \in E \times K$, $(\breve{X}_E \circledast S_{1_{\breve{A}}})(e, k) \neq \phi \Rightarrow \breve{X}_E(e) \times S_{1_{\breve{A}}}(k) \neq \phi$

$\Rightarrow \breve{X} \times S_{1_{\breve{A}}}(k) \neq \phi \Rightarrow S_{1_{\breve{A}}}(k) \neq \phi \Rightarrow S_{1_A}(k) \neq \phi$. So $S_{2_B}(k) = \phi$

$(Since S_{1_A} \cap S_{2_B} = \tilde{\phi} \Rightarrow S_{1_{\breve{A}}}(k) \cap S_{2_B}(k) = \phi) \Rightarrow X_E(e) \times S_{2_B}(k) = \phi \Rightarrow (X_E \circledast S_{2_B})(e, k) = \phi$

$\Rightarrow (\breve{X}_E \circledast S_{1_{\breve{A}}}) \cap (X_E \circledast S_{2_B}) = \tilde{\phi}$

Hence Product of two soft semi $(DW - H)_1$ space is soft semi $(DW - H)_1$.

3.2 In this portion some new results are discussed in Soft Bi-Topological Spaces :

   Definition 20 : Let $\breve{X}$ be a non-empty set and $\tilde{\tau}_1$ and $\tilde{\tau}_2$ be two different topologies on $\breve{X}$. Then $(\breve{X}, \tilde{\tau}_1, \tilde{\tau}_2)$ is called a bi topological space.

   Definition 21 : A soft bi topological space $(\breve{X}, \tilde{\tau}_{1X}, \tilde{\tau}_{2X}, E)$ is said to be Soft semi W-D_2 space or soft semi $W - T_2$ space of type 1 denoted by $(SDW - H)_1$ if it is soft semi $(SDW - H)_1$ With respect to $\tau_{1\breve{X}}$ or soft semi $(SDW - H)_1$ with respect to $\tilde{\tau}_{2\breve{X}}$.

   Definition 22 : A soft bi topological space $(\breve{X}, \tau_{1\breve{X}}, \tau_{2\breve{X}}, E)$ is supposed to be Soft semi W-D-Hausdorf space of type 2 signified by $(SDW - H)_2$ if for every $e_1, e_2 \in E, e_1 \neq e_2$ there occurs soft there exist sD soft open sets $(S_1, A), (S_2, B)$ such that $(S_1, \breve{A}) \in \tilde{\tau}_{1\breve{X}}$ soft $(S_2, \breve{A}) \in \tilde{\tau}_{2\breve{X}}$ such that $S_{1_{\breve{A}}}(e_1) = X$, $S_{2_{\breve{B}}}(e_2) = \breve{X}$ and $(S_1, \breve{A}) \cap (S_2, \breve{B}) = \tilde{\phi}$.

   Theorem 23. Soft subspace of a semi $(SDW - H)_1$ space is semi $(SDW - H)_1$.

   Proof : Let $(\breve{X}, \tau_{1\breve{X}}, \tau_{2\breve{X}}, E)$ be a semi $(SDW - H)_1$ space. Then it is semi $(SDW - H)_1$ with respect to $\tilde{\tau}_{1X}$ *or semi*$(SDW - H)_1$ with respect to $\tilde{\tau}_{2\breve{X}}$. Let $\tilde{Y}$ be a non-vacuous subset of $\breve{X}$. Let $\{\tilde{Y}, \tilde{\tau}_{1\breve{Y}}, \tilde{\tau}_{1\breve{Y}}, E\}$ be a soft subspace of $(X, \tilde{\tau}_{1X}, \tilde{\tau}_{2X}, E)$. From **Theorem19** a soft subspace of semi $(SDW - H)_1$ space is $(SDW - H)_1$. Therefore, $\{\tilde{Y}, \tilde{\tau}_{1\breve{Y}}, \tilde{\tau}_{2\breve{Y}}, E\}$ is *semi*$(SDW - H)_1$ with respect to $\tilde{\tau}_{1Y}$ or *semi*$(SDW - H)_1$ with respect to $\tilde{\tau}_{2Y}$. Hence $\{\tilde{Y}, \tilde{\tau}_{1\breve{Y}}, \tilde{\tau}_{2\breve{Y}}, E\}$ is *semi*$(SDW - H)_1$.

   Theorem 24 : Soft subspace of a *semi*$(SDW - H)_2$ space is *semi*$(SDW - H)_2$.

Proof : Let $(X, \tilde{\tau}_{1\tilde{X}}, \tilde{\tau}_{2\tilde{X}}, E)$ be a $semi(SDW - H)_2$ space. Let Y be a non-vacuous subset of $\tilde{X}$.

Let $\{\tilde{Y}, \tilde{\tau}_{1\tilde{Y}}, \tilde{\tau}_{2\tilde{Y}}, E\}$ be a soft subspace of $(X, \tilde{\tau}_{1\tilde{X}}, \tilde{\tau}_{2\tilde{X}}, E)$ where

$\tilde{\tau}_{1\tilde{Y}} = \{(S_{1\tilde{Y}}, E) : $ sD soft open $(S_1, E) \in \tilde{\tau}_{1\tilde{X}}\}$ &

$\tilde{\tau}_{2\tilde{Y}} = \{(S_{2\tilde{Y}}, E) : $ sD soft open$(S_2, E) \in \tilde{\tau}_{2\tilde{X}}\}$ are supposed to be the relative topologies on Y.

Consider $e_1, e_2 \in E$ $with$ $e_1 \neq e_2$ there occur sD soft open $(S_1, E) \in \tilde{\tau}_{1\tilde{X}}$, sD soft open $(S_2, E) \in \tilde{\tau}_{2\tilde{X}}$

such that $S_{1_A}(e_1) = X, S_{2_B}(e_2) = X$ &$(S_1, E) \,\breve{\cap}\, (S_2, E) = \tilde{\phi}$. Hence $((S_{1_{\breve{A}}})\tilde{Y}, E) \in \tilde{\tau}_{1Y}, ((S_{2_B})\tilde{Y},$

$E) \in \tilde{\tau}_{2\tilde{Y}}$

Also $(S_{1_{\breve{A}}})\tilde{Y}(e_1) = \tilde{Y} \,\breve{\cap}\, S_{1_{\breve{A}}}(e_1)$

$= \tilde{Y} \,\breve{\cap}\, X = Y$

$(S_{2_B})\tilde{Y}(e_2) = \tilde{Y} \,\breve{\cap}\, S_{2_{\breve{B}}}(e_2)$

$= \tilde{Y} \,\breve{\cap}\, X$

$= \tilde{Y}$

$((S_{1_A})Y \,\breve{\cap}\, S_{2_B})\tilde{Y})(e) = ((S_{1_A} \,\breve{\cap}\, S_{2_{\breve{B}}})\tilde{Y})(e)$

$= \tilde{Y} \,\breve{\cap}\, (S_{1_A} \,\breve{\cap}\, S_{2_{\breve{B}}})(e)$

$= \tilde{Y} \,\cap\, \tilde{\phi}(e)$

$= \tilde{Y} \,\cap\, \phi$

$= \phi$

$(S_{1_A})Y \,\cap\, (S_{2_{\breve{B}}})Y = \phi$

Hence $\{\tilde{Y}, \tilde{\tau}_{1\tilde{Y}}, \tilde{\tau}_{2\tilde{Y}}, E\}$is $semi(SDW - H)_2$.

Definition 23 : Let $(X, \tilde{\tau}_{1\tilde{X}}, \tilde{\tau}_{2\tilde{X}}, E)$ be a soft bi-topological space over $\tilde{X}$ & $H \subseteq E$. Then

$\{X, \tilde{\tau}_{1H}, \tilde{\tau}_{2H}, H\}$ is called Soft β p-subspace of $(X, \tilde{\tau}_{1\tilde{X}}, \tilde{\tau}_{2\tilde{X}}, E)$ relative to the parameter set H where

$\tilde{\tau}_{1H} = \{(S_{1_A}) / H : H \subseteq A \subseteq E, S_{1_A} \in \tilde{\tau}_{1\tilde{X}} \; such \, that \; S_{1_A} is$ sD soft open $\}$,

$\tilde{\tau}_{2H} = \{(S_{2_B}) / H : H \subseteq B \subseteq E, S_{2_{\breve{B}}} \in \tilde{\tau}_{2\tilde{X}}\}$ & $(S_{1_A}) / H$, $(S_{2_B}) / H$ are the restriction maps on H.

Theorem 26 : Soft p-subspace of a $semi(SDW - H)_1$ space is semi $semi(SDW - H)_1$.

Proof : Let $(X, \tilde{\tau}_{1\tilde{X}}, \tilde{\tau}_{2\tilde{X}}, E)$ be a $semi(SDW - H)_1$ space. Then it is $semi(SDW - H)_1$ with

respect to $\tilde{\tau}_{1\tilde{X}}$ or $semi(SDW - H)_1$ with respect to $\tilde{\tau}_2$. Let $H \subseteq E$. Let $\{X, \tilde{\tau}_{1H}, \tilde{\tau}_{2H}, H\}$ be a soft

p-subspace of $(X, \tilde{\tau}_{1\tilde{X}}, \tilde{\tau}_{2\tilde{X}}, E)$ relative to the parameter set H. **From the theorem 19,** the soft p-subspace

of $semi(SDW - H)_1$ space $semi(SDW - H)_1$. Therefore, the soft p-subspace of $semi(SDW - H)_1$ is

$semi(SDW - H)_1$ with respect to $\tilde{\tau}_{1H}$ or with respect to $\tilde{\tau}_{2H}$. Hence $(X, \tilde{\tau}_{1H}, \tilde{\tau}_{2H}, H)$ is

$semi(SDW - H)_I$.

Theorem 27 : Soft p-subspace of a $semi(SDW-H)_2$ space is $(SDW-H)_2$.

Proof : Let $(X, \tilde{\tau}_{1X}, \tilde{\tau}_{2X}, E)$ be a $semi(SDW-H)_2$ space. Let $H \subseteq E$. Let $(X, \tilde{\tau}_{1H}, \tilde{\tau}_{2H}, H)$ be a soft p-subspace of $(X, \tilde{\tau}_{1\widehat{X}}, \tilde{\tau}_{2\widehat{X}}, E)$ relative to the parameter set H where

$\tilde{\tau}_{1H} = \{(S_{1\check{A}})/H : H \subseteq \check{A} \subseteq E,$ sD soft open $S_{1\check{A}} \in \tilde{\tau}_{1\widehat{X}}\}$,

$\tilde{\tau}_{2H} = \{(\beta G_B)/H : H \subseteq \check{B} \subseteq E,$ sD soft open $S_{2\check{B}} \in \tilde{\tau}_{2\widehat{X}}\}$.

Consider $h_1, h_2 \in H$, $h_1 \neq h_2$. Then $h_1, h_2 \in E$. Therefore, there exists sD soft open $S_{1\check{A}} \in \tilde{\tau}_{1\widehat{X}}$, (there exists sD soft open $S_{2\check{A}} \in \tilde{\tau}_{2\widehat{X}}$ such that $S_{1\check{A}}(e_1) = X, S_{2\check{B}}(e_2) = \widehat{X}$ and $(S_1, \check{E}) \cap (S_2, \check{B}) = \check{\phi}$. Therefore $(S_{1\check{A}})/)/H \in \tilde{\tau}_{1H}, (S_{2\check{B}})/H \in \tilde{\tau}_{2H}$

Also $((S_{1\check{A}})/H)(h_1) = S_{1\check{A}}(h_1) = \overleftrightarrow{X}$

$((S_{2B})/H)(h_2) = S_{2B}(h_2) = \widetilde{\widetilde{X}}$ and

$((S_{1\check{A}})/H) \cap ((S_{2\check{B}})/H) = (S_{1\check{A}} \cap S_{2\check{B}})/H$

$= \check{\phi}/H$

$= \check{\phi}$

Hence $(X, \tilde{\tau}_{1H}, \tilde{\tau}_{2H}, H)$ *is* $semi(SDW-H)_2$.

Theorem 28 : Product of two semi $(SDW-H)_1$ spaces is $semi(SDW-H)_1$.

Proof : Let $(X, \tilde{\tau}_{1\widehat{X}}, \tilde{\tau}_{2\widehat{X}}, E)$ and $\{\tilde{Y}, \tilde{\tau}_{1\tilde{Y}}, \tau_{2\tilde{Y}}, K\}$ be two $semi(SDW-H)_1$ spaces. Then $(\widehat{X}, \tilde{\tau}_{1\widehat{X}}, \tilde{\tau}_{2\widehat{X}}, E)$ is $semi(SDW-H)_1$ with respect to $\tilde{\tau}_{1\widehat{X}}$ or $semi(SDW-H)_1$ with respect to $\tilde{\tau}_{2\widehat{X}}$ and $\{\tilde{Y}, \tilde{\tau}_{1\tilde{Y}}, \tilde{\tau}_{2\tilde{Y}}, K\}$ is $semi(SDW-H)_1$ with respect to $\tilde{\tau}_{1\tilde{Y}}$ or $semi(SDW-H)_1$ with respect to $\tilde{\tau}_{2\tilde{Y}}$. From theorem 18, the product of two $semi(SDW-H)_1$ spaces is $semi(SDW-H)_1$. Hence the product of two $semi(SDW-H)_1$ spaces is $semi(SDW-H)_1$.

Theorem 29 : Product of two $semi(SDW-H)_2$ spaces is $semi(SDW-H)_2$

Proof : Let $(\widetilde{\widehat{X}}, \tilde{\tau}_{1\widehat{X}}, \tilde{\tau}_{2\widehat{X}}, E)$ and $\{\tilde{Y}, \tilde{\tau}_{1\tilde{Y}}, \tau_{2\tilde{Y}}, K\}$ be two $semi(SDW-H)_2$ spaces. Consider two distinct points $(e_1, k_1), (e_2, k_2 \in E \times K$ either $e_1 \neq e_2$ or $k_1 \neq k_2$. Suppose $e_1 \neq e_2$. Since $(X, \tilde{\tau}_{1X}, \tilde{\tau}_{2X}, E)$ is $semi(SDW-H)_2$, there exist there exists sD soft open $S_{1A} \in \tilde{\tau}_{1\widehat{X}}$, there exists sD soft open $S_{2A} \in \tilde{\tau}_{2\widehat{X}}$ such that $S_{1\check{A}}(e_1) = \widehat{X}, S_{2B}(e_2) = \widehat{X}$ & $S_{1\check{A}} \cap S_{2\check{A}} = \check{\phi}$.

Therefore $S_{1A} \otimes S_{2k} \in \tilde{\tau}_{1X} \otimes \tilde{\tau}_{1Y}, S_{2\check{B}} \otimes Y_K \in \tilde{\tau}_{2\widehat{X}} \otimes \tilde{\tau}_{2Y}$

$(S_{1\breve{A}} \otimes \tilde{Y}_K)\,(e_1, k_1)\ = S_{1\breve{A}}(e_1) \times \tilde{Y}_K(k_1) = \hat{\tilde{X}} \times \tilde{Y}$

$(S_{2\breve{B}} \otimes Y_K)\,(e_2, k_2)\ = S_{2\breve{B}}(e_2) \times Y_K(k_2) = \hat{\tilde{X}} \times \tilde{Y}$

If for any $(e, k) \in (E \times K), (S_{1A} \otimes \tilde{Y}_K)\,(e, k) \neq \phi \Rightarrow S_{1A}(e) \times \tilde{Y}_K(k) \neq \phi$

$\Rightarrow S_1(e) \times \tilde{Y} \neq \phi \Rightarrow S_{1\breve{A}}(e) \neq \phi \Rightarrow S_{2\breve{B}}(e) = \phi$

Since $(S_{1A} \cap S_{2\breve{B}} = \phi \Rightarrow S_{1A}(e) \cap S_{2\breve{B}}(e) = \phi)$

$\Rightarrow S_{2\breve{B}}(e) \times \tilde{Y}_K(k) = \phi \Rightarrow (S_{2B} \otimes \tilde{Y}_K)(e, k)\ = \phi$

$\Rightarrow (S_{1\breve{A}} \otimes \tilde{Y}_K) \cap (S_{2\breve{B}} \otimes \tilde{Y}_K) = \tilde{\phi}$

Assume $k_1 \neq k_2.$ *Since* $\{Y, \tilde{\tau}_{1Y}, \tilde{\tau}_{2Y}, K\}$ is se$mi(SDW - H)_2$, there exists sD open sets $(S_1, \breve{E}) \in$

$\tilde{\tau}_{1\hat{Y}}, (S_2, \breve{B}) \in \tilde{\tau}_{2\hat{Y}}$ such that $S_{1\breve{A}}(k_1) =, \tilde{Y}\ S_{2\breve{B}}(k_2) = \tilde{Y}$ & $\qquad S_{1\breve{A}} \cap S_{2\breve{B}} = \tilde{\phi}.$

Therefore $X_E \otimes S_{1A} \in \tilde{\tau}_{1X} \otimes \tilde{\tau}_{1\hat{Y}}, \hat{\tilde{X}}_E \otimes S_{2\breve{B}} \in \tilde{\tau}_{2X} \otimes \tilde{\tau}_{2\hat{Y}}$

$(\hat{\tilde{X}}_E \otimes S_{1\breve{A}})\,(e_1, k_1)\ = \hat{\tilde{X}}_E(e_1) \times S_{1\breve{A}}(k_1)$

$= \hat{\tilde{X}} \times \tilde{Y}$

$(X_E \otimes S_{2\breve{B}})(e_2, k_2) = \hat{\tilde{X}}_E(e_2) \times S_{2\breve{B}}(k_2)$

$= \hat{\tilde{X}} \times \tilde{Y}$

If for any $(e, k) \in E \times K, (\hat{\tilde{X}}_E \otimes S_{1\breve{A}})(e, k) \neq \phi \Rightarrow \hat{\tilde{X}}_E(e) \times S_{1\breve{A}}(k) \neq \phi$

$\Rightarrow \hat{\tilde{X}} \times S_{1\breve{A}}(k) \neq \phi \Rightarrow S_{1\breve{A}}(k) \neq \phi \Rightarrow S_{2B}(k) = \phi$

$(Since S_{1A} \cap S_{2B} = \tilde{\phi} \Rightarrow S_{1\breve{A}}(k) \cap S_{2B}(k) = \phi) \Rightarrow X_E(e) \times S_{2B}(k) = \phi \Rightarrow (X_E \otimes S_{2B})(e, k)\ = \phi$

$\Rightarrow (\hat{\tilde{X}}_E \otimes S_{1\breve{A}}) \cap (X_E \otimes S_{2B}) = \tilde{\phi}$

Hence, $\left(\hat{\tilde{X}} \times Y, \tilde{\tau}_{1\hat{X}} \otimes \tilde{\tau}_{1Y}, \tilde{\tau}_{2\hat{X}} \otimes \tilde{\tau}_{2\hat{Y}}, E \times K\right)$ is $semi(SDW - H)_2$

4. Conclusion

In this article the concept of Soft semi W-D_2 structure in soft topological spaces is introduced In soft Single topological spaces and in soft bi topological spaces is introduced and some basic properties regarding this concept are demonstrated.

Topology is the most significant branch of mathematics which deals with mathematical structures. Recently, many investigators have deliberated the soft set theory which is originated by Molodtsov[12] and carefully applied to many complications which comprise uncertainties in our social life. Shabir and Naz in[11] familiarized and intensely studied the notion of soft topological spaces. They also deliberate topological structures and demonstrated their several belongings with respect to ordinary points.

In the present work, in this paper the concept of Soft semi W-D_2 structure in soft Single topological

spaces and in soft bi topological spaces and more over some basic properties regarding this concept are demonstrated.

This soft structure would be useful for the growth of the theory of soft topology to bury complex problems, comprising doubts in economics, engineering, medical etc. We hope that these results in this paper will help the researchers for reinforcement the toolbox of soft topology.

References

1. M. I. Ali, F. Feng, X. Liu, W. K. Min and M. Shabir, "On some new operations in soft set theory", *Computers and Mathematics with Applications*, *57,* 1547-1553 (2009).
2. K. V. Babitha and J. J. Sunil, "Soft set relations and Functions", *Comput. Math. Appl*. *60,* 1840-1848 (2010).
3. Basavaraj M. Ittanagi, "Soft Bi topological Spaces", *International Journal of Computer Applications* (0975 8887) Vol. *107,* No. 7, December (2014).
4. J.C. Kelly, "Bi topological Spaces", *Proc. London Math. Soc.*, *13,* 71-81 (1963).
5. P. K. Maji, R.Biswas and A.R.Roy, "Soft Set Theory", *Computers and Mathematics with Applications*, vol. *45,* no. 4-5, pp. 555-562, (2003).
6. D. Molodstov, "Soft Set Theory – First Results", *Computers and Mathematics with Applications*, vol. *37,* no. 4-5, pp. 19-31, (1999).
7. I. LReilly, "On bitopological Separation Properties", Nanta Math., *29,* 14-25 (1972).
8. M. Shabir and M. Naz, "On Soft Topological Spaces", *Computers and Mathematics with Applications*, vol. *61,* no. 7, pp. 1786-1799, (2011).
9. P. Sruthi , V.M. Vijayalakshmi , Dr. A. Kalaichelvi , "Soft W-Hausdorff Spaces" ,*International Journal of Mathematics Trends and Technology*, vol. *43,* no. 1, pp. 16-19, (2017).
10. P.K. Maji, R., Biswas and A. R. Roy, Soft set theory, Comp. Math. Appl., *45,* 555-562 (2003).
11. M. Shabir and .M. Naz, On Soft topological Spaces, Compt, Math, Appl, *61*, 1786 – 1799 (2011).
12. D. A Molodtsov, Soft set theory first results, Comp. Math. Appl., *37,* 19-31 (1999).
13. S. A El-Sheikh and A. M Abd-e-Latif, Characterization of soft b-open sets in soft topological spaces, New theory, *2,* 8-18 (2015).
14. B. Chen, 2013, Soft semi-open sets and related properties in soft topological spaces. Appl. Math. Inf. Sci, *7(1),* 287-294.