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Section A

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Fixed Points for Meir-Keeler Type Contractions in *M-Metric* Spaces with Applications

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Abstract

In this paper, we establish some fixed point theorems using Meir-Keeler type contraction in *M-metric* spaces via Gupta- Saxena type contraction. We also extend very recent results in fixed point theory.

Key words: Fixed point, Meir-Keeler Type Contraction, M-metric spaces.
Mathematical Subject Classification: 54 H10

1. Introduction and Preliminaries

A variational principle is formulated by Ekeland which is the foundation of modern variational calculus. It has many applications in various branches of mathematics, including optimization and fixed point theory[3] and also in nonlinear analysis, since it entails the existence of approximate solutions of minimization problems for a lower semi-continuous function that is bounded from below on complete metric spaces. Also, this principle is also a fruitful tool in simplifying and unifying the proofs of already known theorems and has many generalizations; see Borwein and Zhu[5].

In 1994, Matthews[8] introduced a partial metric space and proved that a partial metric space is differ from metric space in the sense that the distance of a point from itself may not be zero. Matthews[8] proved the Banach contraction principle in this new framework. After that several mathematicians proved many fixed point theorems in partial metric spaces.

Haghi *et al.*[7] in 2013 published a paper which stated that we should 'be careful on partial metric fixed point results' along with very some results therein. They showed that fixed point generalizations to partial metric spaces can be obtained from the corresponding results in metric spaces. For example he proved the following result shows that Theorem 1 in[1] is a consequence of Theorem 2 in[4].

Let (X,p) be a complete partial metric space, $\varphi : [0, +\infty) \to [0, +\infty)$ a continuous and non-decreasing function such that $\varphi(t) < t$ for all $t > 0$ and T a selfmap on X satisfying

$$p(T\,x, T\,y) \le \varphi(M_p(x, y))$$

for all $x, y \in X$. Then T has a unique fixed point.

In 2014, Asadi *et al.*[2] extended the *p*-metric space to an M-*metric* space and proved some fixed point and common fixed point theorems in this spaces. The following result is proved:

Let (X, m) be a complete M-metric space and let $T : X \to X$ be a mapping satisfying the following condition:

$\exists k \in [0,1)$ such that $m(Tx, Ty) \le km(x, y)$ for all $x, y \in X$.

Then T has a unique ûxed point.

In this paper, we establish some of the fixed point theorem for a Meir-Keeler type contraction in *M-metric* spaces via a Gupta-Saxena type contraction. Also, we extend and improve very recent results in fixed point theory.

Definition 1.1. ([8], [11] Definition 1.1) :

A partial metric on a nonempty set X is a function $p: X \times X \to R^+$ such that for all $x, y, z \in X$:

1. $p(x, x) = p(y, y) = p(x, y) \Leftrightarrow x = y$
2. $p(x, x) \le p(x, y)$,
3. $p(x, y) = p(y, x)$,
4. $p(x, y) \le p(x, z) + p(z, y) - p(z, z)$.

A partial metric space is a pair (X, p) such that X is a nonempty set and p is a partial metric on X.

Notation :

The following notations are useful in the sequel:

$(i)\ mxy := \min\{m(x, x), m(y, y)\} = m(x, x) \vee m(y, y)$,

$(ii) Mxy := max\{m(x, x), m(y, y)\} = m(x, x) \wedge m(y, y)$.

Now we want to extend Definition 1.1 as follows.

Definition 1.2.

Let X be a non-empty set. A function $m: X \times X \to R^+$ is called a *M-metric* if the following conditions are satisfied:

$(m1)\ m(x, x) = m(y, y) = m(x, y) \Leftrightarrow x = y$

$(m2)\ m_{xy} \le m(x, y)$

(m3) $m(x, y) = m(y, x)$

(m4) $(m(x, y) - m_{xy}) \leq (m(x, z) - m_{xz}) + (m(z, y) - m_{zy})$

Then the pair (X, m) is called an M-metric space.

According to the above, our definition of the condition (p1) in the definition[8] changes to (m1) and (p2) for $p(x, x)$ is expressed by just $p(y, y)=0$, we may have $p(y, y)\neq 0$, so we improved that condition by replacing it by $\min\{p(x, x), p(y, y)\}\leq p(x, y)$ and also we improved the condition (p4) to the form (m4). In the sequel we present an example that holds for the *M-metric*, but not for the *p*-metric.

Remark 1.3.

For every $x, y \in X$:

(i) $0 \leq M_{xy} + m_{xy} = m(x, x) + m(y, y)$,

(ii) $0 \leq M_{xy} - m_{xy} = |m(x, x) - m(y, y)|$,

(iii) $M_{xy} - m_{xy} \leq (M_{xz} - m_{xz}) + \left(M_{zy} - m_{zy}\right)$

The next examples state that m^s and m^w are ordinary metrices.

Example 1.4. Let m be a M-metric. Put:

(i) $m^w(x, y) = m(x, y) - 2m_{xy} + M_{xy}$

(ii) $m^s(x, y) = m(x, y) - m_{xy}$ when $x \neq y$ and $m^s(x, y) = 0$ if $x = y$.

Then m^s and m^w are ordinary metrices.

Proof: If $m^w(x,y) = 0$ then

$$m(x, y) = 2m_{xy} - M_{xy} \tag{1.1}$$

But from (1.1) and $m_{xy} \leq m(x, y)$ we get

$m_{xy} = M_{xy} = m(x, x) = m(y, y)$ so by (1) get

$m(x, y) = m(x, x) = m(y, y)$ and therefore $x = y$. For the triangle inequality it is enough to - that we consider Remark1.1 and (m4).

In the following example, we present an example of a M-metric which is not a p-metric.

Remark 1.5.

For every $x, y \in X$:

(i) $m(x, y) - M_{xy} \leq m^w(x, y) \leq m(x, y) + M_{xy}$,

(ii) $m(x, y) - M_{xy}) \leq m^s(x, y) \leq m(x, y)$.

Example 1.6.

Let $X = \{1,2,3\}$. Define

$m(1,2) = m(2,1) = m(1,1) = 8$,

$m(1,3) = m(3,1) = m(3,2) = m(2,3) = 7,$

$m(2,2) = 9$ and $m(3,3) = 5,$

So m is an M-metric but m is not a p - metric. Since $m(2,2) \nleq m(1,2), m$ is not a p - metric.
If $D(x,y) = m(x,y) - m_{xy}$ then $m(1,2) = m_{1,2} = 8$ but it means $D(1,2) = 0$ while $1 \neq 2$ while D means is not a metric.

Example 1.7 [2]

Let (X, d) be a metric space, $\emptyset: [0,\infty] \to [\emptyset(0),\infty)$ be a one and nondecreasing or strictly increasing mapping with $\emptyset(0)$, defined such that

$$\emptyset(x+y) \leq \emptyset(x) + \emptyset(y) - \emptyset(0) \qquad \forall x, y \geq 0.$$

Then $m(x,y) = \emptyset\,(d(x,y))$ is a $M-$ metric.

Example 1.8

Let (X,d) be a metric space. Then $m(x,y) = a\,d(x,y) + b$ where $a, b > 0$ is an $M-$ metric, because we can put $\emptyset(t) = at + b.$

Remark 1.9

According to the Example 5.1.4, by the Banach contraction

$$\exists\, k \in [0,1), m(Tx,Ty) \leq k\, m(x,y), \text{ for all } x, y \in X,$$

We have

$$m(Tx,Ty) = ad(Tx,Ty) + b \leq k\,ad(x,y) + k\,b \Longrightarrow d(Tx,Ty) \leq k\,d(x,y) + \frac{b(k-1)}{a},$$

which does not imply that we have the ordinary Banach contraction for all self-maps Ton X. So this states that if the M-metric m and the ordinary metric d even have the same topology, but the Banach contraction of an M-metric, this does not imply the Banach contraction of the ordinary metric d.

Lemma 1.10 ([2]) Every p –metric is a $M-$ metric.

2. Topology for M-metric space :

It is clear that each M-metric p on X generates a T_0 topology on X. The set

$$\{B_m(x,\epsilon) : x \in X, \epsilon > 0\epsilon\},$$

where

$$B_m(x,\epsilon) = \{y \in X : m(x,y) < m_{x,y} + \epsilon\},$$

for all $x \in X, and\ \epsilon > 0$, forms the base of τ_m.

Definition 2.1. Let (X, m) be an m – metric space. Then

(1) A sequence $\{x_n\}$ in an m – metric space (X, m) converges to a point $x \in X$ if and only if

$$\lim_{n\to\infty}\left(m(x_n,x) - m_{x_n,x}\right) = 0 \qquad (2.1)$$

(2) A sequence $\{x_n\}$ in an m – metric space (X, m) is called an m – Cauchy sequence if

$$\lim_{n,m\to\infty}\left(m(x_n, x_m) - m_{x_n,x_m}\right) \text{ and } \lim_{n\to\infty}\left(M(x_n, x_m) - m_{x_n,x}\right) \quad (2.2)$$

in this space exists(and are finite).

(3) An m –metric space (X, m) is said to be complete if every m – Cauchy sequence $\{x_n\}$ in X converges with respect to τ_m, to a point $x \in X$ such that

$$\lim_{n\to\infty}\left(m(x_n, x) - m_{x_n,x}\right) = 0 \text{ and } \lim_{n\to\infty}\left(M(x_n, x) - m_{x_n,x}\right) = 0$$

Lemma 2.2

Let (X, m) be an m –metric space. Then

(i) $\{x_n\}$ is a Cauchy sequence in (X, m) if and only if it is a Cauchy sequence in the metric space (X, m^w).

(ii) An m –metric space (X, m) is complete if and only if the metric space (X, m^w) is complete. Furthermore

$$\lim_{n\to\infty} m^w(x_n, x) = 0 \Leftrightarrow \left(\lim_{n\to\infty} m(x_n, x) - m_{x_n,x}\right) = 0 \text{ and } \left(\lim_{n\to\infty} M(x_n, x) - m_{x_n,x}\right) = 0$$

Likewise the above definition holds also for m^s.

Lemma 2.3

Assume that $x_n \to x \; and \; x_n \to y \; as \; n \to \infty$ in a M – metric space (X, m). Then

$$\lim_{n\to\infty}\left(m(x_n, y_n) - m_{x_n,y_n}\right) = m(x, y) - m_{xy}.$$

Proof: We have

$$\left|\left(m(x_n, y_n) - m_{x_n,y_n}\right) - \left(m(x, y) - m_{x,y}\right)\right| \leq \left(m(x_n, x) - m_{x_n,x}\right) + \left(m(y, y_n) - m_{y,y_n}\right).$$

From lemma 5.2.2 we can deduce the following lemma.

Lemma 2.4

Assume that $x_n \to x \; as \; n \to \infty$ in an M – metric space (X, m).

Then

$$\lim_{n\to\infty}\left(m(x_n, y) - m_{x_n,y}\right) = m(x, y) - m_{x,y}$$

for all $y \in X$.

Lemma 2.5

Assume that $x_n \to x \; and \; x_n \to y \; as \; n \to \infty$ in as n m – metric space (X, m). Then $m(x, y) = m_{xy}$. Further if $m(x, x) = m(y, y)$, then $x = y$.

Proof:

By lemma 2.2 we have

$$0 = \lim_{n\to\infty}\left(m(x_n, x_n) - m_{x_n,x_n}\right) = m(x, y) - m_{xy}.$$

Lemma 2.6 Let $\{x_n\}$ in an m – metric space (X, m) such that $\exists \; r \in [0, 1)$ such that

$$m(x_{n+1}, x_n) \leq r\{x_n\}m(x_n, x_{n-1}) \qquad \forall n \in N \tag{2.3}$$

Then

(a) $\lim_{n\to\infty} m(x_n, x_{n-1}) = 0$

(b) $\lim_{n\to\infty} m(x_n, x_n) = 0$

(c) $\lim_{m,n\to\infty} m_{x_m, x_n} = 0$

(d) $\{x_n\}$ is an m – Cauchy sequence.

Proof: From (2.3) we have,

$$m(x_n, x_{n-1}) \leq rm(x_{n-1}, x_{n-2}) \leq r^2 m(x_{n-2}, x_{n-3}) \leq \cdots \leq r^n m(x_0, x_1)$$

Thus $\lim_{n\to\infty} m(x_n, x_{n-1}) = 0$.
Which implies (a).
To prove (b) , from (m2) and (a) we have

$$\lim_{n\to\infty} \min \{m(x_n, x_n), m(x_{n-1}, x_{n-1})\} = \lim_{n\to\infty} m_{x_n x_{n-1}} \leq \lim_{n\to\infty} m(x_n, x_{n-1}) = 0$$

That is (b) holds.
Clearly, (c) holds, since $\lim_{n\to\infty} m(x_n, x_n) = 0$

Theorem 2.7

The topology τ_m is not Hausdorff.

Theorem 2.8

Let (X, m) be a complete M-metric space and $T: X \to X$ be mapping satisfying the following condition:

$$k \in \left[0, \frac{1}{2}\right) \text{ such that } m(Tx, Ty) \leq k\big(m(x, Tx) + m(y, Ty)\big), \forall\, x, y \in X. \tag{2.4}$$

Then T has a unique fixed point.

3. Main Result and fixed point theorems

The following definition is new version of the definition in[9] for an M-metric space.

Definition 3.1 A Meir-Keeler mapping is a mapping $T: M \to M$ on an M-metric space (X, M) such that $\forall \epsilon > 0\, \exists \delta > 0$ such that

$$\forall\, x, y \in X \; and \; \epsilon \leq m(x, y) < \epsilon + \delta \Longrightarrow m(Tx, Ty) < \epsilon \tag{3.1}$$

Theorem 3.2 Let (X, M) be a complete $M-$ metric space and let T be a mapping from X onto itself satisfying the following condition:

$$\forall \epsilon > 0\, \exists \delta > 0 \text{ such that } \forall\, x, y \in X \; and \; \epsilon \leq m(x, y) < \epsilon + \delta \Longrightarrow m(Tx, Ty) < \epsilon. \tag{3.2}$$

Then T has a unique fixed point $u \in X$. Moreover, for all $x \in X$, the sequence $\{T^n(x)\}$ converges to u.

Theorem 3.3 Let (X, M) be a complete $M-$metric space and let T be a mapping from X onto itself satisfying the following condition:

$$\forall \epsilon > 0 \; \exists \delta > 0 \text{ such that } \forall x, y \in X \text{ and } \epsilon \le kC(x,y) < \epsilon + \delta \Longrightarrow m(Tx, Ty) < \epsilon, \tag{3.3}$$

where $C(x,y) = m(x\,Tx) + m(y,Ty)$

for some $0 < k < \frac{1}{3}$. Then T has a unique fixed point $u \in X$. Moreover, for all $u \in X$, the sequence $\{T^n(x)\}$ converges to u.

Proof: We first observe that (3.3) trivially implies that T is a strict contraction, i.e. $x \neq y \Longrightarrow$

$$m(Tx, Ty) < k\, C(x,y). \tag{3.4}$$

Let $x_0 \in X$ and $x_n = Tx_{n-1}$ so we have

$$C(x_{n-1}, x_n) = m(x_{n-1}, x_n) + m(x_n, x_{n+1})$$

$$< k\big(m(x_{n-1}, x_n) + m(x_n, x_{n+1})\big),$$

$$m(x_n, x_{n+1}) = m(Tx_{n-1}, Tx_n)$$

$$\le k\, C(x_{n-1}, x_n)$$

$$\le k\big(m(x_{n-1}, x_n) + m(x_n, x_{n+1})\big),$$

Therefore

$$m(x_n, x_{n+1}) \le r m(x_{n-1}, x_n), \tag{3.5}$$

where $r = \frac{k}{1-k} < 1$. Now by lemma 2.5, $\{x_n\}$ is a Cauchy sequence, and by completeness of X, $Tx_{n-1} = x_n \to x^* \text{ in } m$ for some $x^* \in X$. since T is a continuous mapping, so $x_n = Tx_{n-1} \to Tx^*$, in m now by lemma 2.4 we find

$$m(x^*, Tx^*) = m_{x^*, Tx^*},$$

$$0 = \lim_{n\to\infty}\big(m(x_n, Tx_n) - m_{x_n, Tx_n}\big) = m(x^*, x^*) - m_{x^*, Tx^*} = m(Tx^*, Tx^*) - m_{x^*, Tx^*}$$

By lemma 2.2 and

$$m(x^*, Tx^*) = m_{x^*, Tx^*} = m(Tx^*, Tx^*) = m(x^*, x^*).$$

So $x^* = Tx^*$. Uniqueness by the contraction (10) is clear.

Put

$$C(x,y) = m(x,y) + \frac{\big(1 + m(x,Tx)\big)m(y.Ty)}{1 + m(x,y)} + \frac{m(x,Tx)\, m(y,Ty)}{1 + m(y,Ty)}$$

Theorem 3.4 Let (X, M) be a complete $M-$metric space and let T be a mapping from X onto itself satisfying the following condition:

$$\forall \epsilon > 0 \; \exists \delta > 0 \text{ such that } \forall x, y \in X \text{ and } \epsilon \le kC(x,y) < \epsilon + \delta \Longrightarrow m(Tx, Ty) < \epsilon, \tag{3.6}$$

for some $0 < k < \frac{1}{3}$. Then T has a unique fixed point $u \in X$. Moreover, for all $x \in X$, the sequence $\{T^n(x)\}$ converges to u.

Proof: We first observe that (3.6) trivially implies that T is a strict contraction, i.e.

$$x \neq y \Longrightarrow m(Tx, Ty) < k\, C(x, y). \tag{3.7}$$

Let $x_0 \in X \; and \; x_n = Tx_{n-1}$ so we have

$$C(x_{n-1}, x_n) = m(x_{n-1}, x_n) + \frac{(1 + m(x_{n-1}, x_n))m(x_n, x_{n+1})}{1 + m(x_{n-1}, x_n)} + \frac{m(x_{n-1}, x_n)m(x_n, x_{n+1})}{1 + m(x_n, x_{n+1})}$$

$$< m(x_{n-1}, x_n) + m(x_n, x_{n+1}) + \frac{m(x_{n-1}, x_n)m(x_n, x_{n+1})}{m(x_n, x_{n+1})}$$

$$< k(2m(x_{n-1}, x_n) + m(x_n, x_{n+1})),$$

$$\begin{aligned} m(x_n, x_{n+1})x_{n-1,} &= m(Tx_{n-1,}, Tx_n) \\ &\leq k\, C(x_{n-1}, x_n) \\ &\leq k(2m(x_{n-1}, x_n) + m(x_n, x_{n+1})), \end{aligned}$$

Therefore

$$m(x_n, x_{n+1}) \leq r m(x_{n-1}, x_n), \tag{3.8}$$

where $r = \frac{2k}{1-k} < 1$. Now by lemma 2.5, $\{x_n\}$ is a Cauchy sequence, and by completeness of X, $Tx_{n-1} = x_n \to x^* \; in \; m$ for some $x^* \in X$. Since T is a continuous mapping , so $x_n = Tx_{n-1} \to Tx^*$, in m now by lemma 2.4 we find

$$m(x^*, Tx^*) = m_{x^*, Tx^*},$$

$$0 = \lim_{n\to\infty}\left(m(x_n, Tx_n) - m_{x_n, Tx_n}\right) = m(x^*, x^*) - m_{x^*, Tx^*} = m(Tx^*, Tx^*) - m_{x^*, Tx^*}$$

By lemma 2.2 and

$$m(x^*, Tx^*) = m_{x^*, Tx^*} = m(Tx^*, Tx^*) = m(x^*, x^*).$$

So $x^* = Tx^*$. Uniqueness by the contraction (3.7) is clear.

Corollary 3.5(Gupta and Saxena[6]*) :*

Let (X, d) be a complete metric space and T be a continuous mapping from X onto itself. Assume that T satisfies

$$\forall\, x, y \in X, x \neq y \Longrightarrow d(Tx, Ty) < k\, C(x, y).$$

where $0 < k < \frac{1}{3}$ is a constant. Then T has a unique fixed point $u \in X$. Moreover, for all $x \in X$, the sequence $\{T^n(x)\}$ converges to u.

4. Applications :

In this section, after an idea of Samet *et al.*[10], we shall state an integral version of the Gupta-Saxena result.

Theorem 4.1

Let (X, m) be an $M-$metric space and let T be a self-mapping defined on X. Assume that there exists a function $\varphi\colon [0,\infty) \to 0,\infty)$ satisfying following condition

(1) $\varphi(0) = 0 and\ t > 0 \Longrightarrow \varphi(t) > 0$

(2) $\emptyset$ is nondecreasing and right continuous;

(3) for every $\epsilon > 0$, there exists $\delta > 0$ such that

$$\epsilon \leq \varphi\big(k\, C(x,y)\big) < \epsilon + \delta \Longrightarrow \varphi\big(m(Tx,Ty)\big) < \epsilon, \tag{4.1}$$

for some $0 < k < \dfrac{1}{3}$ and $\forall\, x, y \in X$, $with\ \ x \neq y$

Then (3.6) is satisfied.

Proof:

Fix $\epsilon > 0\ so\ \varphi(\epsilon) > 0$ Hence by (4.1) $\exists\ \delta_1 > 0$ such that

$$\forall\, x, y \in X\ , with\ \ x \neq y, \varphi(\epsilon) \leq \varphi\big(kC(x,y)\big) < \varphi(\epsilon) + \delta_1 \Longrightarrow \varphi\big(m(Tx,Ty)\big) < \ \varphi(\epsilon) \tag{4.2}$$

According to the right continuity of $\emptyset$

$$\exists\ \delta > 0, \varphi(\epsilon +\ \delta_1) < \varphi(\epsilon) +\ \delta.$$

Noe for $x, y \in X$ with $x \neq y$, and fixed

$$\epsilon \leq k\ C(x,y) < \ \epsilon + \delta, \tag{4.3}$$

Since $\emptyset$ is nondecreasing mapping, we have

$$\varphi(\epsilon) \leq \varphi\big(kC(x,y)\big) <, \varphi(\epsilon +\ \delta_1) < \varphi(\epsilon) +\ \delta.$$

So we get

$$\varphi\big(m(Tx,Ty)\big) < \ \varphi(\epsilon),$$

which implies that $m(Tx,Ty) < \epsilon$.

Corollary 4.2

Let (X, m) be an $M-$ metric space and let T be a self-mapping defined on X. Let $h\colon [0,\infty) \to 0,\infty)$ be a locally integrable function such that

(1) $t > 0 \Longrightarrow \int_0^t h(s)ds > 0;$

(2) For every $\epsilon > 0$, there exists $\delta > 0$ such that

$$\frac{1}{k}\epsilon \leq \int_0^{C(x,y)} h(s)ds < \frac{1}{k}\epsilon + \delta \Longrightarrow \int_0^{\frac{1}{k}m(Tx,Ty)} h(s)ds < \frac{1}{k}\epsilon, \tag{4.4}$$

for some $0 < k < \dfrac{1}{3}$ and $\forall\, x, y \in X$, $with\ \ x \neq y$

Then (3.6) is satisfied.

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References

1. Altun I., Sola F., Simsek H., Generalized contractions on partial metric spaces, Topol. Appl. *157,* 2778–2785 (2010).
2. Asadi, A, Karapýnar, E, Salimi, P: New extension of p-metric spaces with some fixed point results on M-metric spaces. J. Inequal. Appl. 2014, 18 (2014)
3. Aydi, E, Karapýnar, E, Vetro, C: On Ekeland's variational principle in partial metric spaces. *Appl. Math. Inf. Sci. 9(1),* 257-262 (2015).
4. Berinde V., A common fixed point theorem for compatible quasi contractive self-mappings in metric spaces, Appl. Math. Comput. *213,* 348–354 (2009).
5. Borwein, JM, Zhu, QJ: Techniques of Variational Analysis. Springer, New York (2005).
6. Gupta, AN, Saxena, A: A unique fixed point theorem in metric spaces. *Math. Stud. 52,* 156-158 (1984).
7. Haghi, RH, Rezapour, S, Shahzad, N: Be careful on partial metric fixed point results. *Topol. Appl. 160(3),* 450-454 (2013).
8. Matthews, SG: Partial metric topology. Ann. N.Y. Acad. Sci. *728*, 183-197 (1994).
9. Meir, A, Keeler, E: A theorem on contraction mappings. J. Math. Anal. Appl. *28(1-3)*, 326-329 (1969).
10. Samet, B, Vetro, C, Yazidi, H: A fixed point theorem for a Meir-Keeler type contraction through rational expression. J. Nonlinear Sci. Appl. *6,* 162-169 (2013).
11. Shatanawi, W, Postolache, M: Coincidence and fixed point results for generalized weak contractions in the sense of Berinde on partial metric spaces. Fixed Point Theory Appl. 2013, 54 (2013).