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Fixed Point Theorems for Set Valued Contraction Integral Type in Complete Metric Space

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Abstract

In this paper, we shall establish some stability results for Picard and Mann iteration processes in metric space and normed linear space by employing a set-valued contractive condition of integral type.

1. Introduction

In this paper we shall establish some stability results for Picard and Mann iteration processes in metric space and normed linear space by employing a contractive condition of integral type. Our results are generalizations and extensions of some of the results of Berinde[2], Osilike[15], Osilike and Udomene[14], Rhoades[16], Rhoades[18], Harder and Hicks[7], as well as some of the results of the author[8],[12],[13].

2. Preliminaries :

There are several iteration processes in the literature for which the fixed points of operators have been approximated over the years by various authors. In a complete metric space, the Picard iteration process $\{x_n\}_{n=0}^{\infty}$ defined by

$$x_{n+1} = Tx_n \qquad n = 0, 1, \dots\dots\dots\dots\dots\dots \tag{1}$$

has been employed to approximate the fixed points of mappings satisfying the inequality relation

$$d(Tx, Ty) \leq \alpha d(x, y), \qquad \forall\, x, y \in E \text{ and } \alpha \in [0,1). \tag{2}$$

Condition (2) is called the Banach's contraction condition. Any operator satisfying (2) is called strict contraction.

Also, condition (2) is significant in the celebrated Banach's fixed point theorem[1].
In the Banach space setting, we shall state some of the iteration processes generalizing (1) as follows:
For $x_0 \in X$, the sequence $\{x_n\}_{n=0}^{\infty}$ defined by

$$x_{n+1} = (1-\alpha_n)x_n + \alpha_n T x_n, \qquad n = 0, 1, \dots \tag{3}$$

Where $\{\alpha_n\}_{n=0}^{\infty} \subset [0,1]$, is called the Mann iteration process (see Mann [68]).
For $x_0 \in X$, the sequence $\{x_n\}_{n=0}^{\infty}$ defined by

$$\left.\begin{aligned} x_{n+1} &= (1-\alpha_n)x_n + \alpha_n T z_n \\ z_n &= (1-\beta_n)x_n + \beta_n T x_n \end{aligned}\right\} \; n = 0, 1, \dots \tag{4}$$

Where $\{\alpha_n\}_{n=0}^{\infty}$ and $\{\beta_n\}_{n=0}^{\infty}$ are sequences in [0, 1]; is called the Ishikawa iteration process (see Ishikawa [54 (1974)]).

Kannan[10], established an extension of the Banach's fixed point theorem by using the following contractive definition for a selfmap T: there exists $\beta \in (0, \frac{1}{2})$ such that

$$d(Tx, Ty) \le \beta[d(x, Tx) + d(y, Ty)] \qquad \forall\, x, y \in E \tag{5}$$

Chatterjea[5], used the following contractive condition: For a selfmap T: there exists $\gamma \in (0, \frac{1}{2})$ such that

$$d(Tx, Ty) \le \gamma[d(x, Ty) + d(y, Tx)] \qquad \forall\, x, y \in E \tag{6}$$

Zamfirescu[20], established a nice generalization of the Banach's fixed point theorem by combining (2), (5) and (6). That is, for a mapping $T: E \to E$, there exist real numbers α, β, γ satisfying $0 \le \alpha \le 1, 0 \le \beta \le \frac{1}{2}, 0 \le \gamma \le \frac{1}{2}$ respectively such that for each $x, y \in E$, at least one of the following is true:

$$\left.\begin{aligned} &(z_1) \qquad d(Tx, Ty) \le \alpha d(x, y) \\ &(z_2) \qquad d(Tx, Ty) \le \beta[d(x, Tx) + d(y, Ty)] \\ &(z_3) \qquad d(Tx, Ty) \le \gamma[d(x, Ty) + d(y, Tx)]. \end{aligned}\right\} \tag{7}$$

The mapping $T: E \to E$, satisfying (7) is called the Zamfirescu contraction. Any mapping satisfying condition (z_2) of (7) is called a Kannan mapping, while the mapping satisfying condition (z_3) is called Chatterjea operator. The contractive condition (7) implies

$$\|Tx - Ty\| \le 2\delta\|x - Tx\| + \delta\|x - y\|, \quad \forall\, x, y \in E, \tag{8}$$

Where $\delta = max\left\{\alpha, \frac{\beta}{1-\beta}, \frac{\gamma}{1-\gamma}\right\}, \quad 0 \le \delta \le 1.$

The following definition of stability of iteration process due to Harder and Hicks[7], shall be required in the sequel.

2.1 Definition. Let (E, d) be a complete metric space and $T: E \to E$ a selfmap of E. suppose that $F_T = \{p \in E | Tp = p\}$ is the set of fixed points of T: Let $\{\alpha_n\}_{n=0}^{\infty} \subset E$ be the sequence generated by an iteration procedure involving T which is defined by

$$x_{n+1} = f(T, x_n), \qquad n = 0, 1, 2, \dots \tag{9}$$

Where $x_0 \in X$ is the initial approximation and f is some function. Suppose $\{x_n\}_{n=0}^{\infty}$ converges to a fixed point p of T.

Let $\{y_n\}_{n=0}^{\infty} \subset E$ and set $\varepsilon_n = d\big(y_{n+1}, f(T, y_n)\big)$, $\quad n = 0, 1, 2, \ldots\ldots\ldots$ Then, the iteration procedure (9) is said to be $T-$ stable or stable with respect to T if and only if $\lim_{n\to\infty} \varepsilon_n = 0$ implies $\lim_{n\to\infty} y_n = p$.

If in (9), $f(T, x_n) = Tx_n$, $n = 0, 1, 2, \ldots$, then we have the Picard iteration process, while we obtain the Mann iteration process if

$$f(T, x_n) = (1 - \alpha_n)x_n + \alpha_n Tx_n, \quad n = 0, 1, 2, \ldots\ldots\ldots \quad , \alpha_n \in [0, 1].$$

Rhoades[16], extended the results of [7], to the following independent contractive condition: there exists $c \in [0, 1)$ such that

$$d(Tx, Ty) \le c \max\{d(x, y), d(x, Ty), d(y, Tx)\}, \quad \forall\, x, y \in E. \tag{10}$$

Moreover, Osilike[15], generalized and extended some of the results of Rhoades[17], by using a more general contractive definition than those of Rhoades[16,17]. Indeed, he employed the following contractive definition: there exists $a \in [0, 1]$, $\quad L \ge 0$ such that

$$d(Tx, Ty) \le Ld(x, Tx) + ad(x, y), \ \forall\, x, y \in E. \tag{12}$$

In a recent paper of Branciari[4], a generalization of Banach[1], was established. In that paper, Branciari[4], employed the following contractive integral inequality condition: there exists $c \in [0, 1)$ such that $\forall\, x, y \in E$, we have

$$\int_0^{d(f(x), f(y))} \varphi(t)dt \le c \int_0^{d(x,y)} \varphi(t)dt, \tag{13}$$

Where $\varphi : \mathbb{R}^+ \to \mathbb{R}^+$ is a Lebesgue-integrable mapping which is summable, nonnegative and such that for each $\varepsilon > 0, \int_0^{\varepsilon} \varphi(t)dt > 0.$

Rhoades[19], used the conditions

$$\int_0^{d(f(x), f(y))} \varphi(t)dt \le k \int_0^{m(x,y)} \varphi(t)dt, \quad \forall\, x, y \in E, \tag{14}$$

where $m(x, y) = max\left\{d(x, y), d(x, f(x)), d(y, f(y)), \dfrac{d(x, f(y)) + d(y, f(x))}{2}\right\}$,

and $\quad \displaystyle\int_0^{d(f(x), f(y))} \varphi(t)dt \le k \int_0^{M(x,y)} \varphi(t)dt, \quad \forall\, x, y \in E,$ (15)

with $M(x, y) = max\{d(x, y), d(x, f(x)), d(y, f(y)), d(x, f(y)), d(y, f(x))\}$,

where $k \in [0,1)$ and $\varphi : \mathbb{R}^+ \to \mathbb{R}^+$ in both cases is as defined in (14). Condition (16) is the integral form of Ciric's condition in Ciric[6].

Following Branciari[4], and Rhoades[19], we now state the following contractive conditions of integral type which shall be employed in establishing our results.

For a self-mapping $T : E \to E$, there exist a real number $k \in [0,1)$ and monotone increasing functions $v, \psi : \mathbb{R}^+ \to \mathbb{R}^+$ such that $\psi(0) = 0$ and $\forall\, x, y \in E$, we have

$$\int_0^{d(Tx, Ty)} \varphi(t)dv(t) \le \psi\left(\int_0^{d(x, Tx)} \varphi(t)dv(t)\right) + k \int_0^{d(x,y)} \varphi(t)dv(t), \tag{16}$$

where $\varphi : \mathbb{R}^+ \to \mathbb{R}^+$ is a Lebesgue-Stieltjes integrable mapping which is summable, nonnegative and such that for each

$$\varepsilon > 0, \int_0^{\varepsilon} \varphi(t)dv(t) > 0.$$

In this paper, we shall consider the Picard and Mann iteration processes to establish some stability results for self-mappings in metric space and normed linear space by employing the contractive condition of integral type defined in (16).

We shall require the following lemmas in the sequel.

2.2 Lemma. (Berinde[2,3]). If is a real number such that $0 \le \delta < 1$, and $\{\varepsilon_n'\}_{n=0}^{\infty}$ is a sequence of positive numbers such that $\lim_{n\to\infty} \varepsilon_n' = 0$, t then for any sequence of positive numbers $\{u_n\}_{n=0}^{\infty}$ satisfying

$$u_{n+1} \le \delta u_n + \varepsilon_n', \quad n = 0, 1, 2, \dots\dots\dots ,$$

We have $\lim_{n\to\infty} u_n = 0.$

2.3 Lemma. Let (E, d) be a complete metric space and $\varphi : \mathbb{R}^+ \to \mathbb{R}^+$ is a Lebesgue-Stieltjes integrable mapping which is summable, nonnegative and such that for each $\varepsilon > 0, \int_0^{\varepsilon} \varphi(t)dv(t) > 0.$ Suppose that $\{u_n\}_{n=0}^{\infty}, \{v_n\}_{n=0}^{\infty} \subset E$ and $\{a_n\}_{n=0}^{\infty} \subset (0,1)$ are sequence such that

$$\left| d(u_n, v_n) - \int_0^{d(u_n,v_n)} \varphi(t)dv(t) \right| \le a_n,$$

With $\lim_{n\to\infty} a_n = 0.$ Then

$$d(u_n, v_n) - a_n \le \int_0^{d(u_n,v_n)} \varphi(t)dv(t) \le d(u_n, v_n) + a_n. \tag{17}$$

Proof. By letting

$$d(u_n, v_n) - \int_0^{d(u_n,v_n)} \varphi(t)dv(t)$$

and using the definition of modulus function in $|y|$ yields (17).

3. Main Results

3.1 Theorem. Let (E, d) be a complete metric space and $T : E \to E$ a selfmap of E satisfying condition (2.1). Suppose T has a fixed point p. For $x_0 \in E$, let $\{x_n\}_{n=0}^{\infty}$ be the Picard iteration process defined by (1). Let $v, \psi : \mathbb{R}^+ \to \mathbb{R}^+$ is a Lebesgue-Stieltjes integrable mapping which is summable, nonnegative and such that for each $\varepsilon > 0, \int_0^{\varepsilon} \varphi(t)dv(t) > 0.$ Then, the Picard iteration process is T–stable.

Proof. Let $\{y_n\}_{n=0}^{\infty} \subset E \text{ and } \varepsilon_n = d(y_{n+1}, Ty_n).$ Assume $\lim_{n\to\infty} \varepsilon_n = 0.$ Then we shall establish that $\lim_{n\to\infty} y_n = p$ by using condition (16), Lemma 2.2 and the triangle inequality as follows. Let $\{a_n\}_{n=0}^{\infty} \subset (0,1)$ Then, by Lemma 2.2, we have

$$\int_0^{d(y_{n+1},p)} \varphi(t)dv(t) \le d(y_{n+1},p) + a_n$$
$$\le \left(\int_0^{d(p,Tp)} \varphi(t)dv(t)\right) + k\int_0^{d(p,y_n)} \varphi(t)dv(t) + \int_0^{\varepsilon_n} \varphi(t)dv(t) + 3a_n$$
$$= k\int_0^{d(y_n,p)} \varphi(t)dv(t) + \int_0^{\varepsilon_n} \varphi(t)dv(t) + 3a_n. \qquad (18)$$

We can now express (18) in the form $u_{n+1} \le \delta u_n + \varepsilon_n'$,

$0 \le \delta = k < 1$, $u_n = \int_0^{d(y_n,p)} \varphi(t)dv(t)$

And $\varepsilon_n' = \int_0^{\varepsilon_n} \varphi(t)dv(t) + 3a_n$,

$$\lim_{n\to\infty} \varepsilon_n' = \lim_{n\to\infty}\left(\int_0^{\varepsilon_n} \varphi(t)dv(t) + 3a_n\right) = 0,$$

so that by Lemma 2.2 and the fact that $\int_0^{\varepsilon} \varphi(t)dv(t) > 0$, for each $\varepsilon > 0$ we have that $\lim_{n\to\infty} \int_0^{d(y_n,p)} \varphi(t)dv(t) = 0$ from which it follows that $\lim_{n\to\infty} d(y_n,p) = 0$, that is $\lim_{n\to\infty} y_n = p$.

Conversely, let $\lim_{n\to\infty} y_n = p$. Then, by the contractive condition (16), Lemma 2.2 and the triangle inequality again, we have

$$\int_0^{\varepsilon_n} \varphi(t)dv(t) = \int_0^{d(y_{n+1},Ty_n)} \varphi(t)dv(t)$$
$$\le \int_0^{d(y_{n+1},p)} \varphi(t)dv(t) + k\int_0^{d(p,y_n)} \varphi(t)dv(t) + 3a_n \to 0$$

as $n \to \infty$.

Again, using the condition on φ yields $\lim_{n\to\infty} \varepsilon_n = 0$.

3.2 Theorem. Let $(E, \|\cdot\|)$ be a normed linear space and $T: E \to E$ a selfmap of E satisfying condition (16). Suppose T has a fixed point p. For $x_0 \in E$, let $\{x_n\}_{n=0}^{\infty}$ be the Mann iteration process defined by (3), where $\{\alpha_n\}_{n=0}^{\infty}$ is a sequence in [0, 1] such that $0 < \alpha \le \alpha_n \quad (n = 0, 1, \dots \dots)$.

Let $v, \psi : \mathbb{R}^+ \to \mathbb{R}^+$ be monotone increasing functions such that $\psi(0) = 0$ and $\varphi : \mathbb{R}^+ \to \mathbb{R}^+$ is a Lebesgue-Stieltjes integrable mapping which is summable, nonnegative and such that for each $\varepsilon > 0, \int_0^{\varepsilon} \varphi(t)dv(t) > 0$.

Then, the Picard iteration process is $T-$stable.

Proof : Suppose that

$\{y_n\}_{n=0}^{\infty} \subset E$, $\quad \varepsilon_n = \|y_{n+1} - (1-\alpha_n)y_n - \alpha_n Ty_n\|$, $\quad n = 0, 1, 2 \dots \dots \dots ..$,

And let $\lim_{n\to\infty} \varepsilon_n = 0$. Then we shall establish that $\lim_{n\to\infty} y_n = p$ by using condition (16), Lemma 2.2 and the triangle inequality as follows. Let $\{a_n\}_{n=0}^{\infty} \subset (0,1)$ Then, by Lemma 2.2, we have

$$\int_0^{\|y_{n+1}-p\|} \varphi(t)dv(t) \leq [\|y_{n+1} - (1-\alpha_n)y_n - \alpha_n T y_n\| - a_n]$$
$$+ (1-\alpha_n)[\|y_n - p\| - a_n] + \alpha_n[\|Tp - Ty_n\| - a_n] + 3a_n$$
$$\leq [(1-k)\alpha_n]\int_0^{\|y_n-p\|} \varphi(t)dv(t) + \int_0^{\varepsilon_n} \varphi(t)dv(t) + 3a_n$$
$$\leq [(1-k)\alpha]\int_0^{\|y_n-p\|} \varphi(t)dv(t) + \int_0^{\varepsilon_n} \varphi(t)dv(t) + 3a_n. \tag{19}$$

Expressing (19) in the form $u_{n+1} \leq \delta u_n + \varepsilon_n^{'}$,

where $0 \leq \delta = (1-k)\alpha < 1$, $u_n = \int_0^{\|y_n-p\|} \varphi(t)dv(t)$

and $\varepsilon_n^{'} = \int_0^{\varepsilon_n} \varphi(t)dv(t) + 3a_n$,

with $\lim_{n\to\infty} \varepsilon_n^{'} = \lim_{n\to\infty}\left(\int_0^{\varepsilon_n} \varphi(t)dv(t) + 3a_n\right) = 0$,

so that by Lemma 2.2 and the fact that $\int_0^{\varepsilon} \varphi(t)dv(t) > 0$, for each $\varepsilon > 0$ we have that $\lim_{n\to\infty}\int_0^{\|y_n-p\|} \varphi(t)dv(t) = 0$ from which it follows that $\lim_{n\to\infty}\|y_n - p\| = 0$, that is $\lim_{n\to\infty} y_n = p$.

Conversely, let $\lim_{n\to\infty} y_n = p$. Then, by the contractive condition (16), Lemma 2.3 and the triangle inequality again, we have

$$\int_0^{\varepsilon_n} \varphi(t)dv(t) = \int_0^{\|y_{n+1}-(1-\alpha_n)y_n-\alpha_n T y_n\|} \varphi(t)dv(t)$$
$$\leq \int_0^{\|y_{n+1}-p\|} \varphi(t)dv(t) + [1-(1-k)\alpha\int_0^{\|y_n-p\|} \varphi(t)dv(t) + 3a_n \to 0$$

as $n \to \infty$.

3.3 Theorem. Let (X, d) be a complete metric space and $T: E \to E$ having set valued mapping $T: X \to CL(X)$ Let p be a fixed point of T. For $x_0 \in X$, we have a Picard iteration $\{x_n\}_{n=0}^{\infty}$ defined as $x_{n+1} \in Tx_n, n = 0,1,2, \ldots$ If the mapping T satisfies the condition

$$\int_0^{H(Tx,Ty)} \varphi(t)dv(t) \leq k\emptyset\left(\int_0^{D(x,Tx)} \varphi(t)dv(t)\right)\int_0^{D(y,Tx)} \varphi(t)dv(t), \tag{20}$$

such that $\emptyset(0) = 1$ where $v, \emptyset : \mathbb{R}^+ \to \mathbb{R}^+$ be monotone increasing function and $\varphi: \mathbb{R}^+ \to \mathbb{R}^+$ a Lebesgue-Stieltjes integrable mapping which is summable, nonnegative and for each $> 0, \int_0^{\varepsilon} \varphi(t)dv(t) > 0$. Then, the Picard iteration process is T–stable.

Proof: Let $\{y_n\}_{n=0}^{\infty} \subset X \ and \ \varepsilon_n = H(y_{n+1}, Ty_n)$.

Assume $\lim_{n\to\infty} \varepsilon_n = 0$. Then we shall establish that $\lim_{n\to\infty} y_n = p$. Let $\{a_n\}_{n=0}^{\infty} \subset (0,1)$ Then, by Lemma 2.3

and condition (20), we have

$$\int_0^{d(y_{n+1},p)} \varphi(t)dv(t) \le d(y_{n+1},p) + a_n$$

$$\le H(y_{n+1},Ty_n) + D(Ty_n,p) + a_n$$

$$\le H(y_{n+1},Ty_n) + H(Ty_n,Tp) + a_n$$

$$\le \left(\int_0^{H(y_{n+1},Ty_n)} \varphi(t)dv(t)\right) + \int_0^{H(Ty_n,Tp)} \varphi(t)dv(t) + 3a_n$$

$$\le \int_0^{\varepsilon_n} \varphi(t)dv(t) + k\emptyset\left(\int_0^{D(p,Tp)} \varphi(t)dv(t)\right)\int_0^{D(Tp,y_n)} \varphi(t)dv(t) + 3a_n$$

$$\le \int_0^{\varepsilon_n} \varphi(t)dv(t) + k\emptyset(0)\int_0^{d(p,y_n)} \varphi(t)dv(t) + 3a_n$$

$$\le \int_0^{\varepsilon_n} \varphi(t)dv(t) + k\int_0^{d(p,y_n)} \varphi(t)dv(t) + 3a_n \tag{21}$$

We can now express (21) in the form $u_{n+1} \le \delta u_n + \varepsilon_n'$,

$$0 \le \delta = k < 1, \quad u_n = \int_0^{d(y_n,p)} \varphi(t)dv(t)$$

$$\varepsilon_n' = \int_0^{\varepsilon_n} \varphi(t)dv(t) + 3a_n,$$

$$\lim_{n\to\infty} \varepsilon_n' = \lim_{n\to\infty}\left(\int_0^{\varepsilon_n} \varphi(t)dv(t) + 3a_n\right) = 0,$$

so that by Lemma 2.2 and the fact that $\int_0^{\varepsilon} \varphi(t)dv(t) > 0$, for each $\varepsilon > 0$ we have that $\lim_{n\to\infty}\int_0^{d(y_n,p)} \varphi(t)dv(t) = 0$.

From this we can have, $\lim_{n\to\infty} y_n = p$.

Conversely, let $\lim_{n\to\infty} y_n = p$. Then, by the contractive condition (19), Lemma 2.3 and the triangle inequality, we have,

$$\int_0^{\varepsilon_n} \varphi(t)dv(t) = \int_0^{H(y_{n+1},Ty_n)} \varphi(t)dv(t)$$

$$\le \int_0^{d(y_{n+1},p)} \varphi(t)dv(t) + k\int_0^{D(p,Ty_n)} \varphi(t)dv(t) + 3a_n$$

$$\le \int_0^{d(y_{n+1},p)} \varphi(t)dv(t) + k\emptyset\left(\int_0^{D(p,Tp)} \varphi(t)dv(t)\right)\int_0^{D(Tp,y_n)} \varphi(t)dv(t) + 3a_n$$

$$\le \int_0^{d(y_{n+1},p)} \varphi(t)dv(t) + k\emptyset(0)\int_0^{D(Tp,y_n)} \varphi(t)dv(t) + 3a_n$$

$$\leq \int_0^{d(y_{n+1},p)} \varphi(t)dv(t) + k\int_0^{d(p,y_n)} \varphi(t)dv(t) + 3a_n$$
$$\to 0 \ as \ \ n \to \infty.$$

Thus, $\lim_{n\to\infty} \varepsilon_n = 0.$

Hence Proved.

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