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***-Ricci Solitons on Kenmotsu Manifold**

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Abstract

A Ricci soliton is a generalization of an Einstein metric. Theoretical physicists have also been looking into the equation of Ricci soliton in relation with string theory. In the present paper we characterize Kenmotsu manifolds admitting a special type of Ricci soliton, called *-Ricci soliton. The main Theorem of the paper states that if a Kenmotsu manifold M admit *-Ricci soliton then, M is either D-Homothetic to an Einstein manifold or the soliton vector field leaves ϕ invariant.

Key words: Ricci soliton, Kenmotsu manifold, *Einstein manifold*.

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Introduction

Ricci soliton is a natural generalization of an Einstein metric and is defined on a Riemannian manifold (M, g) . A Ricci soliton is a triple (g, V, λ) with g a Riemannian metric, V a vector field, and λ a real scalar such that

$$L_V g + 2S + 2\lambda g = 0 \quad (1.1)$$

where S is a Ricci tensor of M and L_V denotes the Lie derivative operator along the vector field V . The Ricci soliton is said to be shrinking, steady, and expanding accordingly as λ is negative, zero, and positive, respectively¹.

In 1972, Kenmotsu² studied a class of contact Riemannian manifolds satisfying some special conditions and this manifold is known as Kenmotsu manifolds. The authors in³⁻⁷ have studied Ricci solitons in contact and Lorentzian manifolds. G. Kaimakamis and K. Panagiotidou⁸ initiated the notion of *-Ricci soliton where they essentially modified the definition of Ricci soliton by replacing the Ricci tensor Ric in (1.1) with the *-Ricci

tensor Ric^* . A pseudo-Riemannian metric g on a smooth manifold M is called a $*$ -Ricci soliton if there exists a smooth vector field V , such that

$$\frac{1}{2}(\mathcal{L}_V g)(X, Y) + Ric^*(X, Y) = \lambda g(X, Y), \quad (1.2)$$

where

$$Ric^*(X, Y) = \frac{1}{2}(\text{trace}\{\phi, R(X, \phi Y)\}) \quad (1.3)$$

for all vector fields X, Y on M .

The notion of $*$ -Ricci tensor was first introduced by S. Tachibana⁹ on almost Hermitian manifolds and further studied by T. Hamada¹⁰ on real hypersurfaces of non-flat complex space forms.

In the present paper, we have studied $*$ -Ricci soliton on Kenmotsu manifold and prove the following result:

Theorem: Let $M(\phi, \xi, \eta, g)$ be a $(2n + 1)$ -dimensional Kenmotsu manifold. If g is a $$ -Ricci soliton on M , then either M is D-homothetic to an Einstein manifold, or the Ricci tensor of M with respect to canonical paracontact connection vanishes. In the first case, the soliton vector field is Killing and in the second case, the soliton vector field leaves ϕ invariant.*

Preliminaries:

Let M be an almost contact manifold equipped with an almost contact metric structure (ϕ, ξ, η, g) consisting of a $(1,1)$ tensor field ϕ , a vector field ξ , a 1-form η and a compatible Riemannian metric g satisfying

$$\phi = -I + \eta \otimes \xi, \quad \eta(\xi) = 1, \quad \phi(\xi) = 0, \quad \eta \circ \phi = 0, \quad (2.1)$$

$$g(X, Y) = g(\phi X, \phi Y) + \eta(X)\eta(Y) \quad (2.2)$$

$$g(X, \phi Y) = -g(\phi X, Y), \quad g(X, \xi) = \eta(X)$$

for all $X, Y \in \chi(M)$.

an almost contact metric manifold M is called a kenmotsu manifold if it satisfies²

$$(\nabla_X \phi)Y = g(\phi X, Y)\xi - \eta(Y)\phi X, \quad (2.3)$$

Where ∇ is Levi-Civita connection of the Riemannian metric g .

From the above equation it follows that

$$\nabla_X \xi = X - \eta(X)\xi, \quad (2.4)$$

$$(\nabla_X \eta)Y = g(X, Y) - \eta(X)\eta(Y) \quad (2.5)$$

Moreover, the curvature tensor R and Ricci tensor S satisfy²

$$R(\xi, X)Y = \eta(Y)X - g(X, Y)\xi, \quad R(\xi, X)\xi = X - \eta(X)\xi, \quad R(\xi, X)Y = \eta(Y)X - g(X, Y)\xi \quad (2.6)$$

Let (g, V, λ) be a Ricci soliton in an n -dimensional Kenmotsu manifold M ¹¹. From (2.4) we have

$$(\mathcal{L}_V g)(X, Y) = 2[g(X, Y) - \eta(X)\eta(Y)]$$

From (1.1) and with above relation we have¹⁵

$$S(X, Y) = -(\lambda + 1)g(X, Y) + \eta(X)\eta(Y)$$

Which yields

$$QX = -(\lambda + 1)X + \eta(X)\xi$$

$$S(X, \xi) = -\lambda\eta(X) \tag{2.7}$$

$$\text{Or } Q\xi = -\lambda\xi \tag{2.8}$$

for any X, Y on M .

Here, R denotes the curvature tensor of g and S denotes the Ricci tensor defined by $S(X, Y) = g(QX, Y)$, where Q is the Ricci operator.

Lemma 1. Let $M(\varphi, \xi, \eta, g)$ be a Kenmotsu manifold. Then

(i) $\nabla_{\xi}Q = 0$, and (ii) $(\nabla_X Q)\xi = Q\varphi X + \lambda\varphi X$.

Proof: Since ξ is Killing, we have $\mathcal{L}_{\xi}Ric = 0$. This implies $(\mathcal{L}_{\xi}Q)X = 0$ for any vector field X on M . From which it follows that

$$0 = \mathcal{L}_{\xi}(QX) - Q(\mathcal{L}_{\xi}X)$$

$$= \nabla_{\xi}QX + \nabla_{QX}\xi - Q(\nabla_{\xi}X) + Q(\nabla_X\xi)$$

$$= (\nabla_{\xi}Q)X + \nabla_{QX}\xi + Q(\nabla_X\xi).$$

Using (2.4) in the above equation gives $\nabla_{\xi}Q = Q\varphi - \varphi Q$. Since the Ricci operator Q commutes with φ on Kenmotsu manifold, we have (i). Next, taking covariant differentiation of (2.8) along an arbitrary vector field X on M and using (2.4), we obtain (ii). This completes the proof.

If the Ricci tensor of a Kenmotsu manifold M is of the form

$$Ric(X, Y) = Ag(X, Y) + B\eta(X)\eta(Y),$$

for any vector fields X, Y on M , where A and B being constants, then M is called an η -Einstein manifold.

The 1-form η is determined up to a horizontal distribution and hence $D = \text{Kern}\eta$ are connected by $\tilde{\eta} = \sigma\eta$ for a positive smooth function

σ on a paracontact manifold M . This paracontact form $\tilde{\eta}$ defines the structure tensor $(\tilde{\varphi}, \tilde{\xi}, \tilde{g})$ corresponding to η using the condition given in the paper ¹¹. We call the transformation of the structure tensors given by Lemma 4.1 of ¹¹ a gauge (conformal) transformation of paracontact pseudo-Riemannian structure. When σ is constant this is a D-homothetic transformation. Let $M(\varphi, \xi, \eta, g)$ be a paracontact manifold and

$$\tilde{\varphi} = \varphi, \tilde{\xi} = \frac{1}{\alpha}\xi, \tilde{\eta} = \alpha\eta, \tilde{g} = \alpha g + (\alpha^2 - \alpha)\eta \otimes \eta \quad \alpha = \text{const.} \neq 0$$

to be D-homothetic transformation. Then $(\tilde{\varphi}, \tilde{\xi}, \tilde{\eta}, \tilde{g})$ is also a paracontact structure. Using the formula appeared in ¹¹ for D-homothetic deformation, one can easily verify that if $M(\varphi, \xi, \eta, g)$ is a $(2n+1)$ -dimensional $(n > 1)$ η -Einstein Kenmotsu structure with scalar curvature $r \neq 2n$, then there exists a constant α such that $M(\tilde{\varphi}, \tilde{\xi}, \tilde{\eta}, \tilde{g})$ is an Einstein Kenmotsu structure. So we have following result.

Lemma 2. Any $(2n+1)$ -dimensional η -Einstein Kenmotsu manifold with scalar curvature not equal to $2n$ is D-homothetic to an Einstein manifold.

Proof of Theorem :

First, we state and prove some lemmas which will be used to prove Theorem.

Lemma 3. The $*$ -Ricci tensor on a $(2n + 1)$ -dimensional Kenmotsu manifold $M(\varphi, \xi, \eta, g)$ is given by

$$Ric^*(X, Y) = -Ric(X, Y) - (2n - 1)g(X, Y) - \eta(X)\eta(Y) \quad (3.1)$$

for any vector fields X, Y on M .

Proof: The Ricci tensor Ric of a $(2n + 1)$ -dimensional Kenmotsu manifold $M(\varphi, \xi, \eta, g)$ satisfies the relation (c.f. Lemma 3.15 in ¹¹):

$$Ric(X, Y) = \sum_{i=1}^{2n+1} R'(X, \phi Y, e_i, \phi e_i) - (2n - 1)g(X, Y) - \eta(X)\eta(Y) \quad (3.2)$$

for any vector fields X, Y on M . By the skew-symmetric property of φ , we have

$$\sum_{i=1}^{2n+1} R'(X, \phi Y, e_i, \phi e_i) = \sum_{i=1}^{2n+1} R(X, \phi Y, e_i, \phi e_i) = \sum_{i=1}^{2n+1} g(\phi R(X, \phi Y), e_i, e_i)$$

By this, (3.2) becomes

$$\sum_{i=1}^{2n+1} g(\phi R(X, \phi Y), e_i, e_i) = -2Ric(X, Y) - 2(2n - 1)g(X, Y) - 2\eta(X)\eta(Y) \quad (3.3)$$

By (1.3) and (3.3), we have (3.1).

Lemma 4. For a Kenmotsu manifold, we have the following relation

$$(\mathcal{L}_V \eta)(\xi) = -\eta(\mathcal{L}_V \xi) = \lambda. \quad (3.4)$$

Proof: By virtue of Lemma 3, the $*$ -Ricci soliton equation (1.2) can be expressed as

$$(\mathcal{L}_V g)(X, Y) = 2Ric(X, Y) + 2(2n - 1 + \lambda)g(X, Y) + 2\eta(X)\eta(Y). \quad (3.5)$$

Taking $Y = \xi$ in (3.5) and using (2.7) we have $(\mathcal{L}_V g)(X, \xi) = 2\lambda\eta(X)$. Lie-differentiating the equation $\eta(X) = g(X, \xi)$ along V and by (3.5), we have

$$(\mathcal{L}_V \eta)(X) - g(\mathcal{L}_V \xi, X) - 2\lambda\eta(X) = 0. \quad (3.6)$$

Now, Lie-derivative of $g(\xi, \xi) = 1$ along V and equation (3.6) completes proof.

Lemma 5. Let $M(\varphi, \xi, \eta, g)$ be a $(2n + 1)$ -dimensional Kenmotsu manifold. If g is a $*$ -Ricci soliton, then M is an η -Einstein manifold and the Ricci tensor can be written as

$$Ric(X, Y) = -\left[2n - 1 + \frac{\lambda}{2}\right]g(X, Y) + \left[\frac{\lambda}{2} - 1\right]\eta(X)\eta(Y) \quad (3.7)$$

for any vector fields X, Y on M .

Proof: Taking covariant differentiation of (3.5) along an arbitrary vector field Z , we get

$$(\nabla_Z \mathcal{L}_V g)(X, Y) = 2\{(\nabla_Z Ric)(X, Y) - g(X, \varphi Z)\eta(Y) - g(Y, \varphi Z)\eta(X)\}. \quad (3.8)$$

According to Yano¹², we have

$$(\mathcal{L}_V \nabla_Z g - \nabla_Z \mathcal{L}_V g - \nabla_{[V, Z]}g)(X, Y) = -g((\mathcal{L}_V \nabla)(Z, X), Y) - g((\mathcal{L}_V \nabla)(Z, Y), X),$$

for any vector fields X, Y, Z on M .

In view of the parallelism of the pseudo-Riemannian metric g , we have from above relation

$$(\nabla_Z \mathcal{L}_V g)(X, Y) = g((\mathcal{L}_V \nabla)(Z, X), Y) + g((\mathcal{L}_V \nabla)(Z, Y), X). \tag{3.9}$$

From (3.8) and (3.9), we have

$$\begin{aligned} &g((\mathcal{L}_V \nabla)(Z, X), Y) + g((\mathcal{L}_V \nabla)(Z, Y), X) \\ &= 2\{(\nabla_Z Ric)(X, Y) - g(X, \varphi Z)\eta(Y) - g(Y, \varphi Z)\eta(X)\}. \end{aligned} \tag{3.10}$$

Which gives

$$\begin{aligned} g((\mathcal{L}_V \nabla)(X, Y), Z) &= -(\nabla_Z Ric)(X, Y) + (\nabla_X Ric)(Y, Z) \\ &\quad + (\nabla_Y Ric)(Z, X) + 2g(X, \varphi Z)\eta(Y) \\ &\quad + 2g(Y, \varphi Z)\eta(X). \end{aligned} \tag{3.11}$$

Taking ξ in place of Y in (3.11) and Lemma 1, we get

$$(\mathcal{L}_V \nabla)(X, Y) = 2(2n - 1)\varphi X + 2Q\varphi X. \tag{3.12}$$

Differentiating (3.12) covariantly along an arbitrary vector field Y on M and using the relations (2.3) and (2.8), we have

$$\begin{aligned} &(\nabla_Y \mathcal{L}_V \nabla)(X, \xi) + (\mathcal{L}_V \nabla)(X, \varphi Y) \\ &= 2\{(\nabla_Y Q)\varphi X + \eta(X)QY + (2n - 1)\eta(X)Y + g(X, Y)\xi\}. \end{aligned} \tag{3.13}$$

According to Yano¹² we have

$$(\mathcal{L}_V R)(X, Y)Z = (\nabla_X \mathcal{L}_V \nabla)(Y, Z) - (\nabla_Y \mathcal{L}_V \nabla)(X, Z). \tag{3.14}$$

Taking ξ in place of Z in (3.14) and by (3.13), we have

$$\begin{aligned} &(\mathcal{L}_V R)(X, Y)\xi + (\mathcal{L}_V \nabla)(Y, \varphi X) - (\mathcal{L}_V \nabla)(X, \varphi Y) \\ &= 2\{(\nabla_X Q)\varphi Y - (\nabla_Y Q)\varphi X + \eta(Y)QX - \eta(X)QY \\ &\quad + (2n - 1)(\eta(Y)X - \eta(X)Y)\}. \end{aligned} \tag{3.15}$$

Taking ξ for Y in (3.15), then using (2.8), (3.12) and Lemma 1, we have $(\mathcal{L}_V R)(X, \xi)\xi = 4\{QX + (2n - 1)X + \eta(X)\xi\}$. (3.16)

Taking Lie-derivative of (2.6) along V and by (2.5) and (3.4) we have

$$(\mathcal{L}_V R)(X, \xi)\xi = (\mathcal{L}_V \eta)(X)\xi - g(\mathcal{L}_V X, \xi) - 2\lambda X. \tag{3.17}$$

Comparing (3.16) with (3.17), and use of (3.6), gives the required result.

Proof of Theorem: By (3.7), the soliton equation (3.5) can be written as

$$(\mathcal{L}_V g)(X, Y) = \lambda \{g(X, Y) + \eta(X)\eta(Y)\}. \tag{3.18}$$

Taking Lie-differentiation of (3.7) along the vector field V and using (3.5) we have

$$\begin{aligned} (\mathcal{L}_V Ric)(X, Y) &= \left(\frac{\lambda}{2} - 1\right) \{ \eta(Y)(\mathcal{L}_V \eta)(X) + \eta(X)(\mathcal{L}_V \eta)(Y) \} \\ &\quad - \left[\frac{\lambda}{2} + 2n - 1\right] \lambda \{g(X, Y) + \eta(X)\eta(Y)\} \end{aligned} \tag{3.19}$$

Differentiating (3.7) covariantly along an arbitrary vector field Z on M and using (2.4) we have

$$(\nabla_Z Ric)(X, Y) = \left(1 - \frac{\lambda}{2}\right) \{g(X, \phi Z)\eta(Y) + g(Y, \phi Z)\eta(X)\} \quad (3.20)$$

By (3.20), equation (3.11) becomes

$$(\mathcal{L}_V \nabla)(X, Y) = -\lambda \{\eta(Y) \phi X + \eta(X) \phi Y\}. \quad (3.21)$$

Differentiating (3.21) covariantly along an arbitrary vector field Z on M and by (2.3) and (2.4), we have

$$(\nabla_Z \mathcal{L}_V \nabla)(X, Y) = \lambda \{g(Y, \phi Z)\phi X + g(X, \phi Z)\phi Y + g(X, Z)\eta(Y)\xi \\ + g(Y, Z)\eta(X)\xi - 2\eta(X)\eta(Y)Z\}. \quad (3.22)$$

Using (3.22) in (3.14) and using (2.4) we have

$$(\mathcal{L}_V R)(X, Y)Z = \lambda \{g(\phi X, Z)\phi Y - g(\phi Y, Z)\phi X + 2g(\phi X, Y)\phi Z \\ + g(X, Z)\eta(Y)\xi - g(Y, Z)\eta(X)\xi - 2\eta(Y)\eta(Z)X \\ + 2\eta(X)\eta(Z)Y\}. \quad (3.23)$$

Contracting (3.23) over Z , we get

$$(\mathcal{L}_V Ric)(Y, Z) = 2\lambda \{g(Y, Z) - (2n+1)\eta(Y)\eta(Z)\}. \quad (3.24)$$

By (3.19) and (3.24), we have

$$\left(\frac{\lambda}{2} - 1\right) \{\eta(Y)(\mathcal{L}_V \eta)(Z) + \eta(Z)(\mathcal{L}_V \eta)(Y)\} - \left[\frac{\lambda}{2} + 2n - 1\right] \lambda \{g(Y, Z) + \eta(Y)\eta(Z)\} \\ = 2\lambda \{g(Y, Z) - (2n+1)\eta(Y)\eta(Z)\} \quad (3.25)$$

Replacing Y by $\phi^2 Y$ in (3.25) and then using (2.1) and (3.4) we get

$$\left(\frac{\lambda}{2} - 1\right) \left\{ (\mathcal{L}_V \eta)(Y)\eta(Z) = \lambda \left[1 + 2n + \frac{\lambda}{2} \right] g(Y, Z) - 2n\lambda \eta(Y)\eta(Z) \right\} \quad (3.26)$$

By (3.26) and (3.25) and then replacing Z by ϕZ , we have

$$\lambda \left[1 + 2n + \frac{\lambda}{2} \right] g(Y, \phi Z) = 0 \quad (3.27)$$

As $\phi(Y, Z) = g(Y, \phi Z)$ is non-vanishing everywhere on M , so either $\lambda = 0$ or $\lambda = -2(2n+1)$.

Case I: If $\lambda = 0$, from (3.18) we have $\mathcal{L}_V g = 0$, therefore, V is Killing. From (3.7) we have

$$Ric(X, Y) = -(2n-1)g(X, Y) - \eta(X)\eta(Y). \quad (3.28)$$

Contracting the equation (3.28) we have $r = -4n^2$, where r is the scalar curvature of the manifold M . This shows that M is a η -Einstein manifold with scalar curvature $r \neq 2n$. So, M is D-homothetic to an Einstein manifold.

Case II: If $\lambda = -2(2n+1)$, then taking ξ in place of Z in (3.26) and then replace Y by ϕY , the resulting equation gives

$$\left(\frac{\lambda}{2} - 1\right) (\mathcal{L}_V \eta)(\phi Y) = 0.$$

Since $\lambda = -2(2n+1)$, we have $\lambda \neq 2$. Thus we have $(\mathcal{L}_V \eta)(\phi Y) = 0$.

Replacing Y by φY and using (2.1), we have

$$(\mathcal{L}_V \eta)(Y) = -2(2n+1)\eta(X). \quad (3.29)$$

Taking exterior differentiation d on (3.29) we have

$$(\mathcal{L}_V d\eta)(X, Y) = -2(2n+1)g(X, \varphi Y), \quad (3.30)$$

as d commutes with \mathcal{L}_V .

Taking the Lie-derivative of $d\eta(X, Y) = g(X, \varphi Y)$ along the soliton vector field V provides

$$(\mathcal{L}_V d\eta)(X, Y) = (\mathcal{L}_V g)(X, \varphi Y) + g(X, (\mathcal{L}_V \varphi)Y). \quad (3.31)$$

From (3.18) we have

$$(\mathcal{L}_V g)(X, \varphi Y) = -2(2n+1)g(X, \varphi). \quad (3.32)$$

Using (3.30) and (3.32) in (3.31) we have $\mathcal{L}_V \varphi = 0$. Therefore, soliton vector field V leaves φ invariant.

Putting $\lambda = -2(2n+1)$ in (3.7) we have

$$Ric(X, Y) = 2g(X, Y) - (2n+2)\eta(X)\eta(Y). \quad (3.33)$$

Contracting (3.33) we obtain $r = 2n$ (i.e., the manifold M cannot be D-homothetic to an Einstein manifold).

Ricci tensor \widetilde{Ric} of a $(2n+1)$ dimensional Kenmotsu manifold with respect to canonical paracontact connection $\widetilde{\nabla}$ is defined as ¹¹

$$\widetilde{Ric}(X, Y) = Ric(X, Y) - 2g(X, Y) + (2n+2)\eta(X)\eta(Y) \quad (3.34)$$

Using (3.33) in (3.34) we have $\widetilde{Ric}(X, Y) = 0$. Therefore, the Ricci tensor with respect to the connection $\widetilde{\nabla}$ vanishes. This completes the proof of theorem.

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