Gravitational and Electromagnetic Field of an Isolated Proton

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Abstract

The proton is a positively charged massive particle. Therefore it has both gravitational field and electromagnetic field. So, in this work special attention is given to Einstein's gravitational field equations and Maxwell's electromagnetic field equations. The four dimensional metric tensor is used here to solve this problem. The four-dimensional line element is developed in such a way that the metric is solved in two separate ways. The solutions for two fields give some suitable results. Later the formula is extended for the large massive body and gives an interesting result that life cannot survive in a planet of mass just greater than 1.21 time mass of Jupiter planet.

Key words: line element, gravitational field, e-m field, energy momentum tensor, e-m field tensor,

Introduction

Electromagnetic field equations were first unified by Maxwell[1] in the year 1861. Gravitational field equations were first established by Einstein[2] in 1916 publishing the general theory of relativity. The solution of the field equations in empty space was first given by Schwarzschild[3] that was later understood to describe a black hole[4] and in 1963 Kerr[5,6] generalized the solution to rotating black hole. The model of universe was first given by Einstein on the development of his general theory of relativity that later with de-Sitter[7] and finally describe the non-static, isotropic and homogeneous model by Freidman [8] in 1922 as well as by Robertson[9] and Walker[10,11] in 1935 known as FRW model.

During the first three decade of the twentieth century, gravitation and electromagnetism were only two known fundamental forces. After general relativity T Kaluza[12] in 1919 and later in 1926 Oskar Klein[13] tried to unify the relativity as a geometrical theory of gravity and electromagnetic (e-m) fields. The gravitational field due to a charged particle or an electron was first given by Gunnar Nordström[14] and then by G. B. Jeffery[15] in

1921. But their theory failed to establish a correct relation between the electromagnetic field and the gravitational field. Later regarding this problem, a few articles[16-20] published. But still it is an unsolved problem. The author of this paper tries to establish a relation between gravitational field and e-m field.

Mathematical derivation :

         Here considering an isolated proton which has both mass and positive charge. Hence at every state it is combination of two fundamental forces: gravitational force plus e-m force. The charge and mass of the proton considered as +e and m_p. The range of e-m interaction is from infinity to 10^{-8} cm and for gravitational interaction range is from infinity to 10^{-33} cm . The e-m force is responsible for the binding of atoms and mainly it governs all known phenomena of life on earth.

         Let the first attempt to find out the gravitational potential and e-m potential for an isolated proton at rest at origin of our system of coordinates. The most general solution of Einstein gravitational field equations was given by first Schwarzschild for a massive isolated particle situated at origin of the frame of reference.

         The presence of the mass point would modify the line element. However since mass is static and isolated, the line element would be spatially spherically symmetric about the point mass and is static. The most general form of such line element may be expressed as

$$ds^2 = -e^\lambda dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta \, d\phi^2 + e^v dt^2 \tag{1}$$

Here λ and v are the functions of *r* and at infinity λ, v becomes zero.

         Since the proton is at origin of the frame of reference. Hence mass and charge both at origin in the coordinate system. Both forces are inversely proportional to distance. The Einstein field tensor or world tensor is same for all kinds of field. Therefore the author has considered the field equation for electromagnetic field,

$$R_{\mu v} - \frac{1}{2} g_{\mu v} R = -k T_{\mu v} \tag{2}$$

Where k is a constant and is related with the electromagnetic constant. The symbol $T_{\mu v}$ represents the electromagnetic energy momentum tensor like the material energy momentum tensor. But for empty space i.e. no charge in surrounding space the equation (2) becomes,

$$R_{\mu v} = 0 \tag{3}$$

Therefore for e-m field the most general form of line element may be expressed as

$$ds^2 = -e^a dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta \, d\phi^2 + e^b dt^2 \tag{4}$$

Here '*a*' and '*b*' are the functions of '*r*' and at infinity '*a*', '*b*' becomes zero.
Since the same proton gives both e-m and gravitational fields. Hence the line element of the particle is the combination of both (1) and (4) line elements. So

$$ds^2 = -\frac{1}{2}\left[e^a + e^\lambda\right]dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta \, d\phi^2 + \frac{1}{2}\left[e^b + e^v\right]dt^2 \tag{5}$$

The solution of equation (1) for gravitational field in empty space for an isolated particle was given by Schwarzschild as

$$e^\lambda = \left[1 - \frac{2m}{r}\right]^{-1}; \quad e^v = \left[1 - \frac{2m}{r}\right] \tag{6}$$

Here *m* is related with the mass of the particle.

Considering the line element(4) the coordinates are,

$$x^1 = r, x^2 = \theta, x^3 = \phi, x^4 = t.$$

$$g_{11} = -e^a, \quad g_{22} = -r^2, \quad g_{33} = -r^2 \sin^2\theta, \quad g_{44} = e^b$$
$$g^{11} = -e^{-a}, \quad g^{22} = -1/r^2, \quad g^{33} = -1/(r^2\sin^2\theta), \quad g^{44} = e^{-b}$$

Gives,

$$g = |g_{\mu\nu}| = -r^4 \sin^2\theta \ e^{(a+b)}$$
$$\sqrt{-g} = r^2 \sin\theta \ e^{(a+b)}$$

In a similar method the solution of equation (4) is

$$e^a = \left[1 + \frac{B}{r}\right]^{-1}; \quad e^b = \left[1 + \frac{B}{r}\right] \tag{7}$$

Here 'B' is a constant connected with electric charge.

The Maxwell Lorentz equations for e-m field for empty space.

$$\nabla. \vec{E} = 4\pi\rho$$

$$\nabla. \vec{H} = 0$$

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{H}}{\partial t}$$

$$\nabla \times \vec{H} = \frac{1}{c}\frac{\partial \vec{E}}{\partial t} + \frac{4\pi}{c}\vec{J}$$

Where $\vec{E}, \vec{H}, \vec{J}$ respectively denotes electric field intensity, magnetic field intensity, current density. In addition, $\vec{J} = \rho \vec{u}$ if electric charge of density ρ is moving with velocity $\vec{u}$. The field is purely electrostatic and hence the magnetic field intensities are,

$$H_x, H_y, H_z = 0 \tag{8}$$

Now define the general potential K^μ in terms of e-m potential and scalar potential ϕ as,

$$K^\mu = (A_x, A_y, A_z, \Phi)$$

The associate covariant vector K_μ of K^μ is defined as,

$$K_\mu = g_{\mu\nu} K^\nu = g_{\mu\mu} K^\mu; \quad \text{since} \ g_{\mu\nu} = 0 \ \text{for} \ \mu \neq \nu$$

$$K_\mu = (-A_x, -A_y, -A_z, \Phi)$$

For this reason one can write,

As a result the e-m field tensor $F_{\mu\nu}$ can be written as,

$$F^{\mu\nu} = K_{\mu,\nu} - K_{\nu,\mu} = \frac{\partial K_\mu}{\partial x^\nu} - \frac{\partial K_\nu}{\partial x^\mu}$$

(9)

and $\vec{H} = \nabla \times \vec{A}$

In view of this equation and equation (9) gives,

$$A_x, A_y, A_z = 0$$

This means that vanishing of magnetic field intensity implies as vanishing of e-m vector potential. The above argument gives that Φ is a function of 'r' only.

Therefore using (9) we can write,

$$F_{12}, \; F_{23}, \; F_{31}, \; F_{24}, \; F_{34} = 0 \;\; \text{and} \; F_{14} = -\frac{\partial \Phi}{\partial r}$$

(10)

This implies that the only non-vanishing component of $F_{\mu\nu}$ is F_{14} and $F_{14} = -F_{41}$

The current density J^μ can be written as,

$$J^\mu = F_\nu^{\mu\nu} = \frac{\partial F^{\mu\nu}}{\partial x^\nu} + F^{\alpha\nu} \Gamma^\mu_{\alpha\nu} + F^{\mu\alpha} \Gamma^\nu_{\alpha\nu}$$

The value of $F^{\alpha\nu} \Gamma^\mu_{\alpha\nu} = 0$ and we get,

$$\sqrt{-g} \, J^\mu = \frac{\partial}{\partial x^\nu}(\sqrt{-g} \; F^{\mu\nu})$$

This gives us,

$$\sqrt{-g} \, \rho = \frac{\partial}{\partial r}(\sqrt{-g} \; F^{41})$$

But there is no charge and no current in the space surrounding the proton at origin. Therefore,

$$\frac{\partial}{\partial r}(\sqrt{-g} \; F^{41}) = 0$$

(11)

Furthermore,

$$F^{41} = g^{44}g^{11}F_{41} = -F_{41}$$
$$\text{and } F^{14} = g^{11}g^{44}F_{14} = -F_{14}$$

Using above relations in equation (12) gives,

$$e^{-(a+b)/2}r^2 \frac{\partial \phi}{\partial r} = \text{constant} = \varepsilon \;\; \text{(say)}$$

Here ε being an absolute constant. This gives,

$$F_{14} = -F_{41} = E_x = E_r = -\frac{\partial \Phi}{\partial r} = \frac{\varepsilon}{r^2}e^{(a+b)/2}$$

(12)

Since in equation (12) gives $b = -a$, so the exponential terms becomes 1 and equation (12) represents

electromagnetic force. This means that $\varepsilon = q/4\pi$. Here q represents the charge of the proton. Now require a

covariant expressionfor the e-m energy momentum tensor T_μ^ν. But for empty space i.e. there is no charge

surrounding space of the proton then the term,

$$T_\nu^\mu = 0 \tag{13}$$

Therefore in equation (7) we can put $B = 2\varepsilon$, this is done in order to facilitate the physical interpretation of ε as the charge of the particle. Therefore,

$$e^a = \left[1 + \frac{2\varepsilon}{r}\right]^{-1} ; \quad e^b = \left[1 + \frac{2\varepsilon}{r}\right] \tag{14}$$

Therefore we can write,

$$\frac{1}{2}(e^b + e^\nu) = \left(1 + \frac{\varepsilon}{r} - \frac{m}{r}\right) \tag{15}$$

And,

$$\frac{1}{2}(e^a + e^\lambda) = \left[\frac{\left(1 + \dfrac{\varepsilon}{r} - \dfrac{m}{r}\right)}{\left(1 - \dfrac{2m}{r}\right)\left(1 + \dfrac{2\varepsilon}{r}\right)}\right] \tag{16}$$

Hence the line element (5) becomes,

$$ds^2 = -\left[\frac{\left(1 + \dfrac{\varepsilon}{r} - \dfrac{m}{r}\right)}{\left(1 - \dfrac{2m}{r}\right)\left(1 + \dfrac{2\varepsilon}{r}\right)}\right]dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta\, d\phi^2 + \left[1 + \frac{\varepsilon}{r} - \frac{m}{r}\right]dt^2 \tag{17}$$

This is the gravitational and e-m field equation for a proton. The solution becomes singular at $r = 0$; but this singularity also occurs in Newton's theory. However in this metric $r = 2m$ shows also singularity like in Schwarzschild solution. But in equation (17) e-m potential is greater than gravitational potential. Therefore e-m potential can also space-time curved but in opposite of gravitational potential. If the gravity creates concave space-time than e-m potential due to proton creates convex space-time as ε is positive. But at $m = \varepsilon$ gives the flat space time. Hence the time component t within the range of e-m interaction depends up on the combine effect of e-m and gravitational field.

Evaluation of ε and m

To evaluate the value of ε comparing the equation (14) with Newtonian potential gives,

$$\varepsilon = \frac{Kq}{c^2} \tag{18}$$

Therefore,

$$e^b = e^{-a} = \left(1 + \frac{2\varepsilon}{r}\right) = \left(1 + \frac{2Kq}{rc^2}\right) \tag{19}$$

In case of equation (6) the value of m can be found from Newton's potential as,

$$m = \frac{Gm_p}{c^2} \tag{20}$$

So,

$$e^\nu = e^{-\lambda} = \left(1 - \frac{2m}{r}\right) = \left(1 - \frac{2Gm_p}{rc^2}\right) \tag{21}$$

Here G and m_p are the gravitational constant and mass of the proton.

Hence equation (15) and (16) becomes

$$\frac{1}{2}\left(e^b + e^\nu\right) = g_{44} = \left[1 + \frac{Kq}{rc^2} - \frac{Gm_p}{rc^2}\right] \tag{22}$$

And,

$$\frac{1}{2}\left(e^a + e^\lambda\right) = g_{11} = \left[\frac{\left(1 + \dfrac{Kq}{rc^2} - \dfrac{Gm_p}{rc^2}\right)}{\left(1 - \dfrac{2Gm_p}{rc^2}\right)\left(1 + \dfrac{2Kq}{rc^2}\right)}\right] \tag{23}$$

Let the following values are considered for the constants:

$$\left. \begin{array}{l} q = 1.6 \times 10^{-19} C \\[4pt] K = 9 \times 10^{18} dyne\text{-}cm^2/C^2 \\[4pt] G = 6.670 \times 10^{-8} dyne\text{-}cm^2/gm^2 \\[4pt] m_p = 1.67265 \times 10^{-24} gm \end{array} \right\} \tag{24}$$

Putting the above values in equation (18) and (20) gives,

$$\frac{\varepsilon}{m} = \frac{Kq}{Gm_p} = 1.2907 \times 10^{31} \tag{25}$$

The e-m potential is stronger than 1.29×10^{31} times of gravitational potential.

Let us consider another proton comes nearer to the origin particle up to distance 'r' and interacts both electrically and gravitationally. In the above equations we have used gravitational potential energy given by Newton's law since the forces are static and weak. Therefore we have extended the Newton's law in above equations for two protons also.

$$g_{44} = \left[1 + \frac{Kq^2}{rc^2} - \frac{Gm_p m_p}{rc^2} \right] \tag{26}$$

And

$$g_{11} = \left[\frac{\left(1 + \dfrac{Kq^2}{rc^2} - \dfrac{Gm_p m_p}{rc^2} \right)}{\left(1 - \dfrac{2Gm_p m_p}{rc^2} \right) \left(1 + \dfrac{2Kq^2}{rc^2} \right)} \right] \tag{27}$$

In equations (26), (27) the gravitational potential energy is very weak then e-m potential energy. Let we consider isolated particle at rest in origin is a massive body $M = Nm_p$, $(N = 1,2,3,.....\infty)$ which is nothing but the combination of protons. As number of proton increases the mass of the body increases and gravitational force increases. Hence using in equations (26) and (27),

$$g_{44} = \left[1 + \frac{1}{rc^2}(Kq^2 - GMm_p) \right] \tag{28}$$

And

$$g_{11} = \left[\frac{\left\{ 1 + \dfrac{1}{rc^2}(Kq^2 - GMm_p) \right\}}{\left(1 - \dfrac{2GMm_p}{rc^2} \right) \left(1 + \dfrac{2Kq^2}{rc^2} \right)} \right] \tag{29}$$

Results and Discussion

From equation (24) or (25) for flat space-time

$$(Kq^2 - GMm_p) = 0 \tag{30}$$

This gives,

$$M = \frac{Kq^2}{Gm_p} \tag{31}$$

Putting the values from (24) the mass required for flat space time or to stop e-m interaction is equal to $M_{em} = 2.0667735 \times 10^{12} \, gm$.

The values of this 'M_{em}' is so large that cannot exist within the range $r(= 10^{-8} \, cm)$. Density will be very high; hence cannot consider such massive particle. Therefore the author has considered $M'(= \sum_{1}^{N'} m_p = N'm_p)$ such as mass M' is required to stop the e-m interaction and R is considered as the radius

of the massive body.

Now to determine the values of M' one can write,

$$\frac{GM}{r} = \frac{GM'}{R}$$

This gives for e-m interaction

$$M' = M\left(\frac{R}{r}\right) = \frac{Kq^2}{Gm_p}\left(\frac{R}{r}\right) \tag{32}$$

Number of proton contains in mass M is $N (= M / m_p)$ and r is interacting range or atomic radius then volume for N atoms is

$$V = \frac{M}{m_p} \times \frac{4}{3}\pi\, r^3 \tag{33}$$

Therefore density is

$$\rho = \frac{M}{V} = \frac{m_p}{(4/3)\pi\, r^3} \tag{34}$$

Now $\qquad M' = \frac{4}{3}\pi\, R^3 \rho = m_p\left(\frac{R}{r}\right)^3 \tag{35}$

Equating (32) with (35)

$$\frac{R}{r} = \frac{1}{m_p}\left(\frac{Kq^2}{G}\right)^{(1/2)} \tag{36}$$

Putting (36) in equation (35)

$$M' = (1/m_p^2)\left(\frac{Kq^2}{G}\right)^{(3/2)} \tag{37}$$

Using the values of q, K, G, m_p from (24) in (37) to stop e-m interaction between two protons putting as

$M' = M'_{em}$,

$$M'_{em} = 2.29701 \times 10^{30}\, gms = 0.00116\, M_0 \tag{38}$$

Here $M_0 = 1.99 \times 10^{33}$ gm is the mass of sun.

Conclusions

To stop e-m interaction or for flat space time the required mass is $0.00116\, M_0$. The mass of Jupiter planet is 1.898×10^{30} gms and the mass required to stop e-m interaction is just 1.21 times greater than

Jupiter's mass. So in any planet above this mass life cannot survive, because in that planet e-m interaction will be stopped by the gravity. Since the life is nothing but the low energy level e-m interaction. The equation (17) shows that time coefficient may vary due to both gravitational and electromagnetic field. The equation (17) is for a non rotating body. For rotating celestial bodies the above results will be slight differ.

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