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Computation of Nirmala Indices of Some Chemical Networks

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Abstract

Chemical graph theory is a branch of graph theory whose focus of interest is to finding topological indices of chemical graphs which correlate well with chemical properties of the chemical molecules. In this paper, we compute the Nirmala index, first and second inverse Nirmala indices for some chemical networks like silicate networks, chain silicate networks, hexagonal networks, oxide networks and honeycomb networks along with their comparative analysis.

Keywords: Nirmala index, first and second inverse Nirmala indices, chemical networks.
Mathematics Subject Classification: 05C09, 05C92, 92E10.

1. Introduction:

In this paper, we consider finite, simple, connected graphs. Let $G = (V, E)$ be a graph. The degree $d_G(v)$ of a vertex v is the number of vertices adjacent to v. The edge connecting the vertices u and v will be denoted by uv. We refer to[1] for undefined term and notation.

A molecular graph or a chemical graph is a simple graph related to the structure of a chemical compound. Each vertex of this graph represents an atom of the molecule and its edges to the bonds between atoms. A topological index is a numerical parameter mathematically derived from the graph structure. Several topological indices[2] are useful for establishing correlation between the structure of a molecular compound and its physicochemical properties,[3,4,5].

In[6], the Nirmala index and Nirmala exponential of a molecular graph G were introduced and they are defined as

$$N(G) = \sum_{uv \in E(G)} \sqrt{d_G(u) + d_G(v)} \quad \text{and} \quad N(G,x) = \sum_{uv \in E(G)} x^{\sqrt{d_G(u)+d_G(v)}}.$$

In[7], the first and second inverse Nirmala indices of G were introduced and they are defined as

$$IN_1(G) = \sum_{uv \in E(G)} \left[\frac{1}{d_G(u)} + \frac{1}{d_G(v)}\right]^{\frac{1}{2}} \quad \text{and} \quad IN_2(G) = \sum_{uv \in E(G)} \left[\frac{1}{d_G(u)} + \frac{1}{d_G(v)}\right]^{-\frac{1}{2}}.$$

Recently, some Nirmala indices were studied in[8, 9, 10]. Many other topological indices were studied, for example, in [11, 12, 13, 14, 15, 16, 17, 18].

In this paper, we compute the Nirmala index, inverse Nirmala indices for some chemical networks. For more details on some chemical network, we refer to[19].

2. Results for Silicate Networks :

Silicates are obtained by fusing metal oxides or metal carbonates with sand. A silicate network is symbolized by SL_n, where n is the number of hexagons between the center and boundary of SL_n. A silicate network of dimension two is depicted in Figure 1.

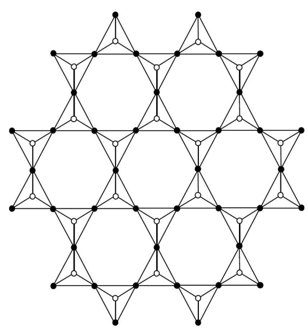

Figure 1. Silicate network of dimension two

In the following theorem, we compute the Nirmala index and its exponential of SL_n.

Theorem 1. Let SL_n be the family of silicate networks. Then

(i) $N(SL_n) = (1+\sqrt{2})54n^2 + (6\sqrt{6} + 18 - 36\sqrt{2})n.$

(ii) $N(SL_n, x) = 6nx^{\sqrt{6}} + (18n^2 + 6n)x^3 + (18n^2 - 12n)x^{3\sqrt{2}}.$

Proof: Let G be the graph of a silicate network SL_n with $|V(SL_n)| = 15n^2 + 3n$ and $|E(SL_n)| = 36n^2$. By algebraic method, in SL_n there are three types of edges based on the degrees of end vertices of each edge as follows:

$E_6 = \{uv \in E(G) \mid d_G(u) = d_G(v) = 3\}$, $|E_6| = 6n$.

$E_9 = \{uv \in E(G) \mid d_G(u) = 3, d_G(v) = 6\}, |E_9| = 18n^2 + 6n.$
$E_{12} = \{uv \in E(G) \mid d_G(u) = d_G(v) = 6\}, |E_{12}| = 18n^2 - 12n.$

By using the definitions and cardinalities of the edge partition of SL_n, we deduce

(i) $N(SL_n) = \sum_{uv \in E(G)} \sqrt{d_G(u) + d_G(v)} = (3+3)^{\frac{1}{2}} 6n + (3+6)^{\frac{1}{2}} (18n^2 + 6n) + (6+6)^{\frac{1}{2}} (18n^2 - 12n).$

After simplification, we get the desired result.

(ii) $N(SL_n, x) = \sum_{uv \in E(G)} x^{\sqrt{d_G(u)+d_G(v)}} = 6nx^{(3+3)^{\frac{1}{2}}} + (18n^2 + 6n)x^{(3+6)^{\frac{1}{2}}} + (18n^2 - 12n)nx^{(6+6)^{\frac{1}{2}}}.$

After simplification, we get the desired result.

In the following theorem, we compute the first and second inverse Nirmala indices of $SL_{n..}$

Theorem 2. Let $SL_{n.}$ be the family of silicate networks. Then

(i) $$IN_1(SL_n) = \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}}\right) 18n^2 + \left(\frac{\sqrt{2}}{\sqrt{3}} + \frac{1}{\sqrt{2}} - \frac{2}{\sqrt{3}}\right) 6n.$$

(ii) $$IN_2(SL_n) = \left(\sqrt{2} + \sqrt{3}\right) 18n^2 + \left(\frac{\sqrt{3}}{\sqrt{2}} + \sqrt{2} - 2\sqrt{3}\right) 6n.$$

Proof: From definitions and by cardinalities of the edge partition of SL_n, we deduce

(i) $IN_1(SL_n) = \sum_{uv \in E(G)} \left[\frac{1}{d_G(u)} + \frac{1}{d_G(v)}\right]^{\frac{1}{2}} = \left(\frac{1}{3} + \frac{1}{3}\right)^{\frac{1}{2}} 6n + \left(\frac{1}{3} + \frac{1}{6}\right)^{\frac{1}{2}} (18n^2 + 6n) + \left(\frac{1}{6} + \frac{1}{6}\right)^{\frac{1}{2}} (18n^2 - 12n)$

After simplification, we get the desired result

(ii) $IN_2(SL_n) = \sum_{uv \in E(G)} \left[\frac{1}{d_G(u)} + \frac{1}{d_G(v)}\right]^{-\frac{1}{2}} = \left(\frac{1}{3} + \frac{1}{3}\right)^{-\frac{1}{2}} 6n + \left(\frac{1}{3} + \frac{1}{6}\right)^{-\frac{1}{2}} (18n^2 + 6n) + \left(\frac{1}{6} + \frac{1}{6}\right)^{-\frac{1}{2}} (18n^2 - 12n).$

After simplification, we get the desired result.

3. Results for Chain Silicate Networks :

We now consider a family of chain silicate networks. This network is symbolized by CS_n and is obtained by arranging n tetrahedral linearly, see Figure 2.

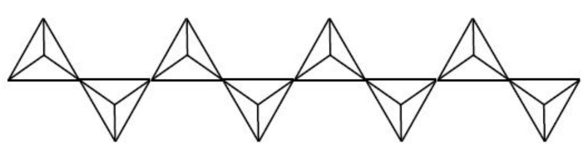

Figure 2. Chain silicate network

In the following theorem, we compute the Nirmala index and its exponential of $CS_{n.}$.

Theorem 3. Let CS_n be the family of chain silicate networks. Then

(i) $N(CS_n) = \left(\sqrt{6} + 12 + 3\sqrt{2}\right)n + 4\sqrt{6} + 6 - 6\sqrt{2}.$

(ii) $N(CS_n, x) = (n+4)x^{\sqrt{6}} + (4n-2)x^3 + (n-2)nx^{3\sqrt{2}}.$

Proof: Let G be the graph of chain silicate networks CS_n with $|V(CS_n)| = 3n + 1$ and $|E(CS_n)| = 6n$. By algebraic method, in CS_n, $n \geq 2$, there are three types of edges based on the degree of the vertices of each edge as follows:

$E_6 = \{uv \in E(G) \mid d_G(u) = d_G(v) = 3\}$, $|E_6| = n + 4$.
$E_9 = \{uv \in E(G) \mid d_G(u) = 3, d_G(v) = 6\}$, $|E_9| = 4n - 2$.
$E_{12} = \{uv \in E(G) \mid d_G(u) = d_G(v) = 6\}$, $|E_{12}| = n - 2$.

By using the definitions and cardinalities of the edge partition of SL_n , we deduce

(i) $$N(CS_n) = \sum_{uv\in E(G)} \sqrt{d_G(u) + d_G(v)} = (3+3)^{\frac{1}{2}}(n+4) + (3+6)^{\frac{1}{2}}(4n-2) + (6+6)^{\frac{1}{2}}(n-2).$$

After simplification, we get the desired result.

(ii) $$N(CS_n, x) = \sum_{uv\in E(G)} x^{\sqrt{d_G(u)+d_G(v)}} = (n+4)x^{(3+3)^{\frac{1}{2}}} + (4n-2)x^{(3+6)^{\frac{1}{2}}} + (n-2)nx^{(6+6)^{\frac{1}{2}}}.$$

After simplification, we get the desired result.

In the following theorem, we compute the first and second inverse Nirmala indices of $CS_{n..}$

Theorem 4. Let $CS_{n.}$ be the family of chain silicate networks. Then

(i) $$IN_1(CS_n) = \left(\frac{\sqrt{2}}{\sqrt{3}} + \frac{4}{\sqrt{2}} + \frac{1}{\sqrt{3}}\right)n + \frac{4\sqrt{2}}{\sqrt{3}} - \frac{2}{\sqrt{2}} - \frac{2}{\sqrt{3}}.$$

(ii) $$IN_2(CS_n) = \left(\frac{\sqrt{3}}{\sqrt{2}} + 4\sqrt{2} + \sqrt{3}\right)n + \frac{4\sqrt{3}}{\sqrt{2}} - 2\sqrt{2} - 2\sqrt{3}.$$

Proof: From definitions and by cardinalities of the edge partition of CS_n , we derive

(i) $$IN_1(CS_n) = \sum_{uv\in E(G)} \left[\frac{1}{d_G(u)} + \frac{1}{d_G(v)}\right]^{\frac{1}{2}} = \left(\frac{1}{3} + \frac{1}{3}\right)^{\frac{1}{2}}(n+4) + \left(\frac{1}{3} + \frac{1}{6}\right)^{\frac{1}{2}}(4n-2) + \left(\frac{1}{6} + \frac{1}{6}\right)^{\frac{1}{2}}(n-2).$$

After simplification, we get the desired result

(ii) $$IN_2(CS_n) = \sum_{uv\in E(G)} \left[\frac{1}{d_G(u)} + \frac{1}{d_G(v)}\right]^{-\frac{1}{2}} = \left(\frac{1}{3} + \frac{1}{3}\right)^{-\frac{1}{2}}(n+4) + \left(\frac{1}{3} + \frac{1}{6}\right)^{-\frac{1}{2}}(4n-2) + \left(\frac{1}{6} + \frac{1}{6}\right)^{-\frac{1}{2}}(n-2).$$

After simplification, we get the desired result.

4. Results for Hexagonal Networks

It is known that there exist three regular plane tilings with composition of same kind of regular polygons such as triangular, hexagonal and square . Triangular tiling is used in the construction of hexagonal networks. This network is symbolized by HX_n, where n is the number of vertices in each side of hexagon. A hexagonal network of dimension six in shown in Figure 3.

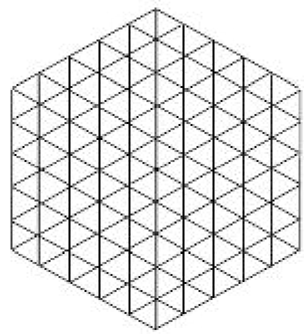

Figure 3. Hexagonal network of dimension six

In the following theorem, we compute the Nirmala index and its exponential of $HX_{n.}$

Theorem 5. Let $HX_{n..}$ be the family of hexagonal networks. Then

(i) $N(HX_n) = 54n^2 + \left(12\sqrt{10} - 186\sqrt{2}\right)n + 12\sqrt{7} + 18 + 144\sqrt{2} - 24\sqrt{10}$.

(ii) $N(HX_n, x) = 12x^{\sqrt{7}} + 6x^3 + (6n-8)x^{2\sqrt{2}} + (12n-24)x^{\sqrt{10}} + (6n^2 - 33n + 30)x^{6\sqrt{2}}$.

Proof : Let G be the graph of hexagonal network HX_n with $|V(HX_n)|=3n^2 - 3n + 1$ and $|E(HX_n)|=9n^2 - 15n + 6$. In HX_n, by algebraic method, there are five types of edges based on the degree of the vertices of each edge as follows:

$E_7 = \{uv \in E(G) \mid d_G(u) = 3, d_G(v) = 4\}, |E_7| = 12.$
$E_9 = \{uv \in E(G) \mid d_G(u) = 3, d_G(v) = 6\}, |E_9| = 6.$
$E_8 = \{uv \in E(G) \mid d_G(u) = d_G(v) = 4\}, |E_8| = 6n - 18.$
$E_{10} = \{uv \in E(G) \mid d_G(u) = 4, d_G(v) = 6\}, |E_{10}| = 12n - 24.$
$E_{12} = \{uv \in E(G) \mid d_G(u) = d_G(v) = 6\}, |E_{12}| = 9n^2 - 33n + 30.$

By using the definitions and cardinalities of the edge partitions of $HX_{n.}$, we deduce

(i) $N\left(HX_n\right) = \sum_{uv \in E(G)} \sqrt{d_G(u) + d_G(v)}$

$$= (3+4)^{\frac{1}{2}} 12 + (3+6)^{\frac{1}{2}} 6 + (4+4)^{\frac{1}{2}} (6n-18) + (4+6)^{\frac{1}{2}} (12n-24) + (6+6)^{\frac{1}{2}} (9n^2 - 33n + 30).$$

After simplification, we get the desired result.

(ii) $N(HX_n, x) = \sum_{uv \in E(G)} x^{\sqrt{d_G(u)+d_G(v)}}$

$$= 12x^{(3+4)^{\frac{1}{2}}} + 6x^{(3+6)^{\frac{1}{2}}} + (6n-18)x^{(4+4)^{\frac{1}{2}}} + (12n-24)x^{(4+6)^{\frac{1}{2}}} + (9n^2-33n+30)x^{(6+6)^{\frac{1}{2}}}.$$

After simplification, we get the desired result.

In the following theorem, we compute the first and second inverse Nirmala indices of $HX_{n.}$.

Theorem 6. Let $HX_{n.}$ be the family of hexagonal networks. Then

(i) $IN_1(HX_n) = \frac{9}{\sqrt{3}}n^2 + \left(\frac{6}{\sqrt{2}} + \frac{6\sqrt{5}}{\sqrt{3}} - \frac{33}{\sqrt{3}}\right)n + \frac{6\sqrt{7}}{\sqrt{3}} - \frac{12}{\sqrt{2}} - \frac{12\sqrt{5}}{\sqrt{3}} + \frac{30}{\sqrt{3}}.$

(ii) $IN_2(HX_n) = 9\sqrt{3}n^2 + \left(6\sqrt{2} + \frac{24\sqrt{3}}{\sqrt{5}} - 33\sqrt{3}\right)n + \frac{24\sqrt{3}}{\sqrt{7}} - 12\sqrt{2} - \frac{48\sqrt{3}}{\sqrt{5}} + 30\sqrt{3}.$

Proof : From definitions and by cardinalities of the edge partitions of $HX_{n.}$, we derive

(i) $IN_1(HX_n) = \sum_{uv \in E(G)} \left[\frac{1}{d_G(u)} + \frac{1}{d_G(v)}\right]^{\frac{1}{2}}$

$$= \left(\frac{1}{3}+\frac{1}{4}\right)^{\frac{1}{2}} 12 + \left(\frac{1}{3}+\frac{1}{6}\right)^{\frac{1}{2}} 6 + \left(\frac{1}{4}+\frac{1}{4}\right)^{\frac{1}{2}} (6n-18) + \left(\frac{1}{4}+\frac{1}{6}\right)^{\frac{1}{2}} (12n-24)$$

$$+ \left(\frac{1}{6}+\frac{1}{6}\right)^{\frac{1}{2}} (9n^2-33n+30).$$

After simplification, we get the desired result.

(ii) $IN_2(HX_n) = \sum_{uv \in E(G)} \left[\frac{1}{d_G(u)} + \frac{1}{d_G(v)}\right]^{-\frac{1}{2}}$

$$= \left(\frac{1}{3}+\frac{1}{4}\right)^{-\frac{1}{2}} 12 + \left(\frac{1}{3}+\frac{1}{6}\right)^{-\frac{1}{2}} 6 + \left(\frac{1}{4}+\frac{1}{4}\right)^{-\frac{1}{2}} (6n-18) + \left(\frac{1}{4}+\frac{1}{6}\right)^{-\frac{1}{2}} (12n-24)$$

$$+ \left(\frac{1}{6}+\frac{1}{6}\right)^{-\frac{1}{2}} (9n^2-33n+30).$$

After simplification, we get the desired result.

5. Results for Oxide Networks

The oxide networks are of vital importance in the study of silicate networks. An oxide network of dimension n is denoted by OX_n. A 5-dimensional oxide network is shown in Figure 4.

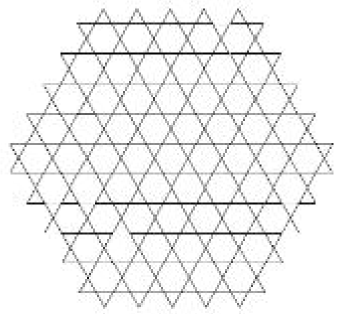

Figure 4. Oxide network of dimension 5

In the following theorem, we compute the Nirmala index and its exponential of OX_n.

Theorem 7. Let OX_n be the family of oxide networks. Then

(i) $N(OX_n) = 36\sqrt{2}n^2 + \left(12\sqrt{6} - 24\sqrt{2}\right)n.$

(ii) $N(OX_n, x) = 12nx^{\sqrt{6}} + (18n^2 - 12n)x^{2\sqrt{2}}.$

Proof: Let G be the graph of oxide network OX_n with $|V(OX_n)| = 9n^2 + 3n$ and $|E(OX_n)| = 18n^2$. In OX_n, by algebraic method, there are two types of edges based on the degree of the vertices of each edge as follows;

$E_6 = \{uv \in E(G) \mid d_G(u) = 2, d_G(v) = 4\}, |E_6| = 12n.$

$E_8 = \{uv \in E(G) \mid d_G(u) = d_G(v) = 4\}, |E_8| = 18n^2 - 12n.$

By using the definitions and cardinalities of the edge partition of OX_n, we deduce

(i) $N\left(OX_n\right) = \sum_{uv \in E(G)} \sqrt{d_G(u) + d_G(v)} = (2+4)^{\frac{1}{2}} 12n + (4+4)^{\frac{1}{2}} (18n^2 - 12n).$

After simplification, we get the desired result.

(ii) $N\left(OX_n, x\right) = \sum_{uv \in E(G)} x^{\sqrt{d_G(u) + d_G(v)}} = 12x^{(2+4)^{\frac{1}{2}}} + (18n^2 - 12n)x^{(4+4)^{\frac{1}{2}}}.$

After simplification, we get the desired result.

In the following theorem, we compute the first and second inverse Nirmala indices of OX_n.

Theorem 8. Let OX_n be the family of oxide networks. Then

(i) $IN_1(OX_n) = \frac{18}{\sqrt{2}}n^2 + \left(6\sqrt{3} - \frac{12}{\sqrt{2}}\right)n.$

(ii) $IN_2(OX_n) = 18\sqrt{2}n^2 + \left(\frac{24}{\sqrt{3}} - 12\sqrt{2}\right)n.$

Proof: From definitions and by cardinalities of the edge partitions of OX_n, we derive

(i) $IN_1(OX_n) = \sum_{uv \in E(G)} \left[\frac{1}{d_G(u)} + \frac{1}{d_G(v)} \right]^{\frac{1}{2}} = \left(\frac{1}{2} + \frac{1}{4} \right)^{\frac{1}{2}} 12n + \left(\frac{1}{4} + \frac{1}{4} \right)^{\frac{1}{2}} (18n^2 - 12n).$

After simplification, we get the desired result.

(ii) $IN_2(OX_n) = \sum_{uv \in E(G)} \left[\frac{1}{d_G(u)} + \frac{1}{d_G(v)} \right]^{-\frac{1}{2}} = \left(\frac{1}{2} + \frac{1}{4} \right)^{-\frac{1}{2}} 12n + \left(\frac{1}{4} + \frac{1}{4} \right)^{-\frac{1}{2}} (18n^2 - 12n).$

After simplification, we get the desired result.

6. Results for Honeycomb Networks

If we recursively use hexagonal tiling in a particular pattern, honeycomb networks are formed. These networks are very useful in chemistry and also in computer graphics. A honeycomb network of dimension n is denoted by HC_n where n is the number of hexagons between central and boundary hexagon. A 4-dimensional honeycomb network is shown in Figure 5.

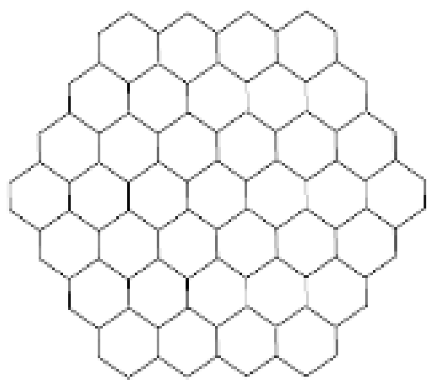

Figure 5. Honeycomb network of dimension four

In the following theorem, we compute the Nirmala index and its exponential of HC_n.

Theorem 9. Let HC_n be the family of honeycomb networks. Then

(i) $N(HC_n) = 9\sqrt{6}n^2 + \left(12\sqrt{5} - 15\sqrt{6}\right)n + 12 - 12\sqrt{5} + 6\sqrt{6}.$

(ii) $N(HC_n, x) = 6x^2 + (12n - 12)x^{\sqrt{5}} + 6 \times 2^n x^{\sqrt{11}} + (9n^2 - 15n + 6)x^{\sqrt{6}}.$

Proof : Let G be the graph of honeycomb network HC_n with $|V(HC_n)| = 6n^2$ and $|E(HC_n)| = 9n^2 - 3n$. In HC_n, by algebraic method, there are three types of edges based on the degree of the vertices of each edge as follows:

$E_4 = \{uv \in E(G) \mid d_G(u) = d_G(v) = 2\}$, $|E_4| = 6$.
$E_5 = \{uv \in E(G) \mid d_G(u) = 2, d_G(v) = 3\}$, $|E_5| = 12n - 12$.
$E_6 = \{uv \in E(G) \mid d_G(u) = d_G(v) = 3\}$, $|E_6| = 9n^2 - 15n + 6$.

By using the definitions and cardinalities of the edge partition of HC_n, we deduce

(i) $N(HC_n) = \sum_{uv\in E(G)} \sqrt{d_G(u)+d_G(v)} = (2+2)^{\frac{1}{2}}6 + (2+3)^{\frac{1}{2}}(12n-12) + (3+3)^{\frac{1}{2}}(9n^2-15n+6).$

After simplification, we get the desired result.

(ii) $N(HC_n, x) = \sum_{uv\in E(G)} x^{\sqrt{d_G(u)+d_G(v)}} = 6x^2 + (12n-12)x^{\sqrt{5}} + (9n^2-15n+6)x^{\sqrt{6}}.$

After simplification, we get the desired result.

In the following theorem, we compute the first and second inverse Nirmala indices of HC_n.

Theorem 10. Let HC_n be the family of honeycomb networks. Then

(i) $IN_1(HC_n) = \frac{9\sqrt{2}}{\sqrt{3}}n^2 + \left(\frac{12\sqrt{5}}{\sqrt{6}} - \frac{15\sqrt{2}}{\sqrt{3}}\right)n + 6 - \frac{12\sqrt{5}}{\sqrt{6}} + \frac{6\sqrt{2}}{\sqrt{3}}.$

(ii) $IN_2(HC_n) = \frac{9\sqrt{3}}{\sqrt{2}}n^2 + \left(\frac{12\sqrt{6}}{\sqrt{5}} - \frac{15\sqrt{3}}{\sqrt{2}}\right)n + 6 - \frac{12\sqrt{6}}{\sqrt{5}} + \frac{6\sqrt{3}}{\sqrt{2}}.$

Proof : From definitions and by cardinalities of the edge partition of HC_n, we derive

(i) $IN_1(HC_n) = \sum_{uv\in E(G)} \left[\frac{1}{d_G(u)} + \frac{1}{d_G(v)}\right]^{\frac{1}{2}} = \left(\frac{1}{2}+\frac{1}{2}\right)^{\frac{1}{2}}6 + \left(\frac{1}{2}+\frac{1}{3}\right)^{\frac{1}{2}}(12n-12) + \left(\frac{1}{3}+\frac{1}{3}\right)^{\frac{1}{2}}(9n^2-15n+6).$

After simplification, we get the desired result

(ii) $IN_2(HC_n) = \sum_{uv\in E(G)} \left[\frac{1}{d_G(u)} + \frac{1}{d_G(v)}\right]^{-\frac{1}{2}} = \left(\frac{1}{2}+\frac{1}{2}\right)^{-\frac{1}{2}}6 + \left(\frac{1}{2}+\frac{1}{3}\right)^{-\frac{1}{2}}(12n-12) + \left(\frac{1}{3}+\frac{1}{3}\right)^{-\frac{1}{2}}(9n^2-15n+6).$

After simplification, we get the desired result.

7. *Data Set of Computed Values :*

In order to find the usefulness of topological index, we have to predict the coefficient of correlation between the physico-chemical properties and the calculated topological indices. For different values of n, the indices are calculated and tabulated below. By using these values, we plot the below graphs and find the coefficient of correlation.

Table - 1. Values of Nirmala type of indices of some chemical networks

Nirmala type of Indices	n = 1	n = 2	n = 3	n = 4	n = 5	n = 6	n = 7
$N(SL_n)$	112.1528	485.0406	1118.6635	2013.0215	2013.0215	4583.9427	6260.5058
$N(CS_n)$	26.0048	44.6969	63.3891	82.0812	100.7733	119.4655	138.1576
$N(HX_n)$	6.4047	-56.6917	-11.7881	141.1155	402.0192	770.9228	1247.8264
$N(OX_n)$	46.3644	194.5523	444.5634	796.3980	1250.0560	1805.5373	2462.8420
$N(HC_n)$	12.0000	68.2267	168.5442	312.9525	501.4517	734.0416	1010.7224

$N(SL_n, x)$	36.0000	1646.2108	18449.1382	106691.3909	421745.6796	1303668.6281	3394072.4443
$N(CS_n, x)$	6.0000	80.7734	690.4598	4001.5985	16567.6633	53600.2137	144956.4051
$N(HX_n, x)$	7.0000	-4148.8559	-166733.2066	-766963.2476	1.2804*10^7	1.9236*10^8	1.3784*10^9
$N(OX_n . x)$	18.0000	472.0372	3348.4524	13540.7286	40079.3290	97288.2232	205890.8075
$N(HC_n, x)$	18.0000	385.1995	2788.4858	13109.9202	49894.9392	169119.5302	533857.6448
$IN_1(SL_n)$	25.3336	96.9077	214.7223	378.7773	589.0728	845.6087	1148.3850
$IN_1(CS_n)$	4.9193	9.1416	13.3639	17.5862	21.8084	26.0307	30.2530
$IN_1(HX_n)$	0.6406	9.1652	28.0820	57.3911	97.0925	147.1862	207.6723
$IN_1(OX_n)$	14.6349	54.7257	120.2724	211.2748	327.7332	469.6473	637.0173
$IN_1(HC_n)$	6.0000	26.7524	62.2018	112.3480	177.1913	256.7314	350.9685
$IN_2(SL_n)$	51.6819	216.6293	494.8423	886.3207	1391.0647	2009.0742	2740.3492
$IN_2(CS_n)$	7.2201	15.8338	24.4474	33.0611	41.6747	50.2884	58.9020
$IN_2(HX_n)$	-0.9716	15.7117	63.5719	142.6090	252.8231	394.2140	566.7819
$IN_2(OX_n)$	22.3417	95.5951	219.7601	394.8369	620.8253	897.7255	1225.5373
$IN_2(HC_n)$	6.0000	33.8423	83.7300	155.6631	249.6416	365.6655	503.7348

8. *Comparative Analysis :*

In this section, we will compare the result of Nirmala type of indices of certain networks in graphical form. Different colors have been used to represent the behavior of the indices for certain networks in the form of graphical lines. And these graphs have been generated by putting different values of n and x, mentioned in above Table.1. A comparison is made in figure. 6 and figure.7, the above mentioned networks are very close at the beginning and then grew. Among the five structure of networks, silicate network (SL_n) is most powerful compare to other structure of networks. The chain silicate (CS_n) grew more slowly than other networks. From the vertical axis of the graph $N(G)$, $IN_1(G)$ and $IN_2(G)$, it is clear that the indices for different networks grew in the following order.

$$CS_n < HX_n \leq HC_n < OX_n < SL_n$$

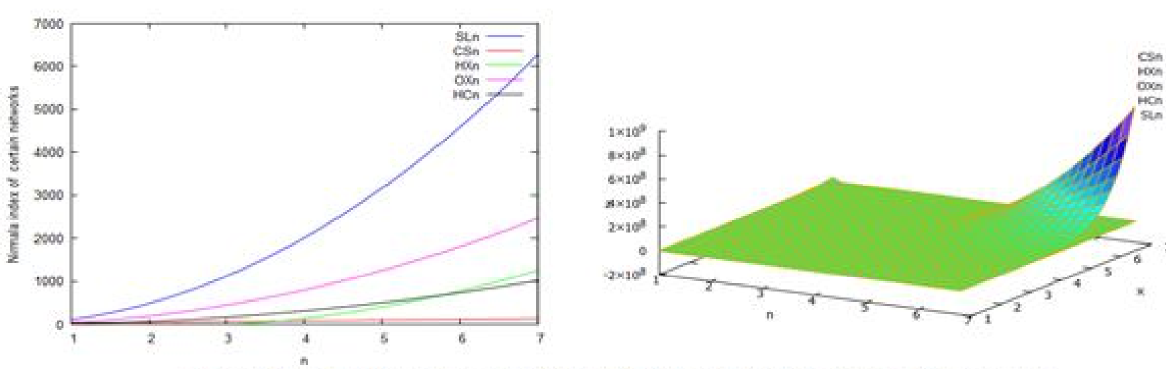

Figure.6: Graphical reprasentation of N(G) and N(G,x) of certain networks.

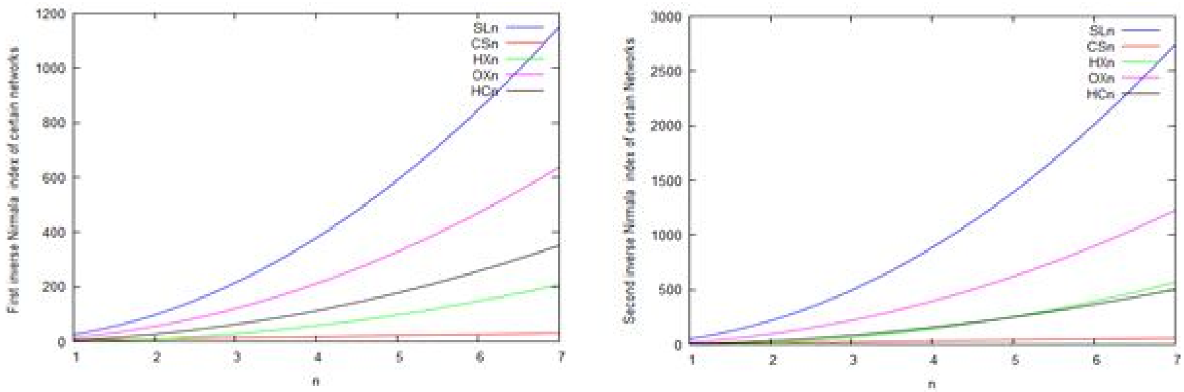

Figure.7: Graphical reprasentation of IN1(G) and IN2(G) of certain networks

9. Conclusion

In this paper, we have computed Nirmala type indices of certain networks. These results are helpful to understand the deep behavior of the networks. The coefficient of correlation of $N(CS_n)$, $IN_1(CS_n)$ and $IN_2(CS_n)$ is 1 which shows that the line is linearly fitted. This indicates that the Nirmala and inverse Nirmala indices are theoretically fit for the chain silicate network (CS_n).

10. Open Problem

Find the values of different types of Nirmala indices of certain classes of chemical graphs and explore some results towards QSPR/QSAR/QSTR Model.

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