ISSN 2231-346X (Print)

JUSPS-A Vol. 34(6), 70-79 (2022). Periodicity-Monthly

Section A

ISSN 2319-8044 (Online)

JOURNAL OF ULTRA SCIENTIST OF PHYSICAL SCIENCES

An International Open Free Access Peer Reviewed Research Journal of Physical Sciences

website:- www.ultrascientist.org

Estd. 1989

Stochastic modelling of a system having two non-identical class of units

PRAVEEN KUMAR SHRIVASTAVA[1], SATYENDRA PRASAD SINGH[2*] and S. K. SINGH[3]

[1]Department of Community Medicine, Chhattisgarh Institute of Medical Sciences, Bilaspur, Chhattisgarh, (India)

[2*]Department of Community Medicine, LSLRAM Govt. Medical College, Bendrachua, Raigarh, Pin code 496001, Chhattisgarh, (India)

[3]Ex. Vice Chancellor, Bastar University, Jagdalpur, Chhattisgarh (India)

*Corresponding Author Email: drspsingh1@hotmail.com

http://dx.doi.org/10.22147/jusps-A/340601

Acceptance Date 11th November, 2022, Online Publication Date 12th December, 2022

Abstract

This paper deals with the study of stochastic modelling of a system having two non-identical class of units. The whole system consists of two class L_1 and L_2 respectively. First class consists only one unit where as second class having two identical units. The whole system is used to process the jobs. Jobs are first processes on L_1 unit (class one) and then goes to L_2 unit (second class). All the failure time distribution are taken as arbitrarily distributed. Using regenerative point technique, various reliability characteristics of interest to the system managers have been evaluated. Finally, some graphs are also plotted in order to highlight the important results.

Key words : Regenerative point technique, Transition Probability, Mean Sojourn Time, MTSF, Availability and Busy Period Analysis.

Introduction

Reliability is desirable and necessary factor in the present-day technology for achieving healthy economic progress of nation. During last three decades, reliability technology has achieved great importance in the development of the technological field. At present reliability is not only a subject of study for scientists and academicians but also a concern to the practicing engineers, manufacturers and economists as well. Now-a-days, this technology is most frequently used in the development of electrical and electronic equipment's on the side and in the study of the problems of transportation and industries on the other. Although, lot of work have been done in the field of reliability but most of it are concerned with hypothetical models. Very few works have been reported by taking practical models and real data. Kochar *et al.*[1] developed a systematic method for investment decision on additional equipment to form a standby or redundant system in a production system. Gupta *et al.*[2] have analysed the availability of two-unit parallel redundant complex system. Dhillion *et al.*[3] have analysed pulverize systems with common cause failures. Kumar *et al.*[4] have analysed a feeding system in sugar industry. Singh et al[5] have studied a stone crushing system having one apron feeder, one grizzly, one primary gyratory crusher. This group of equipment's is used to get iron ores from stones in mining crushing plants. Singh *et al.*[6] have studied the stochastic modelling and analysis of door-extractor system. Singh *et al.*[7] have done the estimation of the reliability characteristics of a plate mill system of an integrated steel plant. Again, Singh *et al.*[8] also have studied the reliability analysis of the wire rod mill system of an integrated steel plant. They have obtained various parameters of the systems which are useful to the system managers and engineers. Very recently M. Savsar[9] has studied the reliability analysis of a flexible manufacturing cell. In the sequence of above study, we have tried to study the Stochastic modelling of a system having two non-identical class of units.

Model :

The purpose of the present research paper is to deal with the stochastic modelling of a system having two classes of units L_1 and L_2 respectively. First class L_1 consist of only one unit and other class L_2 consists of two identical units. The system is used to process the jobs. The rolling mill system having one $_{\text{class l1}}$ and two identical $_{\text{class l2 unit}}$ is one of the examples of such system. In rolling mill machine $_{\text{class l1}}$ (classL_1) is used to convert the cold blooms into red blooms and $_{\text{class l2 unit}}$s (class L_2) are used to convert them into different shapes. Using regenerative part technique, the following measures of system effectiveness are obtained

1. Transition probabilities and mean sojourn times of the system.
2. Mean time to system failure (MTSF).
3. Point wise availability of the system in (0, t] and in the steady state.
4. Expected busy period of the repairman in repair of system in (0, t] and in steady state.
5. Expected time for which the classL_1 unit is under preparation in (0, t] and in the steady state.
6. Expected profit earned by the system in (0, t] and in the steady state.

At last, some graphs are also plotted to highlight the important results.

Description of the system :

1. The job processing system consists of two classes L_1 and L_2 respectively. Class L_1 has only one unit and class L_2 has two identical units. Class L_2 helps class L_1 in processing the jobs in standby

configuration.

2. Whenever class L_1 or both class L_2 fail then L_1 is down in order to repair the system as early as possible. Priority in repair is given to class L_1 unit L_2 the unit over L_2 the units of class L_2.
3. Whenever all the units fail, priority is given to the repair of the units of class L_1 over the unit's class of L_2. Further. If both the units of class two fail, then the preparation of the unit operation is terminated till the repair of a class two unit.
4. There is a single repair facility which is used to repair the failed units on first come first serve basis.
5. Switching device is perfect and instantaneous.
6. Failure time distribution of all the units is taken to be negative exponential whereas all the repair time distributions are taken to be erlang ion and preventive maintenance is taken as arbitrarily.
7. After repair a unit works as new.
   State transition diagram and graphs are shown in figure 1, 2, 3 and 4 respectively.

Notation :

α / γ	Constant failure rate of class L_1 unit / classL_2 unit.
ψ	Constant preparation time of class oneL_1 unit.
Φ	maximum operation time of class L_2 unit.
g_i (t),G_i(t)	p.d.f .and c.d.f. of repair time of classL_1 / class L_2 (i=1,2 units).
a_i (t),A_i(t)	p.d.f. and c.d.f. of time to class L_1 preventive maintenance (i=1,2 units).
b_i (t),B_i(t)	p.d.f. and c.d.f. of time to accomplish class L_1 preventive maintenance (i=1,2 units).

Symbols used for states of the system :

L_1up L_0	: Class one unit is under preparation / under operation.
L_1up/ L_1r	: Class one unit is waiting for preparation / under repair.
L_1 /pm	: Class one unit is preventive maintenance.
L_{20} /L_2g	: Class two unit is under operation/good and non-operative.
L_2ur /L_{2UR}	: Class two unit is under repair / repair continue previous state.
L_{2WR}	: Class two unit is waiting for operation.

Transition probabilities and mean sojourn times :

Simple probabilistic considerations yield the following expressions for transition probabilities:

$$p_{01} = \phi L_0^{-1} \quad ; \quad p_{02} = \alpha L_0^{-1} \quad ; \quad p_{12} = \alpha L_6^{-1}[1 - a^* L_6] \quad ; \quad p_{13} = a^* L_6$$

$$p_{14} = \gamma L_6^{-1}[1 - a^* L_6] \quad ; \quad p_{41} = g_2^* L_6 \quad ; \quad p_{46} = \alpha L_6^{-1}[1 - g_2^* L_6]$$

$$p_{48}^{(7)} = \gamma L_6^{-1}[1 - g_2 L_6] \quad ; \quad p_{25} = p_{35} = p_{50} = p_{68} = 1 \quad ; \quad p_{85} = g_2^*(\psi)$$

$$p_{80}^{(9)} = g_2^* L_0 - g_2^*(L_0 + \psi) \quad ; \quad p_{81}^{(9,10)} = [L_5\{g_2^* L_6 - g_2^*(L_6 + \phi)\} - \phi\{g_2^* L_6 - g_2^*(L_6 + L_5)\}]/L_5$$

$$p_{86}^{(9)} = \alpha(L_0+\psi)\{1-g_2^*L_0\} - \{1-g_2^*(L_0+\psi)\}/L_0(L_0+\psi)$$

$$p_{86}^{(9,10)} = \alpha[\psi(L_6+L_5)\{1-g_2^*L_6\} - L_6L_5(L_6+L_5)\{1-g_2^*(L_6+\phi)\} - \phi L_6(L_6+\phi)\}\{1-g_2^*(L_6+L_5)\}]/L_6 L_5(L_6+\phi)(L_6+L_5)$$

$$p_{88}^{(9,10,7)} = \gamma[L_5(L_6+\phi)(L_6+L_5)\{1-g_2^*L_6\} - L_6L_5(L_6+L_5)\{1-g_2^*(L_6+\phi)\} - \phi(L_6+\phi)(L_6+L_5)\{1-g_2^*L_6\} + \phi L_6(L_6+\phi)\{1-g_2^*(L_6+L_5)\}]/L_6L_5(L_6+\phi)(L_6+L_5) \tag{1-15}$$

Mean sojourn μ_i time in state S_i, which is based on the similar arguments are:

$$\mu_0 = L_0^{-1} \quad ; \quad \mu_1 = L_6^{-1}[1-a^*L_6] \quad ; \quad \mu_2 = \int_0^\infty \bar{G}_1(t)d(t) \quad ; \quad \mu_3 = \int_0^\infty \bar{B}(t)d(t)$$

$$\mu_4 = L_6^{-1}[1-g_2^*L_6] \quad ; \quad \mu_5 = 1/\psi \quad ; \quad \mu_6 = \int_0^\infty \bar{G}_1(t)d(t)$$

$$\mu_8 = [1-g_2^*(\psi)/\psi] \tag{16-23}$$

where: $L_0 = (\alpha+\phi)$, $L_5 = (\alpha+\psi)$ and $L_6 = (\alpha+\gamma)$

Mean time to system failure :

Time to system failure can be regarded as the first passage time to the failed state. To obtain, it we regard the down states S_2, S_3, S_5, S_6, S_8 as absorbing states. Using arguments as for the regenerative process we obtain the following recursive relations for $\pi_i(t)$:

$$\pi_0(t) = Q_{01}(t)\boxed{s}\,\pi_1(t) + Q_{02}(t) \quad ; \quad \pi_1(t) = Q_{14}(t)\boxed{s}\,\pi_4(t) + \sum_{j=2,3} Q_{ij}(t)$$

$$\pi_4(t) = Q_{41}(t)\boxed{s}\,\pi_1(t) + \sum_{j=6,7} Q_{4j}(t) \tag{24-26}$$

Taking Laplace Stieltjes transforms of equations (24 -26) and after solving for $\tilde{\pi}_0(s)$, we have

$$\text{MTSF} = E(T) = -\frac{d}{ds}\tilde{\pi}_0(s)\Big|_{s=0} = \frac{[\mu_0 k_1 + p_{01}(\mu_1 + \mu_4 p_{14})]}{k_1} \tag{27}$$

Where: k_1 + (1-p_{14} p_{41}).

System availability :

$M_i(t)$ is the probability that the system, initially in regenerative state S_i, is up at time t without passing through any other non-regenerative state or returning to itself through one or more non-regenerative states. *i. e.* either it continues to remain in regenerative state S_i or a non-regenerative state including itself.

By probabilistic arguments, we have:

$$M_0(t) = e^{-L_0 t} \quad ; \quad M_1(t) = \overline{A}(t)e^{-L_1 t} \quad ; \quad M_4(t) = e^{L_6 t} \tag{28-30}$$

Recursive relations giving the pointwise availability $A_i(t)$ are :

$A_0(t) = M_0(t) + q_{01}(t) © A_1(t) + q_{02}(t) © A_2(t)$

$A_i(t) = q_{ij}(t) © A_j(t)$, Where $(i,j) = (2,5),(3,5),(5,0),(6,8)$

$A_1(t) = M_1(t) + q_{1j}(t) © A_j(t)$, Where $j = 2,3,4$

$A_4(t) = M_4(t) + q_{4j}(t) © A_j(t) + q_{48}^{(7)}(t) © A_8(t)$, Where $j = 1,6$

$$A_8(t) = q_{80}^{(9)}(t) © A_0(t) + q_{81}^{(9,10)}(t) © A_1 + q_{85}(t) © A_5(t) + q_{86}^{(9)} + q_{86}^{(9,10)}(t) © A_6(t)$$
$$+ q_{88}^{(9,10,7)}(t) © A_8(t) \tag{31-35}$$

Taking Laplace transforms of (31-35) and after solving for $A^*_0(t)$, the steady state availability A_0 is

$$A_0 = \lim_{t\to\infty} A_0(t) = \lim_{s\to 0} s\, A_0^*(s) = \frac{N_2(0)}{D_2'(0)} \tag{36}$$

Where: $N_2(0) = k_3[\mu_0 k_1 - \mu_1 P_{01} + \mu_4 P_{04} p_{14}] - \mu_0 P_{14} P_{81}^{9,10}(1-P_{41})$ and

$$\begin{aligned} D_2'(0) = {} & \mu_0[K_1K_2 + P_{81}^{(9,10)}(1-P_{14}) + \mu_1 P_{01} K_3 + \mu_2[K_3(P_{02} + P_{01}P_{12}) - P_{02}P_{14}P_{41}K_2 - P_{02}P_{14} \\ & P_{81}^{(9,10)}] + \mu_3 P_{01} P_{13} K_3 + n_4 P_{01} P_{14} K_3 + \mu_5[K_3(1 - P_{01}P_{14} - P_{02}P_{14}P_{41}) + (1-P_{41})(P_{01}P_{14} \\ & P_{85} - P_{02}P_{14}P_{81}^{9,10})] + \mu_6[P_{01}P_{14}(1-P_{41})P_{86}^{(9)} + P_{86}^{(9,10)} + P_{01}P_{14}P_{46}K_3 + n_8 P_{01}P_{14}(1-P_{41})] \end{aligned}$$

$$K_2 = P_{80}^{(9)} + P_{85} \quad ; \quad K_3 = K_2 + P_{81}^{(9,10)}]$$

n_4 & n_8 = Unconditional sojorn time in state S_4 and S_8 respectively.

Busy period analysis

Expected busy period analysis of the repairman in the repair in the class one in (0, t]

$B_i^1(t)$is defined as the probability that at time t the repairman is busy with the repair of the class L_1unit given that the system entered regenerative state S_i at t = 0. Developing similar recursive relations as in (31-35) and after solving the resulting recurrence equations of the Laplace transforms for $B_0^{1*}(s)$, in the long run, the fraction of time for which the repairman is busy with repairing the class

L_1 unit is given by:

$$B_0^1 = \lim_{t\to\infty} B_0^1(t) = \lim_{s\to 0} s\,B_0^{1*}(s) = \frac{N_3(0)}{D_2'(0)} \tag{37}$$

where :

$$N_3(0) = \mu_2[K_3(P_{01}P_{12}+P_{02}K_1)-P_{02}P_{14}P_{81}^{(9,10)}(1-P_{41})]+\mu_6P_{01}P_{14}[P_{46}(1-P_{88}^{(9,10,7)}) +P_{48}^{(7)}(P_{86}^{(9)}+P_{86}^{(9,10)}]$$

Expected busy period analysis of the repairman in the repair of the class l_2 unit in (0, t] :

$B_i^2(t)$ is defined as the probability that at time t the repairman is busy with the repair of the class L_2 unit given that the system entered regenerative state Si at T=0. Developing similar recursive relations as in (31-35) and after solving the resulting recurrence equations of the Laplace transforms for $B_0^{2*}(s)$, in the long run, the fraction of time for which the repairman is busy with repairing the class L_2 unit is given by.

$$B_0^2 = \lim_{t\to\infty} B_0^2(t) = \lim_{s\to 0} s\,B_0^{2*}(s) = \frac{N_4(0)}{D_2'(0)} \tag{38}$$

Where: $N_4(0)=n_4P_{01}P_{14}k_3+n_8P_{01}P_{14}(1-P_{41})$

Expected busy period analysis of the repairman in the preparation of the class l_1 unit in (0, t]:

$B_i^3(t)$ is defined as the probability that at time t the repairman is busy with the preparation for the operation of the class l_1, given that the system entered regenerative state Si at t = 0. Developing similar recursive relations as in (31-35) and after solving the resulting recurrence equations of the Laplace transforms for $B_0^{3*}(s)$, in the long run, the fraction of time for which the repairman is busy with preparation of the class l_1 is given by

$$B_0^3 = \lim_{t\to\infty} B_0^3(t) = \lim_{s\to 0} s\,B_0^{3*}(s) = \frac{N_5(0)}{D_2'(0)} \tag{39}$$

Where: $N_5(0)= \mu_5[k_3(1-P_{01}P_{14}-P_{02}P_{14}P_{41})+(1-P_{41})(P_{01}P_{14}P_{85}-P_{02}P_{14}P_{81}^{(9,10)}]+n_8P_{01}P_{14}(1-P_{41})$

Cost analysis :

The cost benefit analysis of the system can be carried out by considering the expected busy period of the repairman in repair of the unit in (0, t].
Therefore,

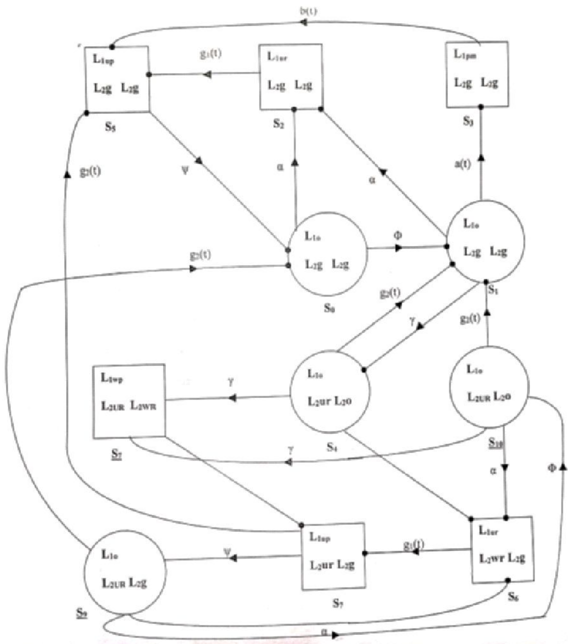

Figure 1. State Transition Diagram

Down state **Up state** **Regenerative point**

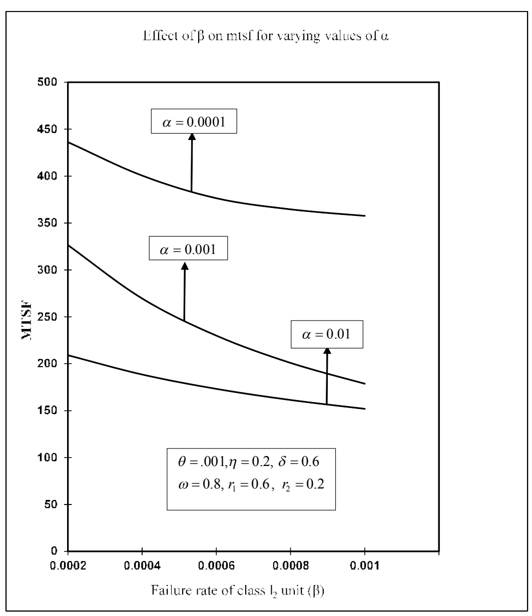

Figure 2

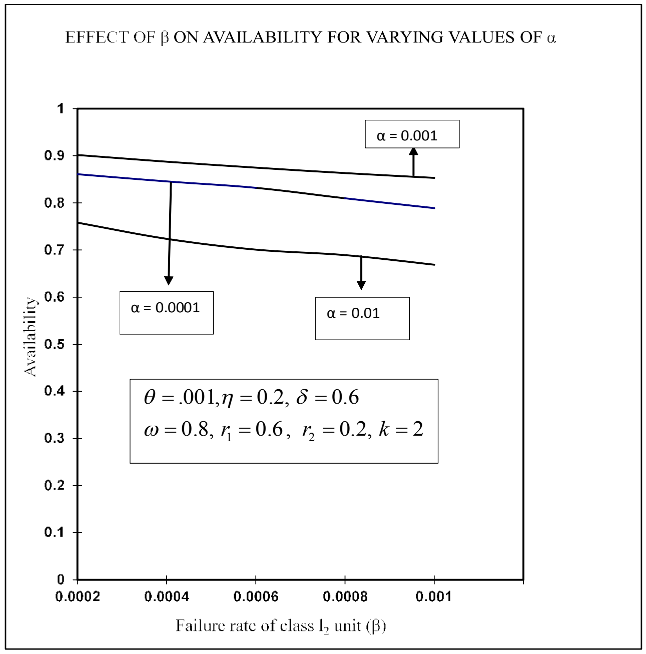

Figure 3

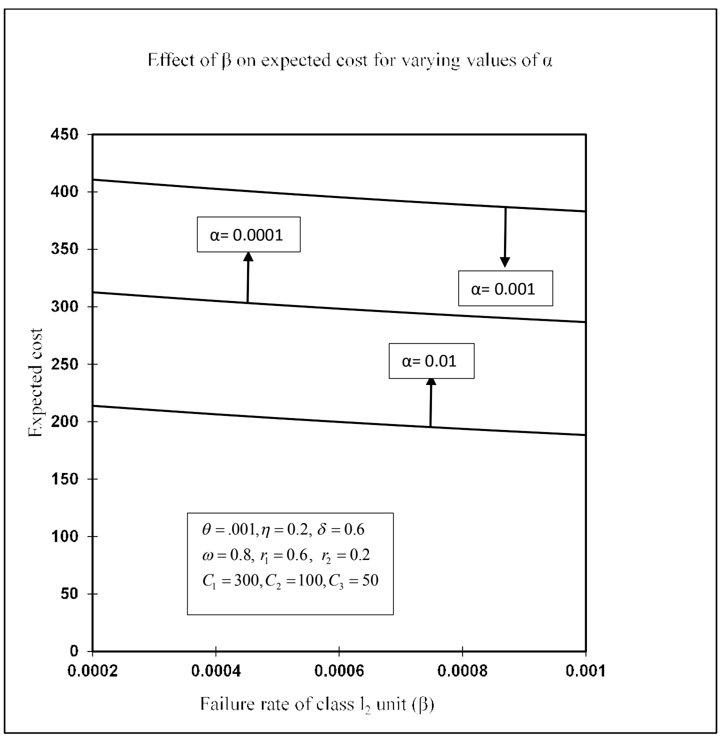

Figure 4

G(t)= Expected total revenue earned by the system in (0, t]-expected repair cost of the repairman in repairing the class l_1 unit in (0, t]-expected repair cost of the repairman in repairing the class l_2 unit in (0, t] -expected cost of the repairman in preparation for operation of the furnace

$$= C_0\ \mu_{up}(t) - C_1 \mu_b^1(t) - C_2 \mu_b^2(t) - C_3 \mu_b^3(t) \tag{40}$$

Where: $\mu_{up}(t) = \int_0^t A_0(t)dt\ ;\ \mu_b^1(t) = \int_0^t B_0^1(t)dt\ ;\ \mu_b^2(t) = \int_0^t B_0^2(t)dt\ ;\ \mu_b^3(t) = \int_0^t B_0^3(t)dt$ (41-44)

The expected total profit per unit of time in steady state is

$$G = \lim_{t\to\infty} \frac{G(t)}{t} = \lim_{s\to 0} s^2\ G^*(s) = C_0\ \mu_{up} - C_1\ \mu_{br} - C_2\ \mu_{br} - C_3 \mu_{bp} \tag{60}$$

Where C_0 is the revenue per unit up time and C_1 and C_2 are the repair cost per unit of time and C_3 is preparation cost

Conclusion and future scope :

This study is very much use full for the engineers and system managers. Equation no. (40) can be used for the cost and benefit analysis with respect to different types failure and repair policies. At last, with the help of programming in C++ some graphs are plotted which clearly explains the characteristics of different parameters such as Mean time to system failure (MTSF), availability and expected with respect to failure rate. In near future the study can be extended as particular case of this study by considering the failure and repair rate as any arbitrary distribution.

Acknowledgement

We are very much thankful to Ex. Vice chancellor of Bastar university, Chhattisgarh for guiding us in programming part to carry out this study. Also we have not received any fund from any organization to carry out this research.

References

1. Kochar Inderpal Singh, “Reliability analysis and investment in electric motors for irrigation”. Microelectronics and Reliability, Vol. *23(1),* 173-174 (1983).
2. Gupta P. P. and Tyagi L., “MTTF and availability evaluation of a two-unit, two-state, standby redundant complex system with constant human failure”. Microelectronics and Reliability, Vol. *26(4),* 647 – 650 (1986).
3. Dhillon B. S. and Natesan J., “Probability analysis of a pulverizer system with common cause failures”. Microelectronics and Reliability, Vol. 22(6), 1121 – 1133 (1987).
4. Kumar Dinesh Singh and Singh S. P., “Availability of the feeding system in the sugar industry”.

Microelectronics and Reliability, Vol. *28(6),* 867 – 871 (1988).
5. Singh S. K. and Sheeba G. Nair, “Stochastic modeling and analysis of stone crushing system used in iron ore mines”. Journal of Ravishankar University, Vol. *8,* 101 – 114 (1995).
6. Singh S. K., Indu Anand and Singh S. P., “Stochastic modeling and analysis of a door-extractor system”. Stochastic Modelling and Applications, Vol. *3,* No. 1, 68 – 87 (2000).
7. Satyendra Prasad Singh, Pragnyabhan Mishra and S. K. Singh, “Estimation of the reliability characteristics of a plate mill system of an integrated steel plant”. Ultra scientist of physical sciences International journal of physical sciences, Vol. *21(2)M*, 585 – 596 (2009).
8. Satyendra Prasad Singh and S. K. Singh, “Reliability analysis of wirerod mill system of an integrated steel plant”. International journal of computer science and information technology (Inter-Disciplinary Journal), Vol. *1(1),* 61-70 (2010).
9. M. Savasar, “Reliability analysis of a flexible manufacturing cell”. Reliability engineering & system safety, Vol. *67,* issue 2, 147 – 152 (2020).