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Estimation of Expected time to Seroconversion of HIV infected when the Antigenic Diversity Thresholds has SCBZ Property

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Abstract

The likelitime at which the seroconversion of HIV infected takes place is a vital event indicating the progression of the infection. The antigenic diversity of the invading antigens due to successive contacts increases and similarly the virulence also increases. The seroconversion occurs if either the total antigenic diversity crosses the threshold level or the total virulence crosses the threshold level. Using the shock model approach the expected time to seroconversion has been derived by several authors. In this paper it is assumed that the antigenic diversity threshold is a random variable which has a distribution that has the change of parameter this is called the SCBZ property. The expected time to seroconversion is obtained using this concept numerical illustrations are also provided.

Key words: Seroconversion, Antigenic Diversity Threshold, Virulence, SCBZ property.

I. Introduction

In the study of HIV infection and progression to AIDS the antigenic diversity of the invading antigens and the virulence of the same play a vital role. There is a threshold level for the antigenic diversity and also for the virulence. If the contribution to antigenic diversity and similarly to virulence crosses the threshold level seroconversion becomes unavoidable. There is contribution to antigenic diversity as well as to the virulence of the invading antigens, due to acquiring more HIV due to sexual contacts and other sources of transmission. It is assumed that the antigenic diversity threshold as well as the virulence threshold of a person undergo changes due to several reasons such as ageing, frequent exposure and transmission of new HIV, under this assumption the expected time to

seroconversion due to contribution to antigenic diversity, similarly due to contribution to virulence, is derived. In doing so the concept of SCBZ property is used. Setting Clock Back to Zero property has been discussed by Raja Rao[4] and Talwalker (1990). A random variable undergoes a change in its distribution after a particular value. This property holds good in the case of human immune system. The antigenic diversity threshold as well as the virulence of random nature threshold undergo changes due to ageing of the infected person. the expected time to seroconversion is derived by using the shock models and cumulative demage process approach due to Easary *et.al.*[3]. Sathiyamoorthi and Kannan[1,2] have obtained the expected time to seroconversion using shock model approach. Elangovan[5] *et. al*. (2010) have obtained the expected time to seroconversion using the similar model.

II. *Assumptions :*

1. A person is exposed to sexual contacts with an infected partner and on each occasion of contact the transmission of HIV takes place.
2. The mode of transmission of HIV on successive occasions results in the contribution to the antigenic diversity of the invading antigens. Also there is increase in the virulence of the invading antigens.
3. As and when the total antigenic diversity crosses a particular level called the antigenic diversity threshold, then the seroconversion takes place. Similarly if the total virulence of the invading antigens crosses the virulence threshold, then the seroconversion will occur.
4. The crossing of both antigenic diversity threshold and virulence threshold simultaneously is considered to be an impossible event.
5. The two thresholds are random variables and are mutually independent.

III. *Notations :*

X_i = a random variable denoting the contribution to antigenic diversity on the i th contact i=1,2,3, ..., k and with P.d.f. g(.) with c.d.f.G(.)

Y_i = The increase in the virulence due to the i th contact, i=1,2,3, ..., k with p.d.f.q (.) and c.d.f Q(.)

Z_1 = a ramdom variable denoting antigenic threshold and has p.d.f h(.) and c.d.f. H(.)

Z_2 = a random variable denoting the virulence threshold with p.d.f m(.) and c.d.f M(.)

T = Time to seroconversion

* = denotes Laplace tranform.

Ui = a random variable denoting the interarrival times between contact with p.d.f of f(.) and c.d.f. F(.)

IV. Results

When the antigenic diversity undergoes changes and satisfies the SCBZ property. The survivor function $S(t)$ is given by

$S(t)=P[T>t]$=P [The antigenic diversity as well as the virulence due to k successive contacts do not cross the respective thresholds]

Now, $\left[\sum_{i=1}^{k} X_i < Z_1 \cap \sum_{i=1}^{k} Y_i < Z_2\right] = P\left[\sum_{i=1}^{k} X_i < Z_1\right].P\left[\sum_{i=1}^{k} Y_i < Z_2\right]$

$S(t)=P$[There are f_k constant in (0,t) and the total antigenic diversity as well as total virulence do not cross the respective thresholds]

$= \sum_{k=0}^{\infty}[F_k(t) - F_{k+1}(t)]\left[\int_0^{\infty} g_k(x)\overline{H(x)}d_x\right]$

$$\left[\int_0^{\infty} q_k(y)\overline{M(y)}d_y\right] \quad (1)$$

Now, the random variable Z_1 denoting the antigenic diversity threshold undergoes changes and satisfies the SCBZ property. $Z_1 \sim h(.)$ with c.d.f $H(.)$.

Z_2 is the virulence threshold and Z_1 does not undergo changes $Z_2 \sim m(.)$ with c.d.f $M(.)$. Now, Since Z_1 follows the SCBZ property.

We have, $h(y) = \theta_1 e^{-\theta_1 y}$ if $y \le \tau_0$

$= \theta_2 e^{-\theta_2 y}\, e^{\tau_0(\theta_2-\theta_1)}$ if $y > \tau_0$

where τ_0 itself is a random variable and $\tau_0 \sim exp(\eta)$. It can be shown that

$h(y) = \frac{(\theta_1-\theta_2)(\eta+\theta_1)}{\eta+\theta_1-\theta_2} e^{-(\theta_1+\eta)y} + \frac{\theta_2\eta}{\eta+\theta_1-\theta_2} e^{-\theta_2 y}$

and hence

$\overline{H(x)} = P.e^{-(\theta_1+\eta)x} + qe^{-\theta_2 x}$

$H(y) = 1 - P.e^{-(\theta_1+\eta)y} - qe^{-(\theta_2)y}$ and also

$\overline{H(y)} = P.e^{-(\theta_1+\eta)y} + qe^{-\theta_2 y}$

Where $P = \frac{(\theta_1-\theta_2)}{\eta+\theta_1-\theta_2}$ and $q = \frac{\eta}{\eta+\theta_1-\theta_2}$

Substituting $\overline{H(x)}$ in (1) we get,

$\therefore S(t) = \sum_{k=0}^{\infty}[F_K(t) - F_{k+1}(t)]$

$\left[\int_0^{\infty} g_k(x)\left[P.e^{-(\theta_1+\eta)x} + qe^{-\theta_2 x}d_x\right]\left[\int_0^{\infty} q_k(y)e^{-\lambda y}d_y\right]\right]d_x d_y$

$\int_0^{\infty} g_k(x)\left[P.e^{-(\theta_1+\eta)x} + qe^{-\theta_2 x}\right] + \cdots + q_k{}^*(\lambda)$

$= P.g_k{}^*(\theta_1+\eta) + qg_k{}^*(\theta_2) + q_k{}^*(\lambda)$

$S(t) = \sum_{k=0}^{\infty}[F_K(t)-F_{k+1}(t)]\ [P.g_k{}^*(\theta_1+\eta) + qg_k{}^*(\theta_2) + q_k{}^*(\lambda)]$

$= 1 - [1 - [g^*(\theta_1+\eta)q^*(\lambda)pq]]$

$\sum_{k=1}^{\infty} F_k(t)(pq)^{k-1}\ [g^*(\theta_1+\eta)q^*(\lambda)]^{k-1}$

$\therefore L(t) = 1 - S(t)$

$= [1 - pq[g^*(\theta_1+\eta)q^*(\lambda)]]$

$$\sum_{k=1}^{\infty}(f^*(s))^{k-1}(pq)^{k-1}[g^*(\theta_1+\eta)q^*(\lambda)]^{k-1}$$

we assume that $f(.) \sim \exp(\eta)$ and $f^*(s) = \frac{\eta}{\eta+s}$

$g(.) \sim \exp(\beta)$ and $g^*(\theta) = \frac{\beta}{\theta+\beta}$

$q(.) \sim \exp(c)$ and $q^*(\lambda) = \frac{c}{\lambda+c}$

$l^*(s) = \left[1 - \frac{pq\beta c}{(\beta+\theta_1+\eta)(c+\lambda)}\right]\sum_{k=1}^{\infty}\left(\frac{\gamma}{\gamma+s}\right)^{k-1}(pq)^{k-1}$

$$\left[\frac{\beta c}{(\beta+\theta_1+\eta)(c+\lambda)}\right]^{k-1}$$

$$= \left[1 - \frac{pq\beta c}{(\beta+\theta_1+\eta)(c+\lambda)}\right]$$

$$\sum_{k=1}^{\infty}\gamma^{k-1}(\gamma+s)^{-(k-1)}(pq)^{k-1}\left[\frac{\beta c}{(\beta+\theta_1+\eta)(c+\lambda)}\right]^{k-1} \quad (2)$$

$\left.\frac{dl^*(s)}{ds}\right|_{s=0} = \left[1 - \frac{pq\beta c}{(\beta+\theta_1+\eta)(c+\lambda)}\right]\sum_{k=1}^{\infty}\gamma^{k-1} -$

$(k-1)(\gamma+s)^{-(k+1-1)}\ (pq)^{k-1}\left[\frac{\beta c}{(\beta+\theta_1+\eta)(c+\lambda)}\right]^{k-1}$

$= \left[1 - \frac{pq\beta c}{(\beta+\theta_1+\eta)(c+\lambda)}\right]\ \sum_{k=1}^{\infty}\gamma^{k-1}$ -(k-1)

$(\gamma+s)^{-k}(pq)^{k-1}\left[\frac{\beta c}{(\beta+\theta_1+\eta)(c+\lambda)}\right]^{k-1}$ (3)

$-\left.\frac{dl^*(s)}{ds}\right|_{s=0} = \left[1 - \frac{pq\beta c}{(\beta+\theta_1+\eta)(c+\lambda)}\right]\sum_{k=1}^{\infty}\gamma^{k}(k-$

$1)(\gamma)^{-k}(pq)^{k-1}\left[\frac{\beta c}{(\beta+\theta_1+\eta)(c+\lambda)}\right]^{k-1}$

$= \left[1 - \frac{pq\beta c}{(\beta+\theta_1+\eta)(c+\lambda)}\right]\sum_{k=1}^{\infty}(k\text{-}1)\left[\frac{pq\beta c}{(\beta+\theta_1+\eta)(c+\lambda)}\right]^{k-1}$

$= \left[1 - \frac{pq\beta c}{(\beta c+\beta\lambda+\theta_1 c+\theta_1\lambda+\eta c+\eta\lambda)}\right]\sum_{k=1}^{\infty}(k-$

$$1)\left[\frac{pq\beta c}{(\beta c+\beta\lambda+\theta_1 c+\theta_1\lambda+\eta c+\eta\lambda)}\right]^{k-1}$$

$$=\left[1-\frac{pq\beta c}{c(\beta+\theta_1+\eta)+\lambda(\beta+\theta_1+\eta)}\right]\sum_{k=1}^{\infty}(k-$$

$$1)\left[\frac{pq\beta c}{c(\beta+\theta_1+\eta)+\lambda(\beta+\theta_1+\eta)}\right]^{k-1} \quad (4)$$

Now take

$$\sum_{k=1}^{\infty}(k-1)\left[\frac{pq\beta c}{c(\beta+\theta_1+\eta)+\lambda(\beta+\theta_1+\eta)}\right]^{k-1}$$

$$=$$

$$\left[\frac{pq\beta c}{c(\beta+\theta_1+\eta)+\lambda(\beta+\theta_1+\eta)}\right]+2\left[\frac{pq\beta c}{c(\beta+\theta_1+\eta)+\lambda(\beta+\theta_1+\eta)}\right]^2+$$

$$3\left[\frac{pq\beta c}{c(\beta+\theta_1+\eta)+\lambda(\beta+\theta_1+\eta)}\right]^3+\ldots\ldots\ldots$$

$$=\left[\frac{pq\beta c}{c(\beta+\theta_1+\eta)+\lambda(\beta+\theta_1+\eta)}\right]$$

$$\left[1+2\left[\frac{pq\beta c}{c(\beta+\theta_1+\eta)+\lambda(\beta+\theta_1+\eta)}\right]\right.$$

$$\left.+3\left[\frac{pq\beta c}{c(\beta+\theta_1+\eta)+\lambda(\beta+\theta_1+\eta)}\right]^2+\cdots..\right]$$

$$=\left[\frac{pq\beta c}{c(\beta+\theta_1+\eta)+\lambda(\beta+\theta_1+\eta)}\right]$$

$$\left[1-\left[\frac{pq\beta c}{c(\beta+\theta_1+\eta)+\lambda(\beta+\theta_1+\eta)}\right]\right]^{-2}$$

$$=\left[\frac{pq\beta c}{c(\beta+\theta_1+\eta)+\lambda(\beta+\theta_1+\eta)}\right]$$

$$\left[\frac{c(\beta+\theta_1+\eta)+\lambda(\beta+\theta_1+\eta)-pq\beta c}{c(\beta+\theta_1+\eta)+\lambda(\beta+\theta_1+\eta)}\right]^{-2}$$

$$=\frac{pq\beta c[c(\beta+\theta_1+\eta)+\lambda(\beta+\theta_1+\eta)]}{[c(\beta+\theta_1+\eta)+\lambda(\beta+\theta_1+\eta)-pq\beta c]^2} \quad (5)$$

Substitute (5) in (4)

$$=\frac{[c(\beta+\theta_1+\eta)+\lambda(\beta+\theta_1+\eta)-pq\beta c]}{c(\beta+\theta_1+\eta)+\lambda(\beta+\theta_1+\eta)}$$

$$\cdot\frac{pq\beta c[c(\beta+\theta_1+\eta)+\lambda(\beta+\theta_1+\eta)]}{[c(\beta+\theta_1+\eta)+\lambda(\beta+\theta_1+\eta)-pq\beta c]^2}$$

Hence

$$E(T)=\left[\frac{pq\beta c}{c(\beta+\theta_1+\eta)+\lambda(\beta+\theta_1+\eta)-pq\beta c}\right]$$

$$\frac{d^2l^*(s)}{ds^2}=\left[\frac{c(\beta+\theta_1+\eta)+\lambda(\beta+\theta_1+\eta)-pq\beta c}{c(\beta+\theta_1+\eta)+\lambda(\beta+\theta_1+\eta)}\right]\sum_{k=1}^{\infty}\gamma^{k-1}-$$

$$(k-1)(-k)(\gamma+s)^{-k-1}\left[\frac{pq\beta c}{c(\beta+\theta_1+\eta)(c+\lambda)}\right]^{k-1}$$

$$\frac{d^2l^*(s)}{ds^2}\Big|_{s=0}=\left[\frac{c(\beta+\theta_1+\eta)+\lambda(\beta+\theta_1+\eta)-pq\beta c}{c(\beta+\theta_1+\eta)+\lambda(\beta+\theta_1+\eta)}\right]$$

$$\sum_{k=1}^{\infty}\gamma^{k-1}k(k-1)(\gamma)^{-k-1}\left[\frac{pq\beta c}{c(\beta+\theta_1+\eta)(c+\lambda)}\right]^{k-1} \quad (6)$$

Now, take from (6)

$$\sum_{k=1}^{\infty}\gamma^{k-1-k-1}k(k-1)\left[\frac{pq\beta c}{c(\beta+\theta_1+\eta)(c+\lambda)}\right]^{k-1}$$

$$=\sum_{k=1}^{\infty}\gamma^{-2}k(k-1)\left[\frac{pq\beta c}{c(\beta+\theta_1+\eta)+\lambda(\beta+\theta_1+\eta)}\right]^{k-1}$$

$$=\frac{1}{\gamma^2}\sum_{k=1}^{\infty}k(k-1)\left[\frac{pq\beta c}{c(\beta+\theta_1+\eta)+\lambda(\beta+\theta_1+\eta)}\right]^{k-1}$$

$$=\frac{1}{\gamma^2}\left[\sum_{k=1}^{\infty}k(k-1)\left[\frac{pq\beta c}{c(\beta+\theta_1+\eta)+\lambda(\beta+\theta_1+\eta)}\right]^{k}\right.$$

$$\left.\left[\frac{pq\beta c}{c(\beta+\theta_1+\eta)+\lambda(\beta+\theta_1+\eta)}\right]^{-1}\right]$$

$$=\frac{1}{\gamma^2\left[\frac{pq\beta c}{c(\beta+\theta_1+\eta)+\lambda(\beta+\theta_1+\eta)}\right]}$$

$$\left[\sum_{k=1}^{\infty}k(k-1)\left[\frac{pq\beta c}{c(\beta+\theta_1+\eta)+\lambda(\beta+\theta_1+\eta)}\right]^{k}\right]$$

$$=\frac{1}{\gamma^2\left[\frac{pq\beta c}{c(\beta+\theta_1+\eta)+\lambda(\beta+\theta_1+\eta)}\right]}\left[2\left[\frac{pq\beta c}{c(\beta+\theta_1+\eta)+\lambda(\beta+\theta_1+\eta)}\right]^2\right.$$

$$\left.+\left[6\left[\frac{pq\beta c}{c(\beta+\theta_1+\eta)+\lambda(\beta+\theta_1+\eta)}\right]^3+\cdots....\right]\right]$$

$$=\frac{2\left[\frac{pq\beta c}{c(\beta+\theta_1+\eta)+\lambda(\beta+\theta_1+\eta)}\right]^2}{\gamma^2\left[\frac{pq\beta c}{c(\beta+\theta_1+\eta)+\lambda(\beta+\theta_1+\eta)}\right]}\left[1+3\left[\frac{pq\beta c}{c(\beta+\theta_1+\eta)+\lambda(\beta+\theta_1+\eta)}\right]+\right.$$

$$\left.\left[6\left[\frac{pq\beta c}{c(\beta+\theta_1+\eta)+\lambda(\beta+\theta_1+\eta)}\right]^2+\cdots....\right]\right]$$

$$=\frac{2pq\beta c}{\gamma^2 c(\beta+\theta_1+\eta)+\lambda(\beta+\theta_1+\eta)}$$

$$\left[1-\frac{pq\beta c}{c(\beta+\theta_1+\eta)+\lambda(\beta+\theta_1+\eta)}\right]^{-3}$$

$$=\frac{2pq\beta c}{\gamma^2 c(\beta+\theta_1+\eta)+\lambda(\beta+\theta_1+\eta)}\left[\frac{c(\beta+\theta_1+\eta)+\lambda(\beta+\theta_1+\eta)-pq\beta c}{c(\beta+\theta_1+\eta)+\lambda(\beta+\theta_1+\eta)}\right]^{-3}$$

$$= \frac{2pq\beta c}{\gamma^2 c(\beta+\theta_1+\eta)+\lambda(\beta+\theta_1+\eta)}.$$

$$\frac{[c(\beta+\theta_1+\eta)+\lambda(\beta+\theta_1+\eta)]^3}{[c(\beta+\theta_1+\eta)+\lambda(\beta+\theta_1+\eta)-pq\beta c]^3}$$

$$= \frac{2pq\beta c[c(\beta+\theta_1+\eta)+\lambda(\beta+\theta_1+\eta)]^2}{\gamma^2[c(\beta+\theta_1+\eta)+\lambda(\beta+\theta_1+\eta)-pq\beta c]^3} \quad (7)$$

Substitute (7) value in (6)

$$= \left[\frac{c(\beta+\theta_1+\eta)+\lambda(\beta+\theta_1+\eta)-pq\beta c}{c(\beta+\theta_1+\eta)+\lambda(\beta+\theta_1+\eta)}\right].$$

$$\frac{2pq\beta c[c(\beta+\theta_1+\eta)+\lambda(\beta+\theta_1+\eta)]^2}{\gamma^2[c(\beta+\theta_1+\eta)+\lambda(\beta+\theta_1+\eta)-pq\beta c]^3}$$

$$E(T^2) = \frac{2pq\beta c[c(\beta+\theta_1+\eta)+\lambda(\beta+\theta_1+\eta)]}{\gamma^2[c(\beta+\theta_1+\eta)+\lambda(\beta+\theta_1+\eta)-pq\beta c]^2}$$

$$V(T) = E(T^2) - [E(T)]^2$$

$$= \frac{2pq\beta c[c(\beta+\theta_1+\eta)+\lambda(\beta+\theta_1+\eta)]}{\gamma^2[c(\beta+\theta_1+\eta)+\lambda(\beta+\theta_1+\eta)-pq\beta c]^2} - \left[\frac{pq\beta c}{c(\beta+\theta_1+\eta)+\lambda(\beta+\theta_1+\eta)-pq\beta c}\right]^2$$

$$= \frac{2pq\beta c[c(\beta+\theta_1+\eta)+\lambda(\beta+\theta_1+\eta)]}{\gamma^2[c(\beta+\theta_1+\eta)+\lambda(\beta+\theta_1+\eta)-pq\beta c]^2} - \frac{[pq\beta c]^2}{[c(\beta+\theta_1+\eta)+\lambda(\beta+\theta_1+\eta)-pq\beta c]^2}$$

$$V(T) = \frac{2pq\beta c[c(\beta+\theta_1+\eta)+\lambda(\beta+\theta_1+\eta)]-\gamma^2[pq\beta c]^2}{\gamma^2[c(\beta+\theta_1+\eta)+\lambda(\beta+\theta_1+\eta)-pq\beta c]^2}$$

Table 1. Variation in E(T) and V(T) for Changes in

θ_1 and $p = 0.6, q = 0.7, \beta = 0.5, c = 0.4, \eta = 0.5, \lambda = 0.5, \gamma = 0.8$

θ1	E(T)	V(T)
0.6	0.0649	0.2017
0.7	0.0607	0.1887
0.8	0.057	0.1772
0.9	0.0537	0.167
1	0.0508	0.158
1.1	0.0481	0.1499
1.2	0.0458	0.1426
1.3	0.0436	0.1359

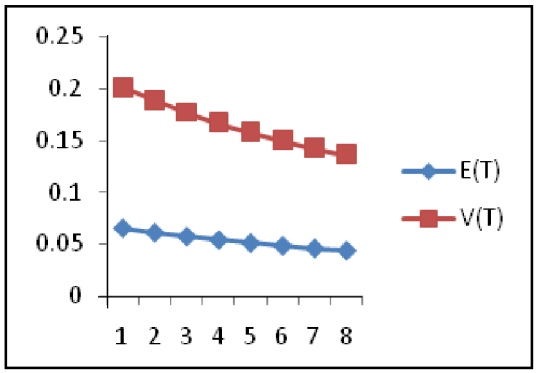

Fig. 1. Variation in E(T) and V(T) for Changes in θ_1

Table 2. Variation in E(T) and V(T) for Changes in β and

$p = 0.6, q = 0.7, \theta_1 = 0.6, c = 0.4, \eta = 0.5, \lambda = 0.5, \gamma = 0.8$

β	E(T)	V(T)
0.5	0.0649	0.1291
0.6	0.0744	0.1865
0.7	0.0831	0.2576
0.8	0.0911	0.282
0.9	0.0984	0.3044
1	0.1052	0.3251
1.1	0.1115	0.3442
1.2	0.1174	0.3619
1.3	0.1228	0.3784

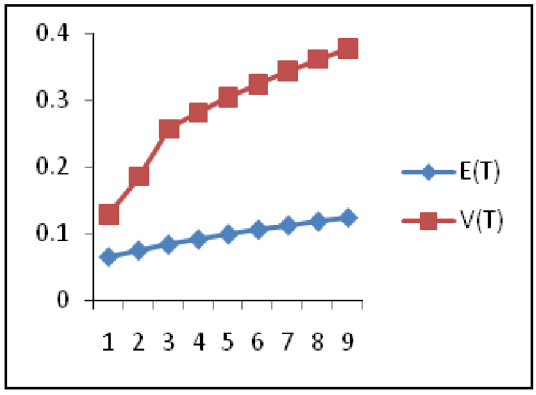

Fig. 2. Variation in E(T) and V(T) for Changes in β

Table 3. Variation in E(T) and V(T) for Changes in λ and

$p = 0.6, q = 0.7, \beta = 0.5, c = 0.4, \eta = 0.5, \theta_1 = 0.6, \gamma = 0.8$

λ	E(T)	V(T)
0.5	0.0649	0.2017
0.6	0.0578	0.1796
0.7	0.052	0.1619
0.8	0.0473	0.1474
0.9	0.04345	0.1352
1	0.0401	0.1249
1.1	0.0372	0.1161
1.2	0.0348	0.1084

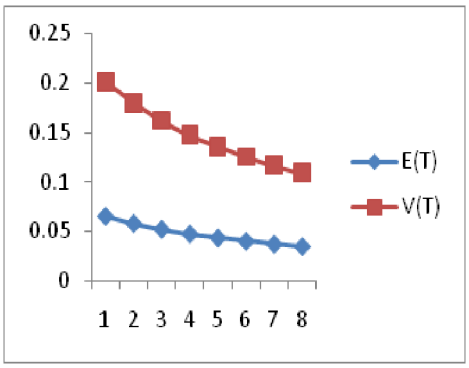

Fig. 3. Variation in E(T) and V(T) for Changes in λ

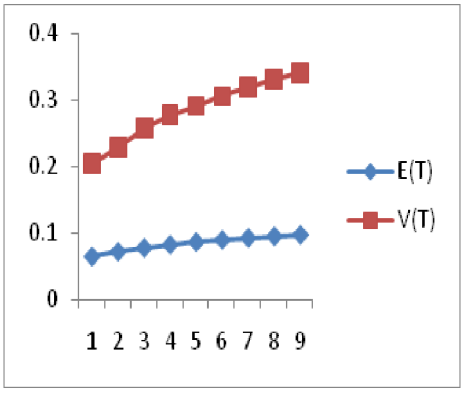

Fig. 4. Variation in E(T) and V(T) for Changes in C

Table 4. Variation in E(T) and V(T) for Changes in C and

$p = 0.6, q = 0.7, \beta = 0.5, \lambda = 0.5, \eta = 0.5, \theta_1 = 0.6, \gamma = 0.8$

C	E(T)	V(T)
0.4	0.0649	0.2047
0.5	0.0722	0.23
0.6	0.0781	0.258
0.7	0.0829	0.278
0.8	0.0869	0.291
0.9	0.0903	0.306
1	0.0932	0.3192
1.1	0.0957	0.3308
1.2	0.0979	0.3412

Table 5. Variation in E(T) and V(T) for Changes in η and

$p = 0.6, q = 0.7, \beta = 0.5, \lambda = 0.5, C = 0.4, \theta_1 = 0.6, \gamma = 0.8$

η	E(T)	V(T)
0.5	0.0649	0.2017
0.6	0.0607	0.1887
0.7	0.057	0.1772
0.8	0.0537	0.167
0.9	0.0508	0.1553
1	0.0481	0.1499
1.1	0.0458	0.1426
1.2	0.0436	0.1359

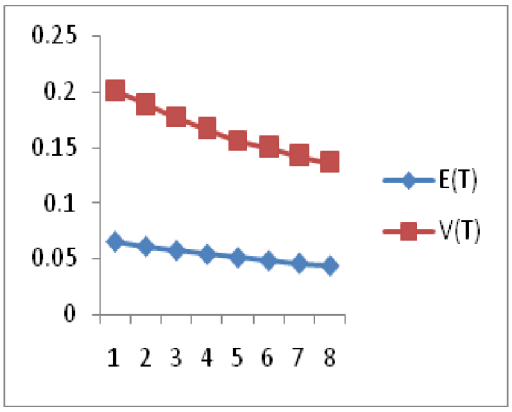

Fig. 5. Variation in E(T) and V(T) for Changes in η

V. Conclusion

The changes in the values of E(T) and V(T) due to changes in various parameters involved in the model is discussed on the basis of the numerical illustration taken up

1. If θ_1 which is the parameter of the exponential distribution prior to the truncation point in the case of the antigenic diversity threshold increases then E(T) and V(T) both decreases. This is due to the fact that the antigenic threshold follows exponential distribution and hence $E(Z_1)$ becomes smaller as increases. Hence it taken less time to cross the antigenic diversity threshold.

2. If β which is the parameter of the exponential distribution for the random variable denoting the contribution to antigenic diversity increases then $E(X) = \frac{1}{\beta}$ decreases. Therefore there is less of contribution to antigenic diversity and show E(T) and V(T) increases as indicated in table 2.

3. If λ increases there is a decrease in E(T) and V(T) this is due to the fact that λ is the parameter of the exponential distribution of the random variable denoting the virulence threshold. So $E(Z_1) = \frac{1}{\lambda}$. It decreases has λ increases, so that, the threshold is smaller and hence E(T) and V(T) decreases.

4. The contribution to virulence in every contact is denoted by the random variable Y. It follows exponential distribution with parameter C. So $E(Y) = \frac{1}{C}$ decreases, as C increases. It means that there is less of contribution to virulence in every contact. Therefore it takes more time to cross the virulence threshold and so E(T) and V(T) increases.

5. The random variable which denotes the interarrival time between contacts follow exponential distribution with parameter η. Therefore the $E(U) = \frac{1}{\eta}$. This decreases as η increases. Therefore the interarrival times between contacts is smaller. Therefore as η increases the contacts will be more frequent and so it takes less time to cross the antigenic diversity threshold. Hence T(T) and V(T) decreases.

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