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Metrics of TOUGMA

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Abstract

In this paper, we present a TOUGMA's Field Equation solutions. A no electrical charge and non-rotating body in vacuum is used to determine the metrics that TOUGMA's field equation allows us. Then we calculate the ricci scalar and tensor components and deduce solutions. Two solutions are found, one real and the other is complex. Those solutions describe others universe properties that still unknow.

Key words : Ricci Tensor, Hermitian black hole, metric of TOUGMA, quantum relativity.

I. Introduction

TOUGMA's Quantum Relativity theory, first published in 2021, is a geometric quantum gravity theory. In this theory quantum and gravity are entangled and theorized to be a space-time curvature and it's dimensions interaction and manifestation caused by α-dimensions massive object[10].

$$[(\alpha-3)R_{uv}-\frac{1}{2}g_{uv}R](1+\frac{2kL_m}{R})-2k(\alpha-3)g_{uv}L_m=T_{uv} \tag{1}$$

The solution imposed spherically symmetric metric, caused by no electrical charge and non rotating n-dimensions massive object in vacum,
to approch such a solution, the method used is calculating Ricci Ten-sor components for a metric general form for some conditions and give their equating to 0. To do this we going to start by calculating

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Christoffel Symbols. After we are going to calculate the Ricci tensor components and solving fields equation and finish by study physics phenomena.

II. **Methods**

Let's take[7 6 9 8 2 4 53]:

$$ds^2 = -e^{2\gamma}c^2dt^2 + e^{2\delta}dr^2 + r^2(d\theta^2 + r^2sin^2\theta d\varphi^2) \ + r^{\alpha-4}d\Omega_{\alpha-4} \tag{2}$$

the g^{uv} are:

$$g^{uv} = diag(-e^{-2\gamma}, e^{-2\delta}, \frac{1}{r^2}, \frac{1}{r^2sin^2}, \frac{1}{r^{\alpha-4}}) \tag{3}$$

then the Christoffel symbols are[1]:

$$\Gamma^u_{ij} = g^{uv}(\frac{\partial g_{vj}}{\partial x^i} + \frac{\partial g_{iv}}{\partial x^j} - \frac{\partial g_{ij}}{\partial x^v}) \tag{4}$$

$$\Gamma^0_{0r} = -e^{-2\gamma(r)}\gamma(r)e^{2\gamma(r)} = \gamma(r) = \Gamma^0_{r0} \tag{5}$$

$$\Gamma^r_{00} = \gamma' e^{-2(\gamma-\delta)} \tag{6}$$

$$\Gamma^r_{rr} = \delta' \tag{7}$$

$$\Gamma^r_{\theta\theta} = -re^{-2\delta} \tag{8}$$

$$\Gamma^r_{\varphi\varphi.} = -rsin^2\delta e^{-2\delta} \tag{9}$$

$$\Gamma^r_{\Omega_{\alpha-4}\Omega_{\alpha-4}} = -(\alpha-4)r^{\alpha-7} \tag{10}$$

$$\Gamma^\theta_{r\theta} = \Gamma^\theta_{\theta r} = \frac{1}{r} \tag{11}$$

$$\Gamma^\theta_{\varphi\varphi} = -cos\theta sin\theta \tag{12}$$

$$\Gamma^\varphi_{r\varphi} = \Gamma^\varphi_{\varphi r} = \frac{1}{r} \tag{13}$$

$$\Gamma^\varphi_{\theta\varphi} = \Gamma^\varphi_{\varphi\theta} = \frac{1}{tan\theta} \tag{14}$$

$$\Gamma^{\Omega_{\alpha-4}}_{r\Omega_{\alpha-4}} = \Gamma^{\Omega_{\alpha-4}}_{\Omega_{\alpha-4}r} = \frac{1}{2r^{\alpha-4}}(-\frac{(\alpha-4)r^{\alpha-5}}{r^{2\alpha-8}}) = -\frac{(\alpha-4)}{r^{2\alpha-7}} \tag{15}$$

and Ricci tensor componets are :

$$R_{00} = e^{2(\gamma-\delta)}[\gamma'' + (\gamma')^2 - \gamma'\delta' + \frac{2\gamma'}{r} - \frac{(\alpha-4)\gamma'}{r^{2\alpha-7}}] \tag{16}$$

and for r≫0

$$R_{00} = e^{2(\gamma-\delta)}[\gamma'' + (\gamma')^2 - \gamma'\delta' + \frac{2\gamma'}{r}] \tag{17}$$

$$R_{rr} = -\gamma'' - (\gamma')^2 + \gamma'\delta' + \frac{2\delta'}{r} - \frac{(\alpha-4)(2n-7)}{r^{2\alpha-6}} - \frac{(\alpha-4)\delta'}{r^{2\alpha-7}} + \frac{(\alpha-4)^2}{r^{4\alpha-14}} \tag{18}$$

and for r≫0

$$R_{rr} = -\gamma'' - (\gamma')^2 + \gamma'\delta' + \frac{2\delta'}{r} \tag{19}$$

for r very large the others components are:

$$R_{\theta\theta} = e^{-2\delta}[r(\delta-\gamma)-1]+1 \tag{20}$$

$$R_{\varphi\varphi} = sin^2\theta e^{-2\delta}[r(\delta-\gamma)-1]+1 \tag{21}$$

$$R_{\Omega_{n-4}\Omega_{n-4}} \simeq 0 \tag{22}$$

the others components are zero. Ricci scalar is given by $R = g^{ij} R^{ij}$:

$$R = -e^{-2\gamma(r)}R_{00} + e^{-2\delta(r)}R_{rr} + \frac{1}{r^2}R_{\theta\theta} \ + \frac{1}{r^2 sin^2\theta} \tag{23}$$

and the calculated :

$$R = 2e^{-2\alpha}[\nu' + (\nu')^2 - \nu'\alpha' + \frac{2}{r}\nu' + \frac{2}{r}(\alpha'-\nu') \ + \frac{2}{r^2}(e^{2\alpha}-1)] \tag{24}$$

III. **Results and Discussion**

Now we are going to resolve TOUGMA's field equation reduce:

$$(\alpha-3)R_{uv} - \frac{1}{2}g_{uv}R - kg_{uv}L_m = T_{uv} \tag{25}$$

with R_{ij} and R, we have :

$$W_{00} = (\alpha - 3)R_{00} - \frac{1}{2}R(-e^{2\gamma}) - kL_m(-e^{2\gamma}) \tag{26}$$

$$W_{rr} = (\alpha - 3)R_{rr} - \frac{1}{2}R(-e^{2\delta}) - kL_m(e^{2\delta}) \tag{27}$$

$$W_{\theta\theta} = (\alpha - 3)R_{\theta\theta} - \frac{1}{2}Rr^2 - kL_m r^2 \tag{28}$$

$$W_{\varphi\varphi} = (\alpha - 3)R_{\varphi\varphi} - \frac{1}{2}Rr^2 sin^2\theta - kL_m r^2 sin^2\theta \tag{29}$$

If we replace R_{ij} and R by their terms, we have :

$$W_{00} = (\alpha-4)[\gamma'' + \gamma'^2 - \gamma'\delta' + \frac{2}{r}\gamma'] \ - \frac{1}{r^2}(e^{2\delta} - 1 + 2r\delta') + kL_m e^{2\gamma} \tag{30}$$

$$W_{rr} = (\alpha-4)[\gamma'' + \gamma'^2 - \gamma'\delta' + \frac{2(\alpha-2)}{(\alpha-4)r}\gamma'] \ - \frac{1}{r^2}(-e^{2\delta} + 1 + 2r\gamma') - kL_m e^{\delta} \tag{31}$$

$$W_{\theta\theta} = r^2 e^{-2\delta}[\gamma'' + \gamma'^2 - \gamma'\delta' + \frac{\alpha-5}{r}(\delta'-\gamma')] \ + e^{-2\delta} + (\alpha-4) - 2kl_m r^2 \tag{32}$$

$$W_{\varphi\varphi} = sin^2(\theta)[r^2e^{-2\delta}[\gamma'' + \gamma'^2 - \gamma'\delta' + \frac{\alpha-5}{r}(\delta' - \gamma')] + e^{-2\delta} + (\alpha-4) - 2kl^m r^2] \quad (33)$$

if $T_{uv} = 0$. TOUGMA' equation become :

$$(\alpha-4)[\gamma'' + \gamma'^2 - \gamma'\delta' + \frac{2}{r}\gamma'] - \frac{1}{r^2}(e^{2\delta} - 1 + 2r\delta') + kL_m e^{2\gamma} = 0 \quad (34)$$

$$(\alpha-4)[\gamma'' + \gamma'^2 - \gamma'\delta' + \frac{2(\alpha-2)}{(\alpha-4)r}\gamma'] - \frac{1}{r^2}(-e^{2\delta} + 1 + 2r\gamma') - kL_m e^{2\delta} = 0 \quad (35)$$

$$r^2e^{-2\delta}[\gamma\text{'} + \gamma'^2 - \gamma'\delta' + \frac{\alpha-5}{r}(\delta' - \gamma')] + e^{-2\delta} + (\alpha-4) - 2kl_m r^2 = 0 \quad (36)$$

$$sin^2\theta[r^2e^{-2\delta}[\gamma\text{'} + \gamma'^2 - \gamma'\delta' + \frac{\alpha-5}{r}(\delta' - \gamma')] + e^{-2\delta} + (\alpha-4) - 2kl^m r^2] = 0 \quad (37)$$

by taken equation 43-45:

$$[(2\alpha-6)r - 2(\alpha-2)r^2]\gamma' - 2r\delta' + kL_m r^2(e^{2\gamma} + e^{\delta}) + 2 = 0 \quad (38)$$

the term $(e^{2\gamma}+e^{\delta})$ sugests that $\delta = -\gamma$ and $\gamma = iN(r)$; then it happens :

$$i[(2\alpha-6)r - 2(\alpha-2)r^2 + 2r]N' + 2kL_m r^2 cos(2N) = -2 \quad (39)$$

if we take this equation wihtout second member, we have:

$$i[(2\alpha-6)r - 2(\alpha-2)r^2 + 2r]N' + 2kL_m r^2 cos(2N) = 0 \quad (40)$$

then

$$i[(2\alpha-6)r - 2(\alpha-2)r^2 + 2r]N' = -2kL_m r^2 cos(2N) \quad (41)$$

and

$$i\frac{N'}{cos(2N)} = -2kL_m \frac{r^2}{[(2\alpha-6)r - 2(\alpha-2)r^2 + 2r]} \quad (42)$$

$$i\int \frac{dN}{cos(2N)} = -2kL_m \int \frac{r^2}{[(2\alpha-6)r - 2(\alpha-2)r^2 + 2r]}dr \quad (43)$$

$$i2ln[tan(N + \frac{\pi}{4})] = -\frac{kL_m}{2(\alpha-2)} \int \frac{r}{[1-r]}dr \quad (44)$$

$$i2ln[tan(N + \frac{\pi}{4})] = -\frac{kL_m}{2(\alpha-2)} \int [1 + \frac{1}{[1-r]}]dr \quad (45)$$

$$i2ln[tan(N + \frac{\pi}{4})] = -\frac{kL_m}{2(\alpha-2)}[1 - ln(1-r)] \quad (46)$$

$$[tan(N + \frac{\pi}{4})]^{2i} = e^{\frac{-kL_m}{2(\alpha-2)}} \frac{e^1}{(1-r)} \quad (47)$$

$$[tan(N + \frac{\pi}{4})] = \frac{e^{\frac{2(\alpha-2)-kL_m}{4i(\alpha-2)}}}{\sqrt{(1-r)}} \quad (48)$$

then the general solution is :

$$N = tan^{-1}[\frac{e^{\frac{2(\alpha-2)-kL_m}{4i(\alpha-2)}}}{\sqrt{(1-r)}}] - \frac{\pi}{4} + C \tag{49}$$

we have the metric :

$$ds^2 = -\exp(2i[tan^{-1}(\frac{e^{\frac{2(\alpha-2)-kL_m}{4i(\alpha-2)}}}{\sqrt{(1-r)}}) - \frac{\pi}{4}])c^2dt^2 + \exp(-2i[tan^{-1}(\frac{e^{\frac{2(\alpha-2)-kL_m}{4i(\alpha-2)}}}{\sqrt{(1-r)}}) - \frac{\pi}{4}])dr^2 + r^2(d\theta^2 + r^2 sin^2\theta d\varphi^2) + r^{\alpha-4}d\Omega_{\alpha-4} \tag{50}$$

$$ds^2 = -\exp(2i[tan^{-1}(\frac{e^{i\frac{[2(\alpha-2)-kL_m]}{4(\alpha-2)}}}{\sqrt{(1-r)}}) - \frac{\pi}{4}])c^2dt^2 + \exp(-2i[tan^{-1}(\frac{e^{i\frac{[2(\alpha-2)-kL_m]}{4(\alpha-2)}}}{\sqrt{(1-r)}}) - \frac{\pi}{4}])dr^2 + r^2(d\theta^2 + r^2 sin^2\theta d\varphi^2) + r^{\alpha-4}d\Omega_{\alpha-4} \tag{51}$$

with dim(r)=α for non rotating space-time.

if the term $(e^{2\gamma}+e^{\delta})$ sugests that $\delta = -\gamma$ and $\gamma = N$; then it happens :

$$[(2\alpha-6)r - 2(\alpha-2)r^2 + 2r]N' + 2kL_m r^2 cosh(2N) = -2 \tag{52}$$

if we take this equation witout second member, we have :

$$[(2\alpha-6)r - 2(\alpha-2)r^2 + 2r]N' + 2kL_m r^2 cosh(2N) = 0 \tag{53}$$

then

$$[(2\alpha-6)r - 2(\alpha-2)r^2 + 2r]N' = -2kL_m r^2 cosh(2N) \tag{54}$$

and

$$\frac{N'}{cosh(2N)} = -2kL_m \frac{r^2}{[(2\alpha-6)r - 2(\alpha-2)r^2 + 2r]} \tag{55}$$

$$\int \frac{dN}{cosh(2N)} = -2kL_m \int \frac{r^2}{[(2\alpha-6)r - 2(\alpha-2)r^2 + 2r]}dr \tag{56}$$

$$2artan(e^N) = -\frac{kL_m}{2(\alpha-2)} \int \frac{r}{[1-r]}dr \tag{57}$$

$$2artan(e^N) = -\frac{kL_m}{2(\alpha-2)} \int [1 + \frac{1}{[1-r]}]dr \tag{58}$$

$$2artan(e^N) = -\frac{kL_m}{2(\alpha-2)}[1 - ln(1 - r)] \tag{59}$$

$$\exp(N) = \tan(-\frac{kL_m}{4(\alpha-2)}[1 - ln(1 - r)]) \tag{60}$$

$$N = \ln(\tan(-\frac{kL_m}{4(\alpha-2)}[1 - ln(1 - r)])) \tag{61}$$

then the general solution is :

$$N = \ln(\tan(-\frac{KL_m}{4(\alpha-2)}[1 - ln(1 - r)])) + C \tag{62}$$

and

$$\gamma = \ln(\tan(-\frac{kL_m}{4(\alpha-2)}[1 - ln(1 - r)])) + C \tag{63}$$

we have the metric :

$$ds^2 = -[\tan(-\frac{kL_m}{4(\alpha-2)}[1 - ln(1-r)])]^2 c^2 dt^2 + \frac{dr^2}{[\tan(-\frac{kL_m}{4(\alpha-2)}[1 - ln(1-r)])]^2} + r^2(d\theta^2 + r^2 sin^2\theta d\varphi^2) + r^{\alpha-4} d\Omega_{\alpha-4} \tag{64}$$

with dim (r)=α for non rotating space-time.

Scope of Future Work :

The solutions of Equation TOUGMA are crucials to understand the properties of universe, they use the quantum lagragian to describe the struture of universe.

The present investigation will be very helpful to the researchers who are engage for the resarch work in cosmological models depending of quantum field.

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References

1. B.H.E... *http ://blackholes.stardate.org/.,.*
2. J.P.M.J. Baez. Gauge fields. *knots and gravity. World Scientific*, 1994.
3. Dhirendra Chhajed and al. Magnetized bianchi type-vi0 string cosmological model for anti-stiff fluid in general relativity. *JUSPS-A Vol. 34(4),* 2022.
4. Y. Choquet-Bruhat. Géométrie différentielle et systèmes extérieurs. *Monographies Universitaires de Mathématiques, Dunod,* 1968.
5. A.F.B. Doubrovine and S. Novikov. Géométrie contemporaine. *Vol. I et Vol. II, Ed. MIRMoscou,* 1979.
6. D. Husemoller. Fibre bundles. *Springer, 1994.,* 1994.
7. Lee. Riemannian manifolds.
8. K. Nomizu. Lie groups and differential geometry. *The Mathematical Society of Japan,* 1956.
9. B. Schutz. Geometrical methods of mathematical physics. *Cambridge University Press,* 1980.
10. TOUGMA and al. The quantum relativity in extra-dimensions. *International Journal of Science and Research (IJSR),* 2021.