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Deriving the physical phenomena of real TOUGMA's solution

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Abstract

In this paper, we derive the physical phenomena that implies the real TOUGMA's metric. We firstly found radial light geodesics and orbits of material bodies and finish by studied their physical characteristics. Due to astrophysical applications, the interest of studying the TOUGMA metric right now is to implement using a metric concrete the new physical concepts that implies this metric.

Key-Words : Ricci Tensor, metric of TOUGMA, quantum relativity.

I. Introduction

Relativistic astrophysics occupies a growing part in contemporary astronomy. Particularly in view of the large amount of data generated are either cosmologies, or involve compact objects (black holes, neutron stars). In the both cases, the theoretical basis of their study was general relativity and quantum mechanics. But TOUGMA have been given recently et unified equation of quantum langrangian and gravity, nonmed Quantum Relativity theory, published in 2021 by TOUGMA and all⁶.

$$[(\alpha - 3)R_{uv} - \frac{1}{2}g_{uv}R](1 + \frac{2kL_m}{R}) - 2k(\alpha - 3)g_{uv}L_m = T_{uv} \quad (1)$$

It has resolved and one of solutions is given by:

$$ds^2 = -[\tan(-\frac{kL_m}{4(\alpha-2)}[1 - \ln(1-r)])]^2 c^2 dt^2 + \frac{dr^2}{[\tan(-\frac{kL_m}{4(\alpha-2)}[1 - \ln(1-r)])]^2} + r^2(d\theta^2 + r^2 \sin^2 \theta d\varphi^2) + r^{\alpha-4} d\Omega_{\alpha-4} \quad (2)$$

The gravitational field of bodies with spherical symmetry is obviously of importance capital in astrophysics. To arrive inunediatly at interesting applications of astrophysical interest, we are going to study the physical concepts of this TOUGMA's metric.

II. Methods

A solution of TOUGMA's equation that can be defined by the existence of a coordinate system $(x^u) = (ct, r, \theta, \varphi, \Omega_{\alpha-4})$, called TOUGMA coordinates, such that the components g_{uy} of the metric tensor g are written there

$$ds^2 = -[\tan(-\frac{kL_m}{4(\alpha-2)}[1 - \ln(1-r)])]^2 c^2 dt^2 + \frac{dr^2}{[\tan(-\frac{kL_m}{4(\alpha-2)}[1 - \ln(1-r)])]^2} + r^2(d\theta^2 + r^2 \sin^2 \theta d\varphi^2) + r^{\alpha-4} d\Omega_{\alpha-4} \quad (3)$$

The first observation that we can made in view of (3) is that the space-time (E, g) is static and spherically symmetric. The metric components are clearly independent of t and, $\vec{\partial}_t \cdot \vec{\partial}_t = -[\tan(-\frac{kL_m}{4(\alpha-2)}[1 - \ln(1-r)])]^2 c^2 < 0$, and then that ∂_t is time-like; we conclude that spacetime is stationary. As for the spherical symmetry, it is immediate because the components g_{uy} given by (3). Moreover, the space-time described by the TOUGMA metric is asymptotical^{5 142} we have in effect if $r=0$

$$[\tan(-\frac{kL_m}{4(\alpha-2)}[1 - \ln(1-r)])]^2 = [\tan(-\frac{kL_m}{4(\alpha-2)})]^2 \quad (4)$$

that we are going to study the physicals phenomena of this function in the next section. as limites we have:

$$\lim_{r \rightarrow -\infty} [\tan(-\frac{kL_m}{4(\alpha-2)}[1 - \ln(1-r)])]^2 = +\infty \quad (5)$$

and

$$\lim_{r \rightarrow 1} [\tan(-\frac{kL_m}{4(\alpha-2)}[1 - \ln(1-r)])]^2 = +\infty \quad (6)$$

A. Finding Radial Light Geodesics :

Let us place ourselves in the frame of the TOUGMA coordinates $(x^u) = (ct, r, \theta, \varphi, \Omega_{\alpha-4})$. A light geodesic is a geodesic of zero length: we must therefore have along this one

$$ds^2 = g_{uv} dx^u dx^v = 0 \quad (7)$$

On the other hand, if we assume the radial geodesic, then $d\theta = 0$ and $d\varphi = 0 = d\Omega_{\alpha-4}$ along it. It happens :

$$-\left[\tan\left(-\frac{kL_m}{4(\alpha-2)}[1-\ln(1-r)]\right)\right]^2 c^2 dt^2 + \frac{dr^2}{\left[\tan\left(-\frac{kL_m}{4(\alpha-2)}[1-\ln(1-r)]\right)\right]^2} = 0 \quad (8)$$

$$-\left[\tan\left(-\frac{kL_m}{4(\alpha-2)}[1-\ln(1-r)]\right)\right]^2 c^2 dt^2 = -\frac{dr^2}{\left[\tan\left(-\frac{kL_m}{4(\alpha-2)}[1-\ln(1-r)]\right)\right]^2} \quad (9)$$

$$\left[\tan\left(-\frac{kL_m}{4(\alpha-2)}[1-\ln(1-r)]\right)\right]^2 c^2 dt^2 = +\frac{dr^2}{\left[\tan\left(-\frac{kL_m}{4(\alpha-2)}[1-\ln(1-r)]\right)\right]^2} \quad (10)$$

$$\left[\tan\left(-\frac{kL_m}{4(\alpha-2)}[1-\ln(1-r)]\right)\right] c dt = \pm \frac{dr}{\left[\tan\left(-\frac{kL_m}{4(\alpha-2)}[1-\ln(1-r)]\right)\right]} \quad (11)$$

$$c dt = \pm \frac{dr}{\left[\tan\left(-\frac{kL_m}{4(\alpha-2)}[1-\ln(1-r)]\right)\right]^2} \quad (12)$$

$$ct = \pm \int_{r_0}^r \frac{dr}{\left[\tan\left(-\frac{kL_m}{4(\alpha-2)}[1-\ln(1-r)]\right)\right]^2} \quad (13)$$

$$ct = \pm \left[\frac{4(\alpha-2)(r-1) \cos^2\left(-\frac{kL_m}{4(\alpha-2)}[1-\ln(1-r)]\right)}{kL_m \tan\left(-\frac{kL_m}{4(\alpha-2)}[1-\ln(1-r)]\right)} \right]_{r_0}^r \quad (14)$$

$$ct = \pm \left[\frac{4(\alpha-2)(r-1) \cos^2\left(-\frac{kL_m}{4(\alpha-2)}[1-\ln(1-r)]\right)}{kL_m \tan\left(-\frac{kL_m}{4(\alpha-2)}[1-\ln(1-r)]\right)} - \frac{4(\alpha-2)(r_0-1) \cos^2\left(-\frac{kL_m}{4(\alpha-2)}[1-\ln(1-r_0)]\right)}{kL_m \tan\left(-\frac{kL_m}{4(\alpha-2)}[1-\ln(1-r_0)]\right)} \right] \quad (15)$$

Due to the \pm , we obtain two families of radial geodesics, which can be classified as following :

- the outgoing geodesics, for which $dr / dt > 0$; their equations are

$$ct = \frac{4(\alpha-2)(r-1) \cos^2\left(-\frac{kL_m}{4(\alpha-2)}[1-\ln(1-r)]\right)}{kL_m \tan\left(-\frac{kL_m}{4(\alpha-2)}[1-\ln(1-r)]\right)} - \frac{4(\alpha-2)(r_0-1) \cos^2\left(-\frac{kL_m}{4(\alpha-2)}[1-\ln(1-r_0)]\right)}{kL_m \tan\left(-\frac{kL_m}{4(\alpha-2)}[1-\ln(1-r_0)]\right)} \quad (16)$$

- incoming geodesice, for which $dr / dt < 0$; their equations are

$$ct = \frac{4(\alpha-2)(r_0-1) \cos^2\left(-\frac{kL_m}{4(\alpha-2)}[1-\ln(1-r_0)]\right)}{kL_m \tan\left(-\frac{kL_m}{4(\alpha-2)}[1-\ln(1-r_0)]\right)} - \frac{4(\alpha-2)(r-1) \cos^2\left(-\frac{kL_m}{4(\alpha-2)}[1-\ln(1-r)]\right)}{kL_m \tan\left(-\frac{kL_m}{4(\alpha-2)}[1-\ln(1-r)]\right)} \quad (17)$$

the physicals phenomena are going to be studied in the next section

B. Orbits of material bodies :

Let us now examine the mass bodies trajectories (orbits) of $m \ll M$ around of the central body of the TOUGMA metric. As we saw in § 2., these trajectories must be time-like geodesics. If

the subsequent trajectory deviates towards one of the two hemispheres separated by this equator, this would represent a break in the spherical symmetry. Thus the particle must remain in the plane and for a specific $\Omega_{\alpha-4}$

$$\theta = \frac{\pi}{2} \quad (18)$$

and

$$\Omega_{\alpha-4} = cste \quad (19)$$

From Kelling vectors we have :

$$\varepsilon = -\frac{c}{m}\vec{\chi}_0\vec{p} = -c^2\vec{\chi}_0\vec{v} \quad (20)$$

$$l = \frac{1}{m}\vec{\chi}_z\vec{p} = c\vec{\chi}_z\vec{v} \quad (21)$$

that implies :

$$\tan^2\left(-\frac{kL_m}{4(\alpha-2)}[1-\ln(1-r)]\right)\frac{dt}{d\tau} \quad (22)$$

$$l = r^2 \sin^2 \theta \frac{d\varphi}{d\tau} \quad (23)$$

the 5-components of the pentavector \vec{v} are

$$v^0 = \tan^{-2}\left(-\frac{kL_m}{4(\alpha-2)}[1-\ln(1-r)]\right)\frac{\varepsilon}{c^2} \quad (24)$$

$$v^\theta = 0 \quad (25)$$

$$v^\varphi = \frac{l}{cr^2} \quad (26)$$

$$v^{\Omega_{\alpha-4}} = 0 \quad (27)$$

by $\vec{v}.\vec{v} = -1$ the renormilization equation, it happends :

$$-\tan^{-2}\left(-\frac{kL_m}{4(\alpha-2)}[1-\ln(1-r)]\right)\frac{\epsilon^2}{c^4} + \tan^{-2}\left(-\frac{kL_m}{4(\alpha-2)}[1-\ln(1-r)]\right)(v^r)^2 + \frac{l^2}{c^2 r^2} = -1 \quad (29)$$

$$-\frac{\epsilon^2}{c^2} + (v^r)^2 = -\left(\frac{l^2}{c^2 r^2} + 1\right) \tan^2\left(-\frac{kL_m}{4(\alpha-2)}[1-\ln(1-r)]\right) \quad (30)$$

$$\frac{1}{2}\left(\frac{dr}{d\tau}\right)^2 = -\frac{1}{2}\left[\left(\frac{l^2}{c^2 r^2} + 1\right) \tan^2\left(-\frac{kL_m}{4(\alpha-2)}[1-\ln(1-r)]\right) - \frac{\epsilon^2}{c^2}\right] \quad (31)$$

$$V_{eff}(r) = -\frac{1}{2}\left[\left(\frac{l^2}{c^2 r^2} + 1\right) \tan^2\left(-\frac{kL_m}{4(\alpha-2)}[1-\ln(1-r)]\right) - \frac{\epsilon^2}{c^2}\right] \quad (32)$$

$$g_{00}(v^0)^2 + g_{rr}(v^r)^2 + g_{\theta\theta}(v^\theta)^2 + g_{\varphi\varphi}(v^\varphi)^2 + g_{\Omega\alpha-4}(v^{\Omega\alpha-4})^2 = -1 \quad (28)$$

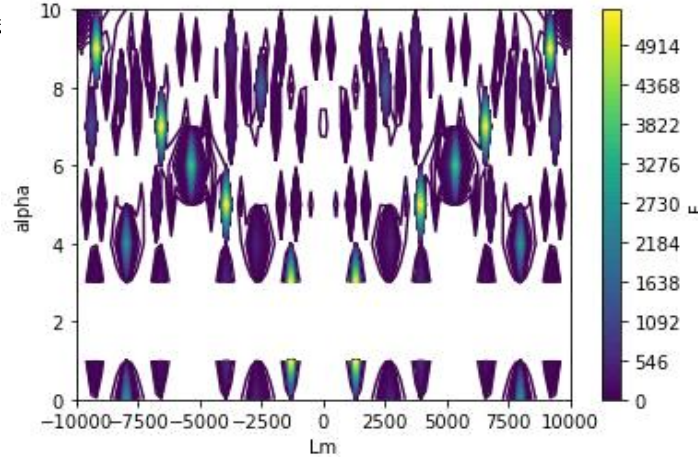
the physicals phenomena of this are going to be studied in the next section

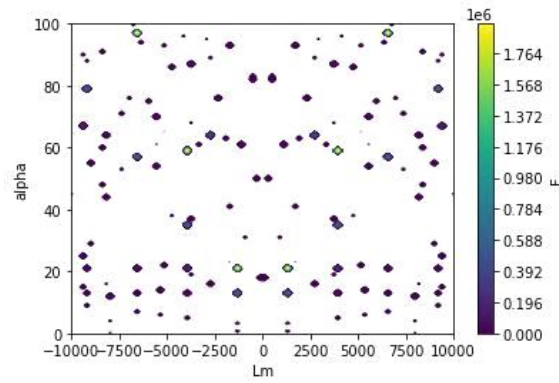
III. Results and Discussion

Now, we are going to give the physicals phenomena of last section equation. The first is

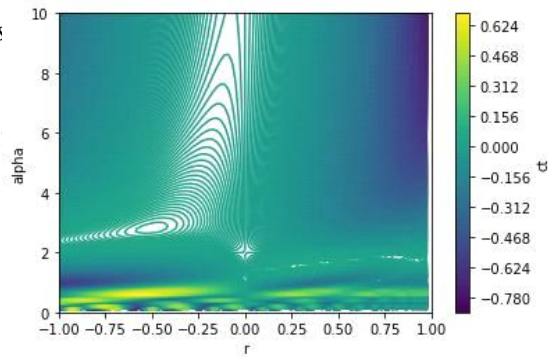
$$\left[\tan\left(-\frac{kL_m}{4(\alpha-2)}\right)\right]^2 \quad (33)$$

Ploted it with L_m and α variables we have We can see in the first figure that there is no universe forms with $\alpha \in [1,3]$ also we can see the best possibility of universes formation according quantum Lagrangian L_m . The $\alpha \in [1,100]$ according L_m and

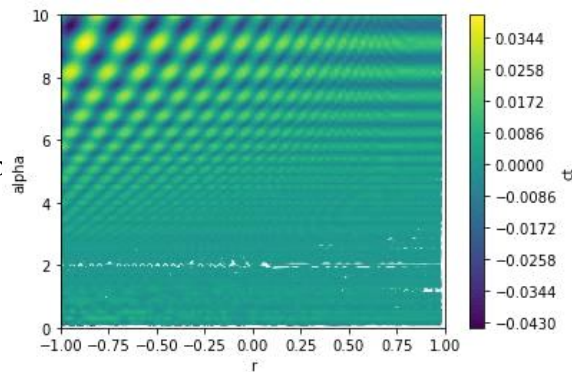


FIG. 2. α and L_m variables with α upperA. *Radial Light Geodesics*

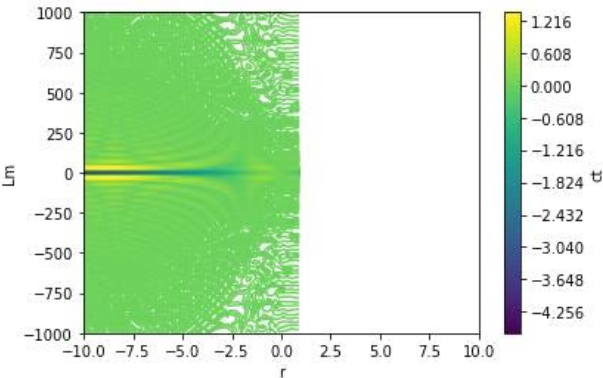
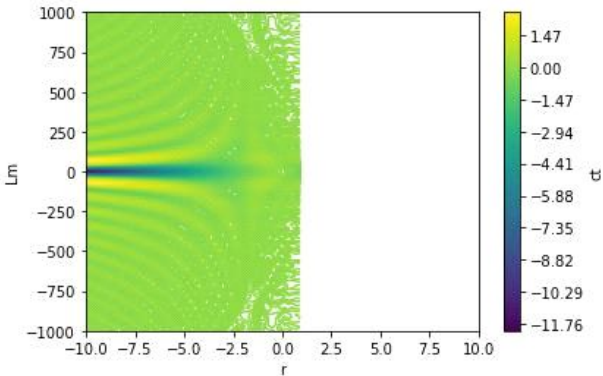
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- for lagrangian



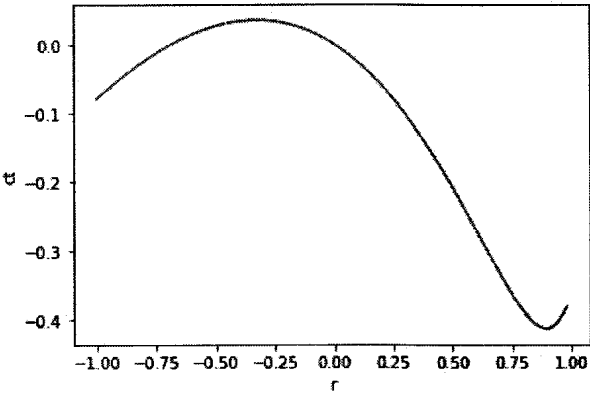
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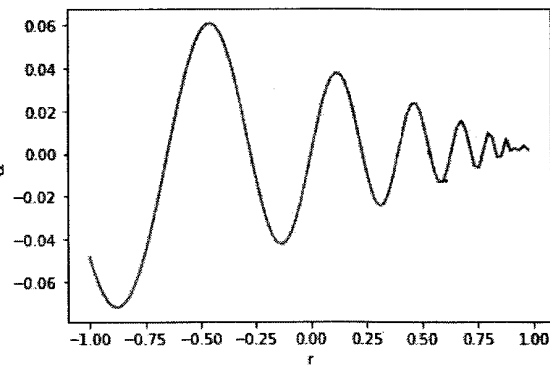
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• for $\alpha = cste$ and L_m k

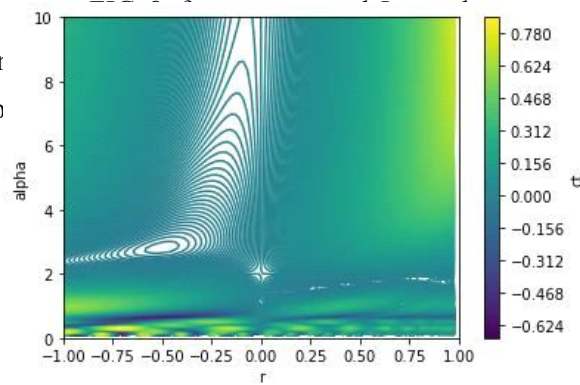


- for $\alpha = cste$ and L_m gra_{ty}

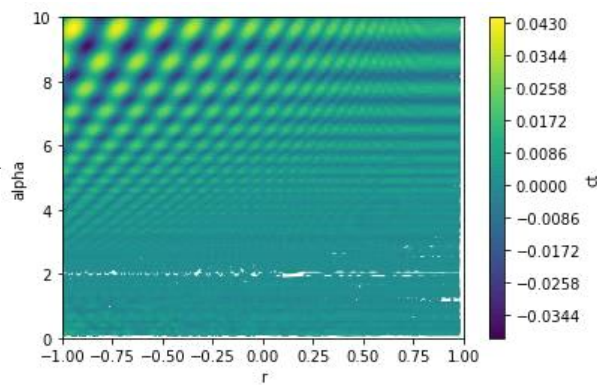


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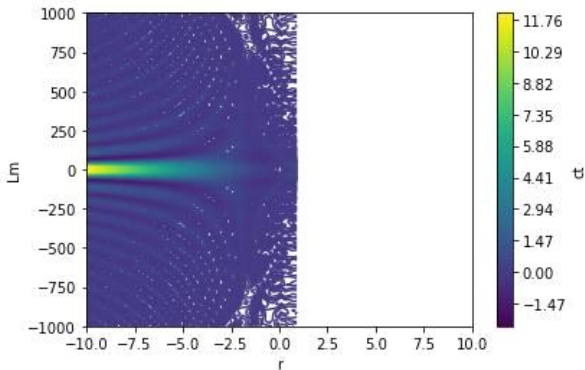
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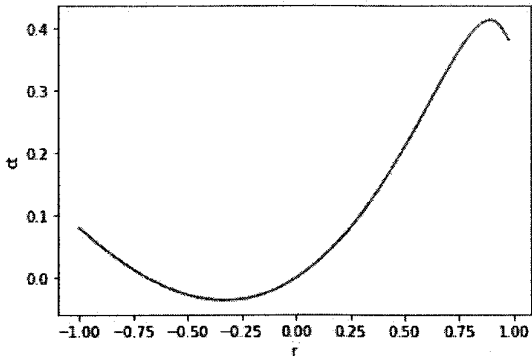
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• for $\alpha = cste$, we represe



• for $\alpha = cste$ and L_m l



• for $\alpha = cste$ and L_m g

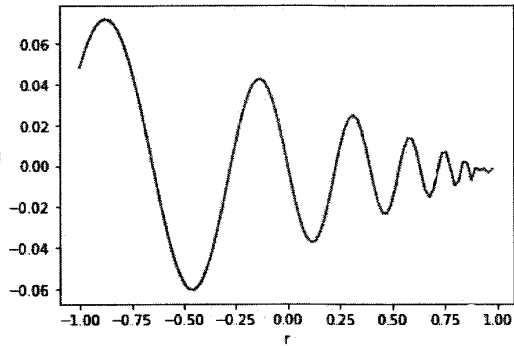


FIG 13. $\alpha = cste$ and L_m grather*B. Orbits of material bodies Geodesics :*

The potential equation is given by :

$$V_{eff}(r) = -\frac{1}{2}[(\frac{l^2}{c^2 r^2} + 1)\tan^2(-\frac{kL_m}{4(\alpha-2)}[1 - \ln(1-r)]) - \frac{\varepsilon^2}{c^2}] \quad (34)$$

$$\frac{dV_{eff}(r)}{dr} = -\frac{1}{2}[(\frac{l^2}{c^2 r^2} + 1)\frac{kL_m}{4(\alpha-2)(1-r)\cos^2(-\frac{kL_m}{4(\alpha-2)}[1 - \ln(1-r)])} - \frac{l^2}{c^2 r^3}\tan^2(-\frac{kL_m}{4(\alpha-2)}[1 - \ln(1-r)])] \quad (35)$$

and the extremums of V_{eff} are given by ³ :

$$0 = -\frac{1}{2}[(\frac{l^2}{c^2 r^2} + 1)\frac{kL_m}{4(\alpha-2)(1-r)\cos^2(-\frac{kL_m}{4(\alpha-2)}[1 - \ln(1-r)])} - \frac{l^2}{c^2 r^3}\tan^2(-\frac{kL_m}{4(\alpha-2)}[1 - \ln(1-r)])] \quad (36)$$

$$(\frac{l^2}{c^2 r^2} + 1)\frac{kL_m}{4(\alpha-2)(1-r)\cos^2(-\frac{kL_m}{4(\alpha-2)}[1 - \ln(1-r)])} = \frac{l^2}{c^2 r^3}\tan^2(-\frac{kL_m}{4(\alpha-2)}[1 - \ln(1-r)]) \quad (37)$$

$$(\frac{l^2}{c^2 r^2} + 1)\frac{kL_m c^2 r^3}{4l^2(\alpha-2)(1-r)} = \sin^2(-\frac{kL_m}{4(\alpha-2)}[1 - \ln(1-r)]) \quad (38)$$

$$\arcsin(\sqrt{(\frac{l^2}{c^2 r^2} + 1)\frac{kL_m c^2 r^3}{4l^2(\alpha-2)(1-r)}}) = -\frac{kL_m}{4(\alpha-2)}[1 - \ln(1-r)] \quad (39)$$

$$\frac{4(\alpha-2)}{kL_m}\arcsin(\sqrt{(\frac{l^2}{c^2 r^2} + 1)\frac{kL_m c^2 r^3}{4l^2(\alpha-2)(1-r)}}) + 1 = \pm \ln(1-r) \quad (40)$$

$$(1-r) = \exp(\pm[\frac{4(\alpha-2)}{kL_m}\arcsin(\sqrt{(\frac{l^2}{c^2 r^2} + 1)\frac{kL_m c^2 r^3}{4l^2(\alpha-2)(1-r)}}) + 1]) \quad (41)$$

$$r = 1 - \exp(\pm[\frac{4(\alpha-2)}{kL_m}\arcsin(\sqrt{(\frac{l^2}{c^2 r^2} + 1)\frac{kL_m c^2 r^3}{4l^2(\alpha-2)(1-r)}}) + 1]) \quad (42)$$

Scope of Future work :

The real TOUGMA's solution is crucials to understand the physical phenomena of universe and it properties what stilled unknow, it describes the matter and energy properties.

The present investigation will be very helpful to the researchers who are engage for the research work in new cosmological models depending of quantum and gravity.

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