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Cauchy Characteristics and Some Fluid Flow Problems

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Abstract

The nature of the differential equation for steady compressible flow is different for subsonic and supersonic flows. For subsonic flow the equation, the equation is of hyperbolic type. In this case the simple wave equation linearised theory. The exact differential equation for the steady flow of a compressible fluid as the same nature as that of linearised theory for steady subsonic flow, the differential equation of the elliptic type and for the steady supersonic flow, the differential equation is of the hyperbolic type. The general theory of the method of characteristics for the case of two independent variables is particularly easy to visualize and computation methods for this case need to be studied extensively. In this paper we discussed some basic properties of irrational flows and boundary conditions that occur in aeronautical engineering problems.

Key words : characteristics, Integral surface, subsonic supersonic flows, Quasi-linear, Mach number.

Nomenclature:

ψ =	Scalar function of x,y	η_y =	Partial derivative η w.r.t. y
u =	Velocity along x direction	$\dot{x}$ =	Partial derivative ξ w.r.t. x
v =	Velocity along y direction	$\dot{y}$ =	Partial derivative ξ w.r.t. y
ψ_x =	Partial derivative ψ w.r.t. x	a =	Speed of the sound
ψ_y =	Partial derivative ψ w.r.t. y	γ =	ratio of specific heats $(=c_p/c_u)$
η_x =	Partial derivative η w.r.t. x	M =	Mach number

1 Introduction

It is extremely difficult to find an exact solution for compressible flow problems[1,2], therefore we need to have approximate methods. Linearisation by the method of small perturbations have been extensively used because of the frequent interest in the flow around these bodies some times this method of linearisation fails for both the transonic and hypersonic regions. Vanarman was the first to obtain the similarity laws for transonic[10] flows where the fluid velocity is very near to the velocity of sound Tsien was the first to obtain the similarity laws for hypersonic flow[4] where the fluid velocity is much higher than the local velocity of the sound velocity. Some problems of hypersonic flows, supersonic and transonic flow[6] need to be studied. In many engineering problems we are interested in the flow around thin body is a uniform stream. In this case we may Linearise some fundamental equation[7]. The boundary conditions occurring in the elliptic equations are difficult from these in the hypersonic equation. For elliptic equations, the problem is to find a function throughout a field where it's value and that of it's normal derivative are given over a portion but not all of a boundary excluding the field[7] which is called Cauchy's problem.

2 *Theory of characteristics for a differential equation of two independent variables. Let the function of the independent variables and satisfy the quasi-linear differential equation:*

Let the function ψ of the independent variables x and y satisfy the quasi-linear differential equation

$$A\psi_{xx} + B\psi_{xy} + C\psi_{yy} + D = 0 \tag{1}$$

where the subscript denotes partial differentiation; A, B, C, and D are functions of $x, y, \psi_{xx}, \psi_{xy}$ and ψ_{yy}. 1 is called quasi-linear because it is linear in the derivative of the highest order, i.e., the second-order derivatives. In the gas-dynamics of in-viscid flow, the differential equations are all of this type, e.g., equations (5.50) and (5.35).

Let

$$\eta(x, y) = constant \tag{2}$$

be the equation of the characteristic curves of the equation 1. Let the family of the characteristic curves 2 be intersected by a second family of curves

$$\xi(x, y) = constant \tag{3}$$

No further specifications are necessary concerning this second family of curves, but in the case of hyperbolic equation, it may be chosen as the second family of characteristics.

We define the interior derivative as the derivative of a function with respect to ξ taken along η = constant, and the exterior derivative as the derivative of a function with respect to η. Thus for exterior derivatives, information concerning the variation of the function beyond η = constant is required.

We may introduce the characteristics in the following three different ways:

1. The characteristic curves can be considered as the loci of possible small discontinuities.
2. The characteristics are the only curves from which an integral surface can be constructed.
3. The continuation of an integral surface beyond a characteristic may become indeterminate.

We shall discuss the characteristics from the above three different ways in the following. All give the same result.

1. *Characteristics as loci of discontinuities of second order:* This is a most natural way for the aerodynamicist to introduce the characteristics of aerodynamic equations. In this case, the characteristics for the supersonic steady flow are the Mach lines along which small disturbances or discontinuities are propagated from the boundary into the interior of the flow field.

Now we assume that there is a discontinuity only in the exterior derivative of the first derivative of the function ψ of equation 1. Then only $\psi_{\eta\eta}$ is discontinuous across η = constant. Since there is a jump $[\psi_{\eta\eta}]$ in $\psi_{\eta\eta}$ across η = constant, the jumps in ψ_{xx}, ψ_{xy} and ψ_{yy} across η = constant are then respectively a characteristic curve

$$[\psi_{xx}] = [\psi_{\eta\eta}]\eta_{x^2}, [\psi_{xy}] = [\psi_{\eta\eta}]\eta_x\eta_y, [\psi_{yy}] = [\psi_{\eta\eta}]\eta_{y^2} \tag{4}$$

because $\psi_{xx} = \psi_{\eta\eta}\eta_{x^2} + \psi_{\xi_n}\xi_x\eta_x + \psi_{\xi\xi}\xi_{x^2} + \psi_\eta\eta_{xx} + \psi_x\xi_{xx}$, etc.

If now the differential equation 1 is written for a fixed value of and small positive $(+\nabla\eta)$ and negative values $(-\nabla\eta)$ of η different from η = 0 characteristics (for simplicity, we may consider the characteristic η = 0 from here on), we obtain by subtraction

$$A[\psi_{xx}] + B[\psi_{xy}] + C[\psi_{yy}] = 0$$

or

$$A\eta_{x^2} + B\eta_x\eta_y + C\eta_{y^2} = 0 \tag{5}$$

Equation 5 is the characteristic condition. If we represent η = constant in parametric from

$$x = x(\xi), \qquad y = y(\xi) \tag{6}$$

then

$$\frac{\eta_x}{\eta_y} = -\frac{\dot{y}}{\dot{x}}$$

where the dot denotes the derivative with respect to ξ.

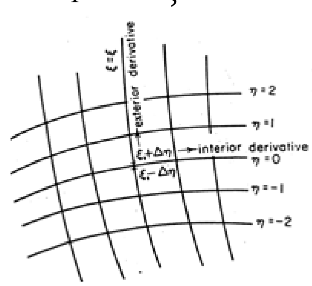

Figure 1: Characteristic curve of a differe4ntial equation

Equation 1 becomes

$$A\dot{y}^2 - B\dot{x}\dot{y} + C\dot{x}^2 \tag{7}$$

or

$$A\left(\frac{dy}{dx}\right)^2 - B\frac{dy}{dx} + C = 0$$

Equations 7 is known as the characteristics equation which give the following two families of characteristic:

$$\left(\frac{dy}{dx}\right)_{1,2} = \frac{B \pm \sqrt{B^2 - 4AC}}{2A} \tag{8}$$

from the values of the functions A, B, and C of equation, we have the classification criteria:

$B^2 - 4AC$	Type of equation	Characteristic Curves
> 0	Hyperbolic	Two real families
< 0	Elliptic	Two imaginary families
= 0	Parabolic	One real family

Since the funotions A, B, C are, in general, variables which take different values in different parts of the field, equation (1) may be of hyperbolic type in a certain region, elliptic in the other, and parabolic in still others. In general, equation (1) is of mixed type.

For example, let us consider the steady two-dimensional flow of compressible inviscid fluid. Here we have

$$A = a^2 - u^2, \qquad B = -2uv, \qquad C = a^2 - v^2 \tag{9}$$

Hence

$$B^2 - 4AC = 4a^2(M^2 - 1) \tag{10}$$

where M is the local Mach number, *ie.* $M = \sqrt{u^2 + v^2/a}$. We see that for supersonic flow M > 1, equation (1) is of hyperbolic type; for subsonic flow M < 1, equation (9) is of elliptic type; and for sonic flow M = 1, equation (9) is of parabolic type. In general, equation (9) is of mixed type. It is also of interest to note that the characteristics for the rotational flow are the same as those for the corresponding irrotational flow equation (10). The only difference in equations (9) and (10) is in D which does not affect the characteristic equation.

From the values of equation (9), we have for supersonic flow M> 1, the following two families of characteristics:

$$\left(\frac{dy}{dx}\right)_{1,2} = \frac{-uv \pm \sqrt{u^2 + v^2 - a^2}}{a^2 - u^2} \tag{11}$$

where is the angle of the velocity vector to the x-axis and is known as the Mach angle, *i.e.*, $\alpha = sin^{-1}(1/M)$ Equation (11) shows that the angle between the characteristic curves and the streamlines is the Mach angle. Hence the characteristics are the Mach lines in the steady two dimensional supersonic flow.

2. *Characteristic strips as elements of integral surfaces :* Let us consider a strip C on the surface of ψ which is associated with a curve $\eta = 0$, and which is defined parametrically by $x(\xi), y(\xi), \psi(\xi, 0) = \psi(\xi), \psi_\xi(\xi, 0) = p(\xi)$ and $\psi_\xi(\xi, 0) = q(\xi)$. We want to find the conditions under which the expression on the left-hand side of equation (1),

$$A\psi_{xx} + B\psi_{xy} + C\psi_{yy} + D \tag{12}$$

can be expressed in terms of the five strip quantities $x(\xi), y(\xi), \psi(\xi), p(\xi)$, and $q(\xi)$ and their internal derivatives. Since the coefficients A, B, C, and D depend only on the quantities x, y, ψ, ψ_ξ, and ψ_η, which are given along the considered strip, we need to investigate only whether and can be expressed in terms of these five strip quantities and their internal derivatives.

From equation (4). We see that if

$$A\eta_{x^2} + B\eta_x\eta_y + C\eta_{y^2} = 0 \tag{13}$$

equation (12) can be expressed in terms of these five quantities and their internal derivatives. Equation (13) is the characteristic condition (5). Only characteristic strips can be considered for the construction of an integral surface from the strips. Of course, that such a construction is indeed possible must be proved.

3. *Indeterminate continuation of an integral surface beyond a characteristic strip:*

The third definition of characteristics is obtained from the consideration of a strip of the first order on the integral surface ψ. We want to know whether the unknown second derivatives $\psi_{xx}, \psi_{xy}, \psi_{yy}$ and the higher derivatives along the strip of the first order can always be uniquely determined from the differential equation (1). If they could be determined, the function ψ might be continued beyond the strip by means of a Taylor series development.

The second derivatives r, s and t can be determined from the following three simultaneous equations:

$$\begin{aligned} A\psi_{xx} + B\psi_{xy} + C\psi_{yy} &= -D \\ \dot{x}\psi_{xx} + \dot{y}\psi_{xy} &= \psi_x \\ \dot{x}\psi_{xy} + \dot{y}\psi_{yy} &= \psi_y \end{aligned} \tag{14}$$

(where the dot denotes differentiation with respect to ξ), provided that the determinant

$$\begin{vmatrix} A & B & C \\ \dot{x} & \dot{y} & 0 \\ 0 & \dot{x} & \dot{y} \end{vmatrix} = A\dot{y}^2 - B\dot{x}\dot{y} + C\dot{x}^2 \tag{15}$$

does not vanish. It is interesting to note that the higher derivatives, too, can be uniquely determined under the same condition (15).

If the determinant vanishes, *i.e.*, if the characteristic condition (7) is satisfied, the derivatives r, s and t are determined, if at all, only within the solution of the homogeneous system of equations which corresponds to equation (14). If the characteristic condition is satisfied, equation (14) admits a solution if the following

$$\begin{vmatrix} A & B & C \\ \dot{x} & 0 & \dot{\psi}_x \\ 0 & \dot{y} & \dot{\psi}_y \end{vmatrix} = A\dot{\psi}_x\dot{y} - B\dot{x}\dot{\psi}_y + C\dot{x}\dot{y} = 0 \tag{16}$$

is also satisfied. This gives the second characteristic differential equation.

The third characteristic equation is

$$\dot{\psi} = \psi_x\dot{x} + \psi_y\dot{y} \tag{17}$$

Equations (7), (16), and (17) are the fundamental differential equations for the characteristic method as applied to the hyperbolic type differential equation. We shall discuss the approximate solution of these equations for flow problems later.

3 *Theory of characteristics for two simultaneous first-order differential equations in two variables :*

Let us consider the following system of quasi-linear, first-order differential equations in two independent and two dependent variables:

$$A_1\frac{\partial u}{\partial x} + B_1\frac{\partial u}{\partial y} + C_1\frac{\partial v}{\partial x} + D_1\frac{\partial v}{\partial y} + E_1 = 0 \tag{18}$$

$$A_2\frac{\partial u}{\partial x} + B_2\frac{\partial u}{\partial y} + C_2\frac{\partial v}{\partial x} + D_2\frac{\partial v}{\partial y} + E_2 = 0$$

where $A_1, A_2, \cdots, E_2$ are functions of *x, y, u* and *v*,

If the functions $A_1, A_2, \cdots, E_2$, are functions of the independent variables x and y but not of the dependent variables u and v, the equations are linear.

In general, the equation (18) is nonlinear. We say it is quasi-linear because it is linear with respect to the first derivatives of u and v. If the equations are homogeneous, ie, $E_1 = E_2 = 0$, and if the other coefficients are functions of u, v but not of x,y explicitly, the equation (18) is said to be reducible and can be transformed into a linear system by interchanging the roles of the independent and dependent variables.

The fundamental equations of the compressible fluid for many cases may be reduced to the form of equation (18). For instance, the fundamental equations for steady irrotational two-dimensional or axially symmetrical flow may be written in the form of equation (11.18) as follows:

$$(u^2 - a^2)\frac{\partial u}{\partial x} + uv\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) + (v^2 - a^2)\frac{\partial v}{\partial y} - \delta\frac{a^2 v}{y} = 0 \tag{19}$$

$$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0$$

where $\delta = 0$ for two-dimensional flow, $\delta = 1$ for axially symmetrical flow, and y is the radial distance in axially symmetrical case. The sound speed a is a given function of the velocity $u^2 + v^2$, ie,

$$a^2 = a_0^2 - \frac{\gamma - 1}{2}(u^2 + v^2) \tag{20}$$

From equation (19), we see that for the two-dimensional case, the equations are reducible and that for the axially symmetrical case , they are not reducible.

Now we consider the characteristics of equation (11.18) in the same manner as that described in $ 2(3), *i.e.*, we consider the continuation of the functions u and y beyond a characteristic.

Let us assume that along a certain curve Σ : $x = x(\xi)$ and $y = y(\xi)$, the values of the functions u and v are given. If we can calculate the first derivatives and higher derivatives from equation (18) and the values of u and v on the curve Σ. the functions u and v might be continued beyond the curve by means of a Taylor series development.

The equations from which we can determine the four first derivatives u_x, u_y, v_x, and v_y are as follows:

$$\begin{aligned} &A_1 u_x + B_1 u_y + C_1 v_x + D_1 v_y = -E_1 \\ &A_2 u_x + B_2 u_y + C_2 v_x + D_2 v_y = -E_2 \\ &\dot{x} u_x + \dot{y} u_y = \dot{u} \\ &\dot{x} v_x + \dot{y} v_y = \dot{v} \end{aligned} \tag{21}$$

We can determine u_x, *etc.*, from equation (21) uniquely except when

$$\begin{vmatrix} A_1 & B_1 & C_1 & D_1 \\ A_2 & B_2 & C_2 & D_2 \\ \dot{x} & \dot{y} & 0 & 0 \\ 0 & 0 & \dot{x} & \dot{y} \end{vmatrix} = \begin{matrix} (B_1 D_2 - D_1 B_2)(dx)^2 \\ -[(A_1 D_2 - A_2 D_1) + (B_1 C_2 - C_1 B_2)]dxdy = 0 \\ +(A_1 C_2 - C_1 A_2)(dy)^2 \end{matrix} \tag{22}$$

Equation (22) is the equation of the characteristics. The solution of equation (22) gives two families of characteristics curves

$$\left(\frac{\partial u}{\partial y}\right)_1 = C'(x, y, u, v) \tag{23}$$

$$\left(\frac{\partial u}{\partial y}\right)_2 = C''(x, y, u, v)$$

These two families of characteristics depend explicitly on the coefficients $A_1, \cdots, D_2$, of equation (18). For instant, for two-dimensional irrotational steady flow of compressible fluid, they are the same characteristics as those given by equation (11).

Along these characteristics, the normal derivatives of u and v are indeterminate and may be discontinuous.

In a manner similar to the case discussed in 1(3), the second condition for characteristics may be written as

$$\begin{vmatrix} B_1 & C_1 & D_1 & E_1 \\ B_2 & C_2 & D_2 & E_2 \\ \dot{y} & 0 & 0 & -\dot{u} \\ 0 & \dot{x} & \dot{y} & -\dot{v} \end{vmatrix} = 0 \tag{24}$$

By the help of equation (23), we have for the case

$$E_1 = E_2 = 0$$

$$\left(\frac{\partial v}{\partial u}\right)_1 = \frac{(B_1C_2 - C_1B_2)C' - (B_1D_2 - D_1B_2)}{(C_1D_2 - C_2D_1)C'} = \Gamma'$$

$$\left(\frac{\partial v}{\partial u}\right)_2 = \frac{(B_1C_2 - C_1B_2)C'' - (B_1D_2 - D_1B_2)}{(C_1D_2 - C_2D_1)C''} = \Gamma'' \tag{25}$$

Equation (25) gives two distinct families of curves in the hodograph plane corresponding to the two families of characteristics C' and C" in the physical plane. For a reducible equation, the hodograph characteristics Γ' and Γ" are determined in advance by equations (18) and independent of the particular initial conditions considered. In general, the characteristics depend on both the differential equations and the initial conditions. ,

Consider again the two-dimensional irrotational steady flow [equation (22)]. We have for the expressions $A_1, .. E_2$

$$A_1 = u^2 - a^2, \quad B_1 = uv, \quad C_1 = uv, \quad D_1 = v^2 - u^2, \quad E_1 = 0$$

$$A_2 = 0, \quad B_2 = -1, \quad C_2 = 1, \quad D_2 = 0, \quad E_2 = 0 \tag{26}$$

The physical characteristics are then

$$C' = tan(\theta - \alpha) \tag{27}$$

$$C'' = tan(\theta + \alpha)$$

which are identical to those obtained in [1], *ie*, equation (11). The corresponding hodograph characteristics are

$$\left(\frac{\partial v}{\partial u}\right)_1 = C''$$

$$\left(\frac{\partial v}{\partial u}\right)_2 = C' \tag{28}$$

We may integrate equation (28) to get a universal function $f(u, v) = 0$ for the hodograph characteristics. This function is useful in the graphical method of finding the flow field of a supersonic irrotational steady two-dimensional flow. It was first used by Busemann and will be discussedin next section.

4 *Characteristic equations :*

From this point on we shall consider only the cases with two real families of characteristics, *ie*, hyperbolic type equations. From the last two sections, we see that the characteristic equations obtained from these two points of view are exactly the same. Hence from a given flow problem, we need to consider either a second-order partial differential equation or a system of two first-order differential equations.

We shall have the following two families of characteristics:

$$\left(\frac{\partial v}{\partial u}\right)_1 = C''(x, y, \psi, \psi_x, \psi_y)$$

$$\left(\frac{\partial v}{\partial u}\right)_1 = C'(x, y, \psi, \psi_x, \psi_y) \tag{29}$$

The characteristic equations which hold along these characteristic curves are equations (8), (16), and (17). For the first family of characteristics, with as parameter, we have the following three relations along the characteristic C':

$$y_\xi - C'x_\xi = 0$$
$$AC'\psi_{x\xi} + C\psi_{y\xi} + Dy_\xi = 0$$
$$\psi_\xi - \psi_x x_\xi - \psi_y v_\xi = 0 \tag{30}$$

For the second family of characteristics, with η as a parameter, we have another three relations along the characteristics C:

$$y_\eta - C'x_\eta = 0$$
$$AC'\psi_{x\eta} + C\psi_{y\eta} + Dy_\eta = 0 \tag{31}$$
$$\psi_\eta - \psi_x x_\eta - \psi_y v_\eta = 0$$

If we choose ξ and η as the independent variables, we have the five unknowns $x(\xi,\eta), y(\xi,\eta), \psi(\xi,n), \psi_x(\xi,n)$, and $\psi_y(\xi,n)$ and the above equations (30) and (31). However one of these six equations is automatically satisfied if the other five are satisfied. This statement may be proved as follows:

Multiplying equation (30) by y_η equation (31) by y_ξ, and subtracting the resultant equations, we have

$$A(C'\psi_{x\xi}y_\eta - C''\psi_{x\eta}y_\xi) + C(\psi_{y\xi}y_\eta - \psi_{y\eta}y_\xi) = 0 \tag{32}$$

Dividing equations (32) by $C'C" = C/A$ and using the relations (30a) and (31a), we obtain

$$\psi_{x\xi}x_\eta + \psi_{y\xi}y_\eta = \psi_{x\eta}x_\xi + \psi_{y\eta}y_\xi \tag{33}$$

Differentiating equation (30c) with respect to η gives

$$\psi_{\xi\eta} - \psi_{x\eta} - \psi_{y\eta}y_\xi - \psi_x x_{\xi\eta} - \psi_y y_{\xi\eta} = 0 \tag{34}$$

while using equation (33), equation (34) becomes

$$\frac{\partial}{\partial \xi}(\psi_\eta - \psi_x x_\eta - \psi_y y_\eta) = 0 \tag{35}$$

It follows that

$$\psi_\eta - \psi_x x_\eta - \psi_y y_\eta = h(\eta)$$

where $h(\eta)$ is an arbitrary function of η only.

Consider a boundary curve which is intersected by the family of curves = constant. If equation (31c) is satisfied along this boundary, the function $h(\eta)$ vanishes. Equation (31c) is then also satisfied in the interior. The initial or boundary conditions must be prescribed so that equation (3lc) is satisfied along the boundary. In practice the boundary is usually a streamline which may be assigned the value $\psi = 0$, so that equation (31c) is automatically satisfied.

If the value of the function ψ and its first derivatives are given along a certain curve in the region considered, we may approximately calculate the values of the function of ψ at points in the neighborhood of the given curve by finite difference methods. This is the method of characteristics. Before discussing the practical procedure, we would like to point out some fundamental concepts and properties associated with the characteristics.

5 *Some fundamental properties of characteristics :*

The first important concept of characteristics is the domain of dependence.

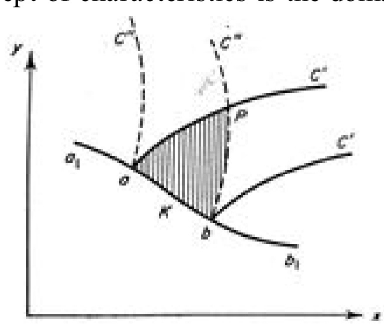

Figure 2: Domain of dependence

Consider an ordinary curve K given in parametric form by $x = x_0(\xi),\ \ y = y_0(\xi)$, An ordinary curve is one which intersects any single characteristic not more than once (Fig. 2). We assume that ψ, ψ_x and ψ_y are given along the curve K, and that they are continuous functions. These are the initial values of the problem. Consider a segment ab on this curve. Through point a there are two characteristics C' and $C"$. Through point *b* there are also two characteristics. As shown in Fig. 11.2, the C' characteristics through a will intersect the $C"$ characteristic through b. The intersection point is *P*. The values of the functions in the curved triangle Pab are uniquely determined by the initial values along the segment ab and are unaffected by the values outside ab. The segment ab of *K* is known as the domain of dependence of the point *P*.

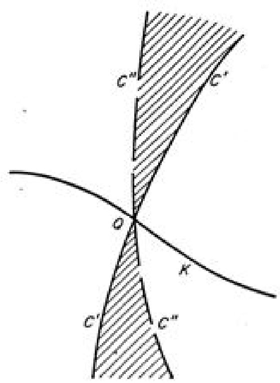

Figure 3: Range of influence

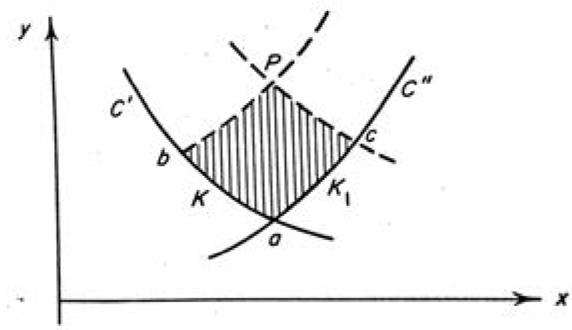

Figure 4: Characteristic initial value problem

The second important concept is the range of influence of a point Q on the initial curve K. This is defined as the totality of points in the xy-plane which are influenced by the initial values at Q. It is evident that the range of influence is made up of all points P whose domains of dependence include Q. Hence the two curved triangular regions included between the two characteristics through Q are the range of influence of Q (Fig. 3).

6 Conclusion

If there are discontinuities in the second derivatives of y along an initial or boundary lines K, they are propagated along the characteristics through the points on K where the discontinuities originate. Thus in two-dimensional steady supersonic flow, the characteristics are the Mach lines along which the discontinuities in the derivatives of velocity components propagate.

If the initial values are given along one characteristic only, the problem is indeterminate. However, if we are given the initial values along two characteristics K of C'' and K_1 of C', as in Fig. 4, the problem is uniquely determined. For instance, if ab of K and ac of K_1 are given, the value of in the quadrilateral region $Pabc$ is uniquely determined.

7 *Scope of Future Work :*

The investigations that are carried out in this article throw the light on how the nature of second order partial differential equations is relevent to study the nature of the fluid flows. These results are helpful for further research both analytically and numerically in hydro dynamics and allied subjects.

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