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Magnetohydrodynamic flow in a vertical annular channel in presence of a uniform axial magnetic field

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Abstract

we have discussed the flow of an electrically conducting fluid in a tall vertical closed annular channel under magnetic field along the decreasing direction of gravitational acceleration. The expression for non-dimensional axial velocity and temperature are obtained and shown graphically for varying values of ratio of two radii. we have discussed flow of a viscous incompressible electrically conducting fluid in a tall vertical closed annular channel in the presence of uniform magnetic field in vertical direction.

Key word : Annular channel Electrically conducting fluid, Axial magnetic Field.

1. Introduction

Flows in cylindrical geometry configurations are of importance in many engineering fields like electrical motors, lubrication and heat transfer equipment. MHD flow in circular pipe, where the applied field is in the radial direction has been treated by Pai[8]. The study of the flow of a viscous incompressible fluid between two coaxial cylinders was first undertaken by Couette[2] with a view to measuring the viscosity of the fluid. Hughes and Young[4] discussed the flow between two coaxial rotating cylinders with a radial magnetic field.

Sinha and Choudhary[10], Jain and Bansal[5] considered the flow of a viscous incompressible fluid between two coaxial rotating or non- rotating cylinders with the walls of the cylinder being either solid or porous. Jain and Bansal[5] considered the flow of a viscous incompressible fluid between two coaxial rotating porous cylinders. Jain and Mehta[6] obtained exact solution in a closed form of the hydromagnetic equations for an incompressible viscous and electrically conducting fluid flow through an annulus with porous walls in the presence of a transverse radial magnetic field. Syam Babu[11] discussed the flow of a viscous incompressible conducting fluid between two coaxial rotating porous cylinders under the influence of a uniform radial magnetic field. Pillai, Varma and Babu[9] studied the flow of viscous conducting incompressible fluid between two coaxial rotating porous cylinders with the outer cylinder bounded by a permeable bed, while Arora and Gupta[1] considered the MHD flow between two rotating coaxial cyliders under radial magneticfield $\frac{H_O R_1}{r}$. Globe[3] considered the laminar steady- state magneto-hydrodynamic flow in an annular channel. Kapur and Jain[7] discussed the flow of an electrically conducting incompressible fluid in the annular region of two concentric infinite cylinders under an applied radial magnetic field.

In this paper we have discussed flow of a viscous incompressible electrically conducting fluid in a tall vertical closed annular channel in the presence of uniform magnetic field in vertical direction. Thomas *et al.*[12] investigated the influence of mass transfer on the global stability of the rotating- disk boundary layer. Baranova, V.E. *et al.*[13] focused on the production of a uniform magnetic field using a system of axial coils for the purpose of calibrating magnetometers. Mason *et al.*[14] examines the phenomena of magnetoconvection in a rotating spherical shell when subjected to a uniform axial magnetic field. The authors investigates the behavior and dynamics of such a system and its implications for geophysical and astrophysical fluid dynamics. Khine, Y.Y.[15] *et al.* explored the thermoelectric magnetohydrodynamic effects that ovvur during the Bridgman semiconductor crystal growth process when a uniform axial magnetic field is applied. Nicolas *et al.*[16] investigated the magnetohydrodynamic stability of cylindrical liquid bridges when subjected to a uniform axial magnetic field. Cooper, R.G. *et al.*[17] explored the phenomenon of subcritical dynamos in the context of rapidly rotating planar convection. Xiao R. *et al.*[18] investigates the development of an X- band dual- mode relativistic backward wave oscillator. FL Braghin *et al.*[19] investigates the effects of weak magnetic fields on the form factors of pions and constituent quarks. Rawat S.B. *et al.*[20] examines the influence of axial magnetic fields in a magnetic mutipole line cusp ion source. Masale M. *et al.*[21] investigates the scattering rates of electrons and longitudinal optical phonos in a hollow cylinder. Zhang G. *et al.*[22] investigates the impact of non-uniform magnetic fields on the radial oscillation of electron beams in a low- magnetic-field foilless diode. Cristian Vllavicencio[23] investigate axial coupling constant within the context of a magnetic backgrtound.

2. Basic Equations And Formulation Of The Problem :

Let us consider the flow of an electrically conducting fluid in a tall vertical annular channel of radii R_1 and $R_2 \left(R_2 > R_1\right)$. The inner and outer cylinders are kept at constant temperature T_1 and T_2 respectively $\left(T_2 > T_1\right)$. An external uniform magnetic field $\vec{B}\left(0,0,B_0\right)$ is applied along the axis of the cylinders. Since the external magnetic field is vertical, there is no interaction between the base flow and the magnetic field. The flow may be realized in the middle portion of a sufficiently long vertical layer of fluid where the end effects are negligible.

The equation of continuity $\frac{\partial \bar{v}_z}{\partial \bar{z}} = 0$ yields

$$\bar{v}_z = \bar{w}_o\left(\bar{r}\right) \tag{1}$$

The equation of motion and energy under the usual Boussinesq approximations are :

$$-\frac{1}{\rho}\frac{d\bar{p}}{d\bar{z}} + \upsilon\left(\frac{d^2\bar{w}_o}{d\bar{r}^2} + \frac{1}{\bar{r}}\frac{d\bar{w}_o}{d\bar{r}}\right) + g\beta\left(T - T_1\right) = 0 \tag{2}$$

$$\frac{d^2T}{d\bar{r}^2} + \frac{1}{\bar{r}}\frac{dT}{d\bar{r}} = 0 \tag{3}$$

The boundary conditions are

$$\bar{r} = R_1 : \bar{w}_o = 0, T = T_1$$

$$\bar{r} = R_2 : \bar{w}_o = 0, T = T_2 \tag{4}$$

Where υ is the kinematic viscosity, β is the coefficient of volume expansion, g is the gravitational acceleration.

3. Solution of the Problem :

Let us introduce the following non- dimensional variables :

$$w_o = \frac{\bar{w}_o}{g\beta\left(T_2 - T_1\right)h^2/\upsilon}, r = \frac{\bar{r}}{h}, z = \frac{\bar{z}}{h}$$

$$p = \frac{\bar{p}}{\rho g\beta\left(T_2 - T_1\right)h}, \theta_0 = \frac{T - T_1}{T_2 - T_1}, \tag{5}$$

Where $h = \frac{R_2 - R_1}{2}$

Introducing the above non- dimensional variables, the equations (2) and (3) reduce to

$$-\frac{dp}{dz} + \frac{d^2w_o}{dr^2} + \frac{1}{r}\frac{dw_o}{dr} + \theta_o = 0 \tag{6}$$

$$\frac{d^2\theta_o}{dr^2} + \frac{1}{r}\frac{d\theta_o}{dr} = 0 \tag{7}$$

The corresponding boundary conditions are

$$r = \frac{R_1}{h} : w_o = 0, \theta_o = 0$$
$$r = \frac{R_2}{h} : w_o = 0, \theta_o = 1 \tag{8}$$

Finally, we introduce the notation

$$R = \frac{R_1}{R_2}, r_1 = \frac{2R}{1-R}, r_2 = \frac{2}{1-R}, \tag{9}$$

Let us assume that

$$\frac{dp}{dz} = A \text{ (constant)} \tag{10}$$

Then the equation (6), reduces to

$$\frac{d^2 w_o}{dr^2} + \frac{1}{r}\frac{dw_o}{dr} + \theta_o = A \tag{11}$$

And the boundary condition become

$$r = r_1 : w_o = 0, \theta_o = 0$$
$$r = r_2 : w_o = 0, \theta_o = 1 \tag{12}$$

Since the channel is closed, the fluid flux through any cross- section must be zero :

$$\int_{r_1}^{r_2} r w_o(r)\, dr = 0 \tag{13}$$

Solving the equation (11), (7) and (13) with the help of boundary conditions (12), we get

$$w_o = c_1\left[1 - \frac{2r^2}{r_1^2 + r_2^2}\right] + c_2\left[\log r - \frac{2\left(r_2^2 \log r_2 - r_1^2 \log r_1\right) r^2}{r_2^4 - r_1^4} + \frac{r^2}{r_1^2 + r_2^2}\right]$$
$$- \frac{r^2}{4\left(r_2^4 - r_1^4\right)\log R}\left\{r_2^4 \log r_2 - r_1^4 \log r_1 - \frac{1}{4}\left(r_2^4 - r_1^4\right)\right\} + \frac{r^2 \log r}{4 \log R} \tag{14}$$

Where

$$c_1 = c_3 \left[\begin{aligned} &\left\{\log r_1 - \frac{2r_1^2\left(r_2^2 \log r_2 - r_1^2 \log r_1\right)}{r_2^4 - r_1^4} + \frac{r_1^2}{r_1^2 + r_2^2}\right\} \times \left\{\frac{r_1^4 r_2^2}{4\left(r_2^4 - r_1^4\right)} + \frac{r_2^2}{16 \log R}\right\} \\ &- \left\{\log r_2 - \frac{2r_2^2\left(r_2^2 \log r_2 - r_1^2 \log r_1\right)}{r_2^4 - r_1^4} + \frac{r_2^2}{r_1^2 + r_2^2}\right\}\left\{\frac{r_1^2 r_2^4}{4\left(r_2^4 - r_1^4\right)} + \frac{r_1^2}{16 \log R}\right\} \end{aligned}\right] \tag{15}$$

$$c_2 = c_3\left[-\frac{r_1^2 r_2^2}{4\left(r_1^2+r_2^2\right)}-\frac{r_2^2-r_1^2}{16\log R}\right] \tag{16}$$

$$c_3 = \frac{r_1^2+r_2^2}{\left(r_2^2-r_1^2\right)\left(1+\log r_1 r_2\right)-2\left(r_2^2\log r_2-r_1^2\log r_1\right)} \tag{17}$$

and pressure is given by

$$A=\frac{dp}{dz}=-\frac{8}{r_2^4-r_1^4}\begin{bmatrix} c_1\left(r_2^2-r_1^2\right)+c_2\left\{r_2^2\log r_2-r_1^2\log r_1-\dfrac{\left(r_2^2-r_1^2\right)}{2}\right\} \\ +\dfrac{1}{8\log R}\left\{r_2^4\log r_2-r_1^4\log r_1-\dfrac{\left(r_2^4-r_1^4\right)}{4}\right\}\end{bmatrix}+\frac{1+\log r_1}{\log R} \tag{18}$$

$$\theta_o=-\frac{\log\left(\dfrac{r}{r_1}\right)}{\log R} \tag{19}$$

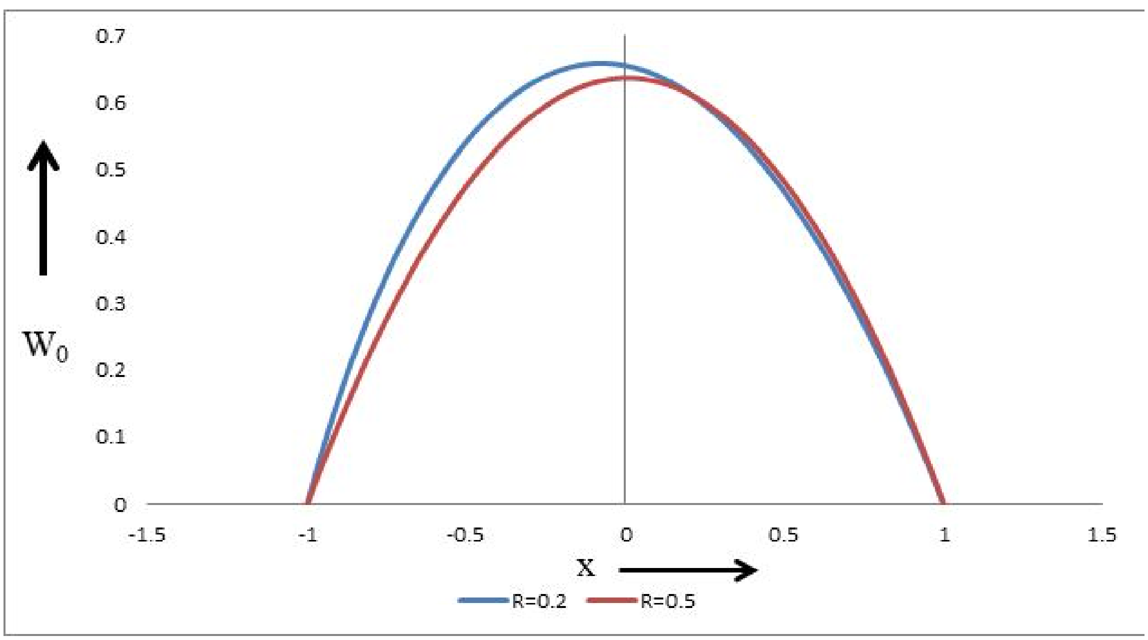

Fig. 1. Non-dimensional base velocity w_o vs x

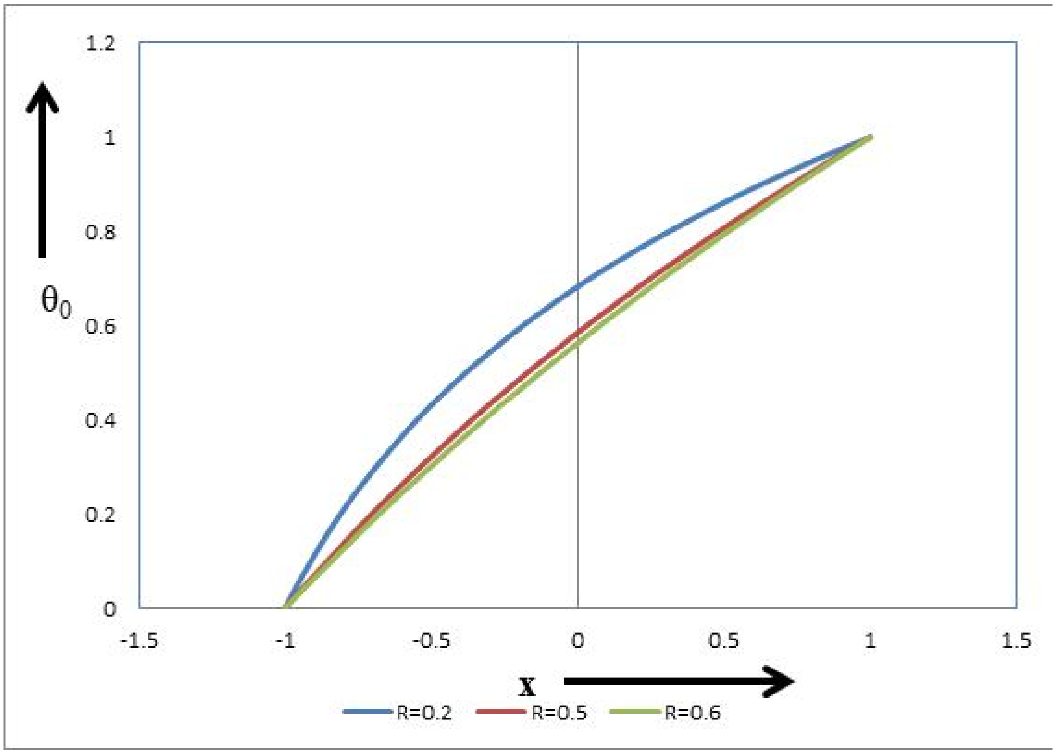

Fig. 2. Temperature profiles θ vs x

4. Numerical Discussion :

The non-dimensional base velocity (14) and temperature profiles (19) are shown in figure 1 and 2 respectively in terms of radial coordinate $x = r - \frac{1+R}{1-R}$. The magnetic field does not affect the velocity and temperature distribution. Also temperature decreases with increase in R, the ratio of inner to outer radii. The maximum and minimum temperature remains same for all values of R. It has been observed that as R increases, the parabolic distribution of θ_o assumes a linear form. The velocity near the cooled surface increases, while the velocity near hotter surface decreases. The velocity increases with the increase in R.

5. Conclusion

We found that the magnetic field does not affect the velocity and temperature distribution. Als temperature decreases with the increase in R, whereas maximum and minimum values of temperature remains the same for all values of R. Flows in cylindrical geometry configurations are of importance in many engineering fields like electrical motors, lubrication and heat transfer equipment.

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