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Higher Dimensional Bianchi V Models in $f\,(R,T)$ Theory with Perfect Fluid Distribution

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Abstract

In this paper we have studied five-dimensional Bianchi type-V spacetime in the $f\,(R,T)$ theory. To solve Einstein's field equations, we have assumed that the Hubble parameter is inversely proportional to r^{th} power of the scale factor α i.e. $H \propto \frac{1}{\alpha^r}$. We have obtained a singular model for $r > 0$, non-singular model for $r = 0$. The physical behavior of the models has also been discussed using some physical parameters.

Key words : Bianchi type-V, $f(R,T)$ gravity, deceleration parameter, Higher dimension.

Introduction

General relativistic cosmological models provide a framework for investigation of the evolution of the Universe. Physical cosmology is based on the Friedmann-Robertson-Walker (FRW) world models, which describe the Universe as completely homogeneous and isotropic during the entire process of cosmological expansion. Einstein general theory of relativity explain most of the gravitational theories of the universe but it does not appropriate for some of the important problems in cosmology like as the accelerating expansion phase of the universe. The recent scenario of accelerated expansion of the universe supported by astronomical observation[1] has been playing an important role in modern cosmology. It is now proved from theoretical and observational facts that our universe is in the phase of accelerated expansion.

It is now believed that dark energy is the best candidate to explain cosmic acceleration. The results of the Wilkinson microwave anisotropy probe (WMAP)[2] and Plank indicate that the universe consists 68.5% dark energy, 26.5% dark matter and 5% baryonic matter.

In order to explain the accelerated expansion of the universe two theories are usually chosen. First one is by constructing the various dark energy components like cosmological constant[3], quintessence[4], k-essence[5], phantom energy[6] etc. The second one is to modify Einstein's theory of gravitation. The astronomical observation of supernova experiments[7-8] also suggests that the universe is expanding. Cosmic microwave background radiation[9] and large-scale structure[10] provide indirect evidence for late time accelerated expansion of the universe. In view of late time acceleration of the universe and existence of the dark energy and dark matter, many useful modified theories of gravity have been developed and studied. The modification in Einstein theory of gravitation is based on the Einstein-Hilbert action and we obtain alternative theories of gravitation such as $f(R)$ gravity[11], $f(T)$ gravity[12], $f(G)$ gravity[13], $f(R,G)$ gravity[14], $f(R,T)$ gravity etc. where R,T,G are the scalar curvature, the torsion scalar and the Gauss-Bonnet scalar respectively. Bertolami *et al.*[15] have proposed a generalization of $f(R)$ theory of gravity, by including an explicit coupling of arbitrary function of Ricci scalar R with the matter Lagrangian density L_m. The generalization of $f(R)$ gravity by introducing the trace of energy momentum tensor has become a most popular theory to represent the expansion nature of the universe, known as $f(R,T)$ theory of gravity proposed by Harko *et al.*[16].

The quadrature forms of metric functions for Bianchi Type-V cosmological models with perfect fluid and viscous fluid distribution have been obtained by following the work of Saha[17], Singh and Chaubey[18]. The solution of Einstein filed equations for homogenous but anisotropic models have been obtained by large number of authors[19-20] in various contexts by using different techniques. Shamir *et al.*[21] obtained the exact solutions of Bianchi type-I & V models in $f(R,T)$ gravity by taking $f(R,T)=R+2f(T)$. Reddy *et al.*[22] have investigated the LRS Bianchi type-II universe in $f(R,T)$ theory of gravity. Tiwari and Mishra[23] have investigated Bianchi type-V cosmological model in $f(R,T)$ theory. Jokweni *et.al.*[24] explored LRS Bianchi type-I model filled with strange quark matter in $f(R,T)$ theory.

Higher Dimensional cosmological model plays an important role in many phases of early stage of evolution of cosmological problems. The extra dimension plays a physical role and they being too small are unobservable. Kaluza[25] and Klein[26] have done noticeable work by introducing an idea of higher dimension spacetime of the universe. Reddy and Rao[27] have studied several phases of five dimensional spacetime in bi-metric theory and variable mass theory. Rathore and Mandawat[28] have investigated five-dimensional Bianchi type-I string cosmological model in Brans-Dicke theory. Samanta and Dhal[29] have studied higher dimensional cosmological model with perfect fluid in $f(R,T)$ theory. Ladke *et al.*[30] discussed five-dimensional exact solution of Bianchi type-I spacetime in $f(R,T)$ theory of gravity. Sahoo *et al.*[31] obtained the exact solutions for five dimensional LRS Bianchi type-I spacetime in $f(R,T)$ theory.

Inspiring from the above analysis, we have obtained solution of five-dimensional Bianchi type-V spacetime by considering $f(R,T)=f_1(R)+f_2(T)$ with the use of variation law of Hubble parameter[32]. Physical and geometrical properties for the models are also discussed.

2. *Five-Dimensional Field Equation in f(R,T)Theory of gravity :*

The $f(R,T)$ theory of gravity is the generalization or modification of general relativity. $f(R,T)$ gravity formulated by Harko[16] whose field equations are derived from the action principal.

$$S = \int \left(\frac{1}{16\pi} f(R,T) + L_m\right)\sqrt{-g}\, d^5x \tag{2.1}$$

Where $f(R,T)$ is an arbitrary function of the Ricci scalar R and the trace T of the energy momentum tensor T_{ij} of the matter. L_m is the matter Lagrangian density and g is the metric determinant of fundamental tensor g_{ij}. Here we consider $G = c = 1$.

By varying the above equation (2.1) with respect to g_{ij}, we obtain the field equation of $f(R,T)$ gravity in covariant tensor form as

$$f_R(R,T)R_{ij} - \frac{1}{2} f(R,T) g_{ij} + \left(g_{ij}\Box - \nabla_i\nabla_j\right) f_R(R,T) = 8\pi T_{ij} - \left(T_{ij} + \Theta_{ij}\right) f_T(R,T) \tag{2.2}$$

where $f_R(R,T) \equiv \frac{\partial f(R,T)}{\partial R}$, $f_T(R,T) \equiv \frac{\partial f(R,T)}{\partial T}$, $T_{ij} = -\frac{2}{\sqrt{-g}}\frac{\partial(\sqrt{-g}L_m)}{\partial g^{ij}}$, $\Box \equiv \nabla^i\nabla_i$ is the D'Alembert operator, ∇_i is the covariant derivative and R_{ij} is the Ricci tensor and Θ_{ij} is defined as

$$\Theta_{ij} = g_{ij}L_m - 2T_{ij} - 2g^{lk}\frac{\partial^2 L_m}{\partial g^{ij}\partial g^{lk}} \tag{2.3}$$

Here the energy-momentum tensor is considered to be perfect fluid which is defined as

$$T_{ij} = (p+\rho)u_i u_j - p g_{ij} \tag{2.4}$$

Where $u^i = (0,0,0,0,1)$ is the velocity in the co-moving coordinates which satisfies the condition $u^i u_i = 1$. ρ and p are the energy density and pressure of the fluid, respectively. Here the matter Lagrangian can be taken as $L_m = -p$ in equation (2.3)

$$\Theta_{ij} = -p g_{ij} - 2T_{ij} \tag{2.5}$$

Also, above field equation (2.2) takes the form

$$f_R(R,T)R_{ij} - \frac{1}{2} f(R,T) g_{ij} + \left(g_{ij}\Box - \nabla_i\nabla_j\right) f_R(R,T) = 8\pi T_{ij} + \left(T_{ij} + p g_{ij}\right) f_T(R,T) \tag{2.6}$$

Three explicit functional form of $f(R,T)$ have also been considered in Harko *et al.*[16] Which are

$$f(R,T) = \begin{cases} R + 2f(T) \\ f_1(R) + f_2(T) \\ f_1(R) + f_2(R)f_2(T) \end{cases} \tag{2.7}$$

In this paper we have considered that $f(R,T) = f_1(R) + f_2(T)$. Therefore, the gravitational field equation (2.6) becomes

$$f_1^{'}(R)R_{ij} - \frac{f_1(R)g_{ij}}{2} + \left(g_{ij}\Box - \nabla_i\nabla_j\right)f_1^{'}(R) = 8\pi T_{ij} + f_2^{'}(T)T_{ij} + \left(f_2^{'}(T)p + \frac{f_2(T)}{2}\right)g_{ij} \tag{2.8}$$

where prime denotes differentiation with respect to argument. Here we consider a particular form of the function $f_1(R) = \lambda_1 R$ and $f_2(T) = \lambda_2 T$ where λ_1 and λ_1 are arbitrary parameters. So that $f(R,T) = \lambda_1 R + \lambda_2 T$.

Now, equation (2.8) can be rewritten as

$$\lambda_1 R_{ij} - \frac{1}{2}\lambda_1 R g_{ij} + \left(g_{ij}\Box - \nabla_i\nabla_j\right)\lambda_1 = 8\pi T_{ij} + \lambda_2 T_{ij} + \lambda_2\left(p + \frac{T}{2}\right)g_{ij} \tag{2.9}$$

Since $\left(g_{ij}\Box - \nabla_i\nabla_j\right)\lambda_1 = 0$, we get

$$\lambda_1 G_{ij} = 8\pi T_{ij} + \lambda_2 T_{ij} + \lambda_2\left(p + \frac{T}{2}\right)g_{ij} \tag{2.10}$$

Where $G_{ij} = R_{ij} - \frac{1}{2}Rg_{ij}$ is the Einstein tensor. Equation (2.10) reduces to

$$G_{ij} - \frac{\lambda_2}{\lambda_1}\left(p + \frac{T}{2}\right)g_{ij} = \left(\frac{8\pi + \lambda_2}{\lambda_1}\right)T_{ij} \tag{2.11}$$

Since Einstein equation with cosmological constant.

$$G_{ij} - \Lambda g_{ij} = -8\pi T_{ij} \tag{2.12}$$

We have chosen a negative small value for the arbitrary λ_1, so that we have same sign for the equations (2.11) and (2.12) in right hand side. The term $\frac{\lambda_2}{\lambda_1}\left(p + \frac{T}{2}\right)$ can now be regarded as a cosmological constant. Hence, we write

$$\Lambda \equiv \Lambda(T) = \frac{\lambda_2}{\lambda_1}\left(p + \frac{T}{2}\right) \tag{2.13}$$

The dependence of cosmological constant (Λ) on the trace T of the energy momentum tensor T_{ij} have been proposed by Poplawski[33], where the cosmological constant in the gravitational Lagrangian is a function of trace of energy momentum tensor. In this paper we have considered the perfect fluid distribution case, so the trace of energy momentum tensor $T = \rho - 4p$ for our model, equation (2.13) becomes to

$$\Lambda = \frac{\lambda_2}{\lambda_1}\left(\frac{\rho}{2} - p\right) \tag{2.14}$$

Now, from the equations (2.11) and (2.13) we have

$$G_{ij} = \left(\frac{8\pi + \lambda_2}{\lambda_1}\right) T_{ij} + \Lambda g_{ij} \tag{2.15}$$

3 Metric and Field Equations in V_5 :

In this section we have obtained exact solution of five-dimensional Bianchi type-V space time in $f(R,T)$ theory of gravity. The line element for Bianchi type-V in V_5 is given by

$$ds^2 = dt^2 - A^2(t)dx^2 - e^{2mx}\{B^2(t)dy^2 + C^2(t)dz^2 + D^2(t)du^2\} \tag{3.1}$$

Where m is constant and A, B, C, D are function of cosmic time t. The corresponding Ricci scalar is given by

$$R = 2\left[\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\ddot{D}}{D} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} + \frac{\dot{A}}{A}\frac{\dot{D}}{D} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} + \frac{\dot{B}}{B}\frac{\dot{D}}{D} + \frac{\dot{C}}{C}\frac{\dot{D}}{D} - \frac{6m^2}{A^2}\right] \tag{3.2}$$

From equation (2.15), cosmological field equations for the metric (3.1) are as follows

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\ddot{D}}{D} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{D}}{CD} + \frac{\dot{D}\dot{B}}{DB} - \frac{3m^2}{A^2} = \left(\frac{8\pi + \lambda_2}{\lambda_1}\right)p - \Lambda \tag{3.3}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\ddot{D}}{D} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{C}\dot{D}}{CD} + \frac{\dot{D}\dot{A}}{DA} - \frac{3m^2}{A^2} = \left(\frac{8\pi + \lambda_2}{\lambda_1}\right)p - \Lambda \tag{3.4}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{D}}{D} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{D}}{BD} + \frac{\dot{D}\dot{A}}{DA} - \frac{3m^2}{A^2} = \left(\frac{8\pi + \lambda_2}{\lambda_1}\right)p - \Lambda \tag{3.5}$$

$$\frac{\dot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} - \frac{3m^2}{A^2} = \left(\frac{8\pi + \lambda_2}{\lambda_1}\right)p - \Lambda \tag{3.6}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{D}}{CD} + \frac{\dot{D}\dot{A}}{DA} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{D}}{BD} - \frac{6m^2}{A^2} = -\left(\frac{8\pi + \lambda_2}{\lambda_1}\right)\rho - \Lambda \tag{3.7}$$

$$3\frac{\dot{A}}{A} = \frac{\dot{B}}{B} + \frac{\dot{C}}{C} + \frac{\dot{D}}{D} \tag{3.8}$$

Here dot (.) represents a derivative with respect to cosmic time t.

Now, integrating equation (3.8) and absorbing the integration constant into A, we get

$$A^3 = BCD \tag{3.9}$$

The average scale factor $a(t)$, the spatial volume V are defined as

$$V = \alpha^4(t) = ABCD \tag{3.10}$$

From equations (3.3) - (3.6) and (3.10), we get the following four relations respectively:

$$\frac{A}{B} = p_1 \exp\left(q_1 \int \frac{dt}{\alpha^4}\right) \tag{3.11}$$

$$\frac{B}{C} = p_2 \exp\left(q_2 \int \frac{dt}{\alpha^4}\right) \tag{3.12}$$

$$\frac{C}{D} = p_3 \exp\left(q_3 \int \frac{dt}{\alpha^4}\right) \tag{3.13}$$

$$\frac{A}{D} = p_4 \exp\left(q_4 \int \frac{dt}{\alpha^4}\right) \tag{3.14}$$

Where $p_1, p_2, p_3, p_4, q_1, q_2, q_3$ and q_4 are constants of integration.

Now, from equations (3.11) - (3.14) we obtain

$$\frac{A}{C} = p_1 p_2 \exp\left((q_1 + q_2) \int \frac{dt}{\alpha^4}\right) \tag{3.15}$$

$$\frac{B}{D} = p_2 p_3 \exp\left((q_2 + q_3) \int \frac{dt}{\alpha^4}\right) \tag{3.16}$$

Using $\alpha^4(t) = ABCD$, we have

$$4(t) = \alpha_1 \alpha \exp\left(\beta_1 \int \frac{dt}{\alpha^4}\right) \tag{3.17}$$

$$B(t) = \alpha_2 \alpha \exp\left(\beta_2 \int \frac{dt}{\alpha^4}\right) \tag{3.18}$$

$$C(t) = \alpha_3 \alpha \exp\left(\beta_3 \int \frac{dt}{\alpha^4}\right) \tag{3.19}$$

$$D(t) = \alpha_4 \alpha \exp\left(\beta_4 \int \frac{dt}{\alpha^4}\right) \tag{3.20}$$

Where

$$\alpha_1 = [p_1 p_4 (p_1 p_2)]^{\frac{1}{4}}, \quad \alpha_2 = \left[\frac{1}{p_1} p_2 (p_2 p_3)\right]^{\frac{1}{4}},$$

$$\alpha_3 = \left[\frac{1}{p_2} p_3 \left(\frac{1}{p_1 p_2}\right)\right]^{\frac{1}{4}}, \quad \alpha_4 = \left[\frac{1}{p_3} \frac{1}{p_4} \left(\frac{1}{p_2 p_3}\right)\right]^{\frac{1}{4}} \tag{3.21}$$

and

$$\beta_1 = \frac{q_1 + q_4 + (q_1 + q_2)}{4}, \quad \beta_2 = \frac{-q_1 + q_2 + (q_2 + q_3)}{4},$$

$$\beta_3 = \frac{-q_2 + q_3 - (q_1 + q_2)}{4}, \quad \beta_4 = \frac{-q_3 - q_4 - (q_2 + q_3)}{4} \tag{3.22}$$

The constants $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ and $\beta_1, \beta_2, \beta_3, \beta_4$ satisfy the following two relations:

$$\alpha_1\alpha_2\alpha_3\alpha_4 = 1, \qquad \beta_1 + \beta_2 + \beta_3 + \beta_4 = 0 \tag{3.23}$$

Substituting equation (3.10) in equations (3.17) - (3.20), we obtain

$$\alpha_1 = 1,\ \beta_1 = 0 \text{ and put } \alpha_2 = m_1,\ \alpha_3 = m_2,\ \alpha_4 = m_3,\ \beta_2 = n_1,\ \beta_3 = n_2,\ \beta_4 = n_3 \tag{3.24}$$

Again, substituting equation (3.24) in equations (3.17) - (3.20), the metric coefficients in terms of average scale factor can be written as

$$A(t) = \alpha \tag{3.25}$$

$$B(t) = m_1\alpha \exp\left(n_1 \int \frac{dt}{\alpha^4}\right) \tag{3.26}$$

$$C(t) = m_2\alpha \exp\left(n_2 \int \frac{dt}{\alpha^4}\right) \tag{3.27}$$

$$D(t) = m_3\alpha \exp\left(n_3 \int \frac{dt}{\alpha^4}\right) \tag{3.28}$$

Constants m_1, m_2, m_3 and n_1, n_2, n_3 satisfy the following two relations:

$$m_1m_2m_3 = 1, \qquad n_1 + n_2 + n_3 = 0 \tag{3.29}$$

The modified gravity field equations can be solved, if we assume that Hubble parameter (H) is inversely proportional to the r^{th} power of scale factor $\alpha(t)$. This gives

$$H = l\alpha^{-r}; \qquad l > 0, r \geq 0 \tag{3.30}$$

4 Some Important Physical Quantities :

The average scale factor and the spatial volume are defined respectively as under

$$a = (ABCD)^{\frac{1}{4}}, \qquad V = \alpha^4 = \mathrm{ABCD} \tag{4.1}$$

The generalized mean Hubble parameter H is defined by

$$H = \frac{\dot{\alpha}}{\alpha} = \frac{1}{4}\sum_{i=1}^{4} H_i \tag{4.2}$$

Where $H_1 = \frac{\dot{A}}{A}, H_2 = \frac{\dot{B}}{B}, H_3 = \frac{\dot{C}}{C}$ and $H_4 = \frac{\dot{D}}{D}$ are the directional Hubble parameters in the directions of x, y, z and u respectively.

The anisotropy parameter A_m is defined as

$$A_m = \frac{1}{4}\sum_{i=1}^{4}\left(\frac{H_i - H}{H}\right)^2 \tag{4.3}$$

The expansion scalar θ and shear scalar σ are defined as

$$\theta = u^i_{;i} = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} + \frac{\dot{D}}{D} \tag{4.4}$$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} \tag{4.5}$$

Where σ_{ij} is shear tensor which is defined as

$$\sigma_{ij} = \frac{1}{2}(\nabla_i u_j + \nabla_j u_i) - \frac{1}{4}\theta g_{ij} \tag{4.6}$$

Shear scalar σ^2 in term of Hubble parameters is defined as

$$\sigma^2 = \frac{1}{2}\left(\sum_{i=1}^{4} H_i^2 - 4H^2\right) \tag{4.7}$$

The deceleration parameter q is defined as under

$$q = -1 + \frac{d}{dt}\left(\frac{1}{H}\right) \tag{4.8}$$

From equation (3.30) and (4.2), we obtain average scale factor $a(t)$ as,

$$a(t) = \begin{cases}(lrt + k_1)^{1/r} & ,r > 0\\ k_2 \exp(lt) & ,r = 0\end{cases} \tag{4.9}$$

Where k_1 and k_2 are constant of integration.

$$\text{Put} \quad (lrt + k_1) = \tau \tag{4.10}$$

5 Five-Dimensional Model of The Universe When $r > 0$

For $r > 0$, using equations (3.24) - (3.27), we obtain

$$A(t) = \tau^{1/r} \tag{5.1}$$

$$B(t) = m_1 \tau^{1/r} \exp\left[\frac{n_1}{l(r-4)}\tau^{(r-4)/r}\right], \quad r \neq 4 \tag{5.2}$$

$$C(t) = m_2\tau^{1/r}\exp\left[\frac{n_2}{l(r-4)}\tau^{(r-4)/r}\right], \quad r \neq 4 \tag{5.3}$$

$$D(t) = m_3\tau^{1/r}\exp\left[\frac{n_3}{l(r-4)}\tau^{(r-4)/r}\right], \quad r \neq 4 \tag{5.4}$$

For this solution, the metric (3.1) becomes

$$\begin{aligned} ds^2 &= dt^2 - \tau^{\frac{2}{r}}dx^2 \\ &- \tau^{\frac{2}{r}}e^{2mx}\left\{m_1^2\exp\left(\frac{2n_1}{l(r-4)}\tau^{\frac{r-4}{r}}\right)dy^2 + m_2^2\exp\left(\frac{2n_2}{l(r-4)}\tau^{\frac{r-4}{r}}\right)dz^2\right. \\ &\left.+ m_3^2\exp\left(\frac{2n_3}{l(r-4)}\tau^{\frac{r-4}{r}}\right)du^2\right\} \end{aligned} \tag{5.5}$$

For the model $r > 0$, the spatial volume (V), Hubble parameter (H) and expansion scalar (θ) are respectively given by

$$V = \alpha^4 = \tau^{4/r} \tag{5.6}$$

$$H = \frac{l}{\tau} \tag{5.7}$$

$$\theta = \frac{4l}{\tau} \tag{5.8}$$

The deceleration parameter q, shear scalar σ^2, anisotropy parameter (A_m) and anisotropy for the model can be expressed as

$$q = r - 1 \tag{5.9}$$

which is constant.

$$\sigma^2 = \frac{(n_1^2 + n_2^2 + n_3^2)}{2\tau^{8/r}} \tag{5.10}$$

$$A_m = \frac{(n_1^2 + n_2^2 + n_3^2)}{4l^2\tau^{(8-2r)/r}} \tag{5.11}$$

$$\frac{\sigma}{\theta} = \frac{(n_1^2 + n_1^2 + n_1^2)^{1/2}}{4\sqrt{2}\,l\tau^{(4-r)/r}} \tag{5.12}$$

Now, from equations (5.1) - (5.4) and (3.3) - (3.7), we obtain pressure (p) and energy density (ρ) of the universe

$$\begin{aligned} p(t) = \frac{-\lambda_1}{(8\pi+\lambda_2)(16\pi+5\lambda_2)}&\Big[2(n_1n_2+n_2n_3+n_3n_1)(8\pi+2\lambda_2)\tau^{-\frac{8}{r}} + 3m^2(16\pi+\lambda_2)\,\tau^{-\frac{2}{r}} \\ &+ 3l^2\{(r-2)16\pi + (3r-4)\lambda_2\}\,\tau^{-2}\Big] \end{aligned} \tag{5.13}$$

$$\begin{aligned} \rho(t) = \frac{-\lambda_1}{(8\pi+\lambda_2)(16\pi+5\lambda_2)}&\Big[2(n_1n_2+n_2n_3+n_3n_1)(8\pi+3\lambda_2)\tau^{-\frac{8}{r}} - 6m^2(16\pi+3\lambda_2)\,\tau^{-\frac{2}{r}} \\ &+ 6l^2\{16\pi + (r+2)\lambda_2\}\,\tau^{-2}\Big] \end{aligned} \tag{5.14}$$

From equations (5.13), (5.14) and (2.14), we obtain the expression for cosmological constant

$$\Lambda(t) = \frac{\lambda_2}{(16\pi + 5\lambda_2)}\left[(n_1 n_2 + n_2 n_3 + n_3 n_1)\,\tau^{-\frac{8}{r}} + 12m^2\,\tau^{-\frac{2}{r}} + 6l^2(r-3)\,\tau^{-2}\right] \tag{5.15}$$

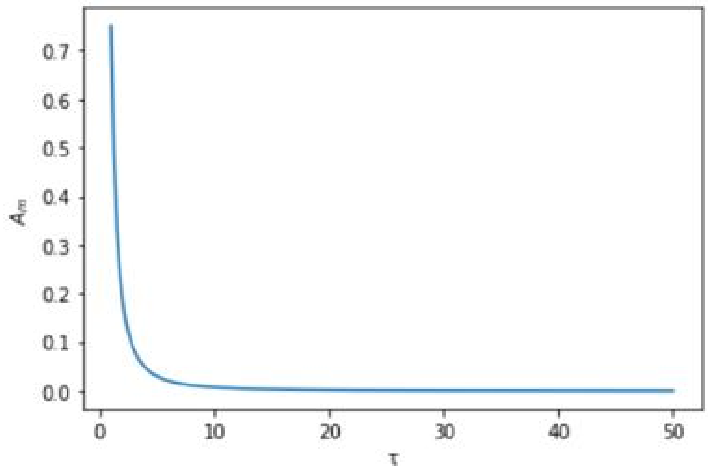

Figure 1: Plot of A_m versus τ for $n_1 = n_2 = n_3 = 1, r = 2$

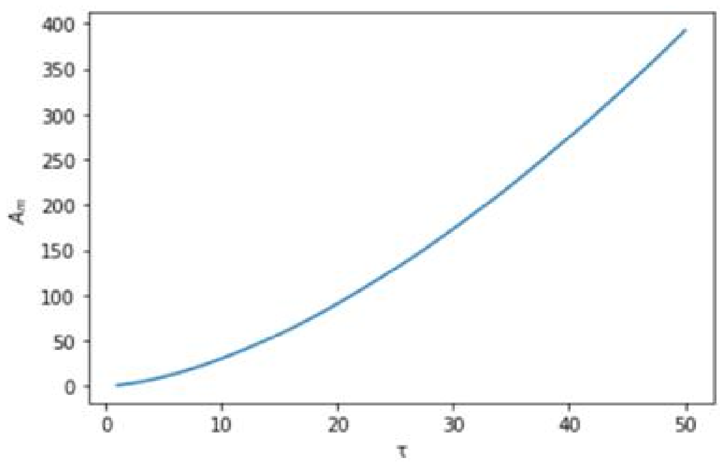

Figure 2: Plot of A_m versus τ for $n_1 = n_2 = n_3 = 1, r = 20$

6 Conclusion for model $r > 0$

The model has a singularity at $t = -\frac{k_1}{lr}$ i.e. $\tau = 0$ known as point type singularity. From equations (5.1) - (5.4) we observe that the scale factors A, B, C and D are vanish at this point of singularity.

From the model we observe that the spatial volume (V) is zero and expansion scalar (θ) is infinite at $\tau = 0$ which shows that the universe starts evolving with zero volume at $\tau = 0$ which is big bang scenario.

The Hubble parameter (H) and shear scalar σ^2 are infinitely large at $\tau = 0$. The physical quantities pressure (p), energy density (ρ) and cosmological constant (Λ) diverge at $\tau = 0$. In the large time i.e. as $t \to \infty$, Hubble parameter (H), expansion scalar (θ), shear scalar (σ^2), pressure, density and cosmological constant becomes negligible. As $t \to \infty$, spatial volume (V) become infinite.

From equation (5.9), we find that $q = r - 1$, which is constant. A positive sign of q i.e. $r > 1$ corresponds to the decelerating phase of expansion whereas for $r < 1$, q is negative which shows that the universe in the model is in accelerating phase.

For $0 < r < 4$, from figure (1) at $\tau = 0$, anisotropy parameter (A_m) and anisotropy $\left(\frac{\sigma}{\theta}\right)$ diverge which indicating that the universe was anisotropic model at the initial time. $A_m \to 0,\ \frac{\sigma}{\theta} \to 0\ as\ t \to \infty$. This shows that the universe in the model turns isotropic at late time, which agrees Collins and Hawking[34].

For $r > 4$, from figure (2) anisotropy parameter and anisotropy are proportional to τ which shows that universe was isotropic at initial time and turns into anisotropic model at late time.

7 Five-Dimensional Model of The Universe When ($r = 0$) :

In this section we study the five-dimensional model of the universe for $r = 0$. For this model average scale factor gives as $\alpha(t) = k_2 \exp(lt)$.
For $r = 0$, equations (3.25) - (3.28), gives

$$A(t) = k_2 \exp(lt) \tag{7.1}$$

$$B(t) = m_1 k_2 \exp(lt)\, exp\left\{-\frac{n_1}{4l\, k_2^4} e^{-4lt}\right\} \tag{7.2}$$

$$C(t) = m_2 k_2 \exp(lt)\, exp\left\{-\frac{n_2}{4l\, k_2^4} e^{-4lt}\right\} \tag{7.3}$$

$$D(t) = m_3 k_2 \exp(lt)\, exp\left\{-\frac{n_3}{4l\, k_2^4} e^{-4lt}\right\} \tag{7.4}$$

For these solutions, the metric (3.1) becomes

$$\begin{aligned} ds^2 &= dt^2 - k_2^2\, e^{2lt} dx^2 \\ &\quad - e^{2mx} k_2^2\, e^{2lt}\left[m_1^2 \exp\left(\frac{-n_1}{2lk_2^4} e^{-4lt}\right) dy^2 + m_2^2 \exp\left(\frac{-n_2}{2lk_2^4} e^{-4lt}\right) dz^2 \right. \\ &\quad \left. + m_3^2 \exp\left(\frac{-n_3}{2lk_2^4} e^{-4lt}\right) du^2\right] \end{aligned} \tag{7.5}$$

In case of $r = 0$, spatial volume (V), Hubble parameter (H), expansion scalar (θ) and deceleration parameter q take the form

$$V = k_2^4 \, e^{4lt} \tag{7.6}$$

$$H = l \tag{7.7}$$

$$\theta = 4l \tag{7.8}$$

$$q = -1 \tag{7.9}$$

The shear scalar σ^2 obtained as

$$\sigma^2 = \left(\frac{n_1^2 + n_2^2 + n_3^2}{2k_2^8}\right) e^{-8lt} \tag{7.10}$$

The anisotropy parameter A_m becomes

$$A_m = \left(\frac{n_1^2 + n_2^2 + n_3^2}{4l^2 \, k_2^8}\right) e^{-8lt} \tag{7.11}$$

Now, from equations (3.3) - (3.7) and (7.1) - (7.4), we obtain pressure (p) and energy density (ρ) of the universe for the model $r = 0$

$$\begin{aligned} p(t) = \frac{-\lambda_1}{(8\pi + \lambda_2)(16\pi + 5\lambda_2)} [& 2k_2^{-8}(n_1 n_2 + n_2 n_3 + n_3 n_1)(8\pi + 2\lambda_2)\, e^{-8lt} \\ & + 3m^2 \, k_2^{-2}(16\pi + \lambda_2)\, e^{-2lt} - 12l^2(8\pi + \lambda_2)] \end{aligned} \tag{7.12}$$

$$\begin{aligned} \rho(t) = \frac{-\lambda_1}{(8\pi + \lambda_2)(16\pi + 5\lambda_2)} [& 2k_2^{-8}(n_1 n_2 + n_2 n_3 + n_3 n_1)(8\pi + 3\lambda_2)\, e^{-8lt} \\ & - 6m^2 \, k_2^{-2}(16\pi + \lambda_2)\, e^{-2lt} + 12l^2(8\pi + \lambda_2)] \end{aligned} \tag{7.13}$$

From equations (7.12), (7.13) and (2.14), we obtain the expression for cosmological constant

$$\Lambda(t) = \frac{\lambda_2}{(16\pi + 5\lambda_2)} [k_2^{-8}(n_1 n_2 + n_2 n_3 + n_3 n_1)\, e^{-8lt} + 9m^2 \, k_2^{-2} \, e^{-2lt} - 18l^2] \tag{7.14}$$

8 Conclusion for model $r = 0$

This model shows a non-singular behavior and does not possess any singularity during the phase of expansion. Model also follow the accelerated expansion as $\frac{\dot{V}}{V} > 0$ with $l > 0$. The metric coefficients A, B, C, D and spatial volume (V) of the universe increase exponentially with the cosmic time t.

We observe that pressure (p), energy density (ρ), cosmological constant (Λ), anisotropy parameter (A_m) and shear scalar (σ^2) decrease exponentially with cosmic time *t*. Since $\frac{\sigma}{\theta} \to 0$ as $t \to \infty$, which shows that the universe attains isotropy at later stages. The deceleration parameter q = − 1, shows accelerating expansion of the universe. We shall search more solutions for other higher dimensional space time in *f(R,T)* theory by taking various functional form of *f(R,T)* as per Harko *et al.*[16].

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