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Bianchi Type-VIII Inflationary Cosmological Models with Flat Potential and Stiff perfect Fluid Distribution in General Relativity

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Abstract

The objective of this paper is to investigate the behaviour of Bianchi Type-VIII Inflationary Cosmological Models with Flat Potential for Stiff Fluid Distribution. To find the solution of thes models we consider that the expansion θ is proportional to the shear σ, which leads to $B = A^n$ and potential $V(\phi)$ as a constant. The behaviour of these models from physical and geometrical aspects are also discussed.

Key words : Bianchi Type-VIII, Inflationary Universe, Stiff Fluid, General Relativity.

1 Introduction

Exponential expansion in the early universe is termed as inflation. This theory comes in the beginning of the 1980s and today receives a lot of attention. The inflationary epoch comprises the first part of the electro weak epoch following the grand unification epoch. It was going on from 10^{-33} to 10^{-32} seconds. In that inflationary period, the universe continuously expanded. Inflationary universes allows a potential solution to the creation of structure problem in Big-Bang cosmology like Horizon, Flatness and Magnetic monopole problems. Guth[11] proposed the idea of early inflationary phase in respect to grand unified field theory in that symmetry breaking phase transition occur with decrease of temperature at the very early stage of evolution of universe and suggested that rapid expansion is due to false vacuum as confirmed by Cosmic Microwave Background radiation. The inflationary scenario explains several mysteries of modern cosmology like flatness problem, horizon problem, homogeneity and isotropy etc. as mentioned by Liddle and Lyth[15].

The inflationary cosmology is not substitution for the Hot Big Bang model but probably link-on that occurs at very early times without disturbing any of its success. When we saw the universe in smaller scale, then we find neither homogeneous nor isotropic. We expect that these properties have in the early universe. The sufficiency of spatially homogeneous and isotropic Fridman-Roberston – Walker (FRW) models for characterizing the present state of universe, is no basis for expecting suitable for characterizing the early stages of evolution of universe. Cosmological models which are spatially homogeneous but not isotropic have important role in the explanation of the universe at its early stage of evolution.

Bianchi type-VIII inflationary cosmological models plays a major role for relativistic studies as these models allows not only expansion but also shear and rotation in general, these models are anisotropic. Bali and Swati[5] have studied Bianchi type-VIII inflationary universe with massless scalar field and find the cosmological parameters in hyperbolic function. Ram and Priyanka[16] have studied Bianchi Type-II Inflationary Models with Stiff Matter and Decaying Cosmological Term. Balli and Poonia[7] studied Bianchi Type-III inflationary cosmological model with bulk viscosity and flat potential. Bali and Kumari[6] investigated Chaotic inflationary scenario in spatially homogeneous Bianchi type-V space–time. Sing and Kumar[20] have investigated Bianchi Type-II inflationary models with constant deceleration parameter in general relativity.

Anninos *et al.*[4] has discussed the significance of inflation for isotropization of the universe. Many authors viz. Abbott and Wise[1], Adhav *et al.*[2], Albrecht and Steinhardt[3], Bali *et al.*[8,9,10], Reddy *et al.*[17,18], Katore *et al.*[14], Jat *et al.*[12,13] studied inflationary cosmological models in different Bianchi space-times with self interacting scalar field in General Relativity.

Anisotropic cosmological models as well as the so-called Bianchi cosmologies are of significant theoretical importance. Due to the most general everexpanding Bianchi cosmologies, Bianchi Type-VIII cosmological models are special interesting. Bianchi type space-time play a very important role in explanation of the early evolution of the universe. In general, the study of Bianchi type II, VIII, and IX universes are most important due to familiar solution of FRW universe with positive curvature. The de-Sitter universe, the Taub-Nut solutions etc. correspond Bianchi type II, VIII and IX space-times. Shri and Singh[19] have studied Bianchi type-II, VIII and IX cosmological models with matter and electromagnetic fields.

Motivated by the above mentioned studies, we investigate inflationary scenario in Bianchi Type-VIII space-time for a massless scalar field with flat potential using the condition that shear σ is proportional to expansion (θ). The physical and geometrical aspects of these model are also discussed. We find that model represents accelerating universe and spatial volume increases exponentially representing inflationary scenario.

2 The Metric and Field Equations

The Bianchi type-VIII line-element in the form as-

$$ds^2 = dt^2 - B^2dx^2 - A^2dy^2 - \left[A^2sinh^2y + B^2cosh^2y\right]dz^2 - 2B^2\,coshy\,dx\,dz \tag{1}$$

where A and B are functions of time t only.
We assume the co-ordinates to be co-moving so that

$$v^1 = v^2 = v^3 = 0 \text{ and } v^4 = 1 \tag{2}$$

In case of gravity minimally coupled to a scalar field $V(\phi)$, as given by Stein Schabes, we have

$$S = \int \sqrt{-g}\left[R - \frac{1}{2}g^{ij}\partial_i\phi\partial_j\phi - V(\phi)\right]d^4x \tag{3}$$

Einstein's field equation is given by

$$R_{ij} - \frac{1}{2}Rg_{ij} = -T_{ij} \tag{4}$$

with

$$T_{ij} = (\rho + p)v_iv_j - pg_{ij} + \partial_i\phi\partial_j\phi - \left[\frac{1}{2}\partial_k\phi\partial\phi^k + V(\phi)\right]g_{ij} \tag{5}$$

Here ρ is the energy density, p is pressure, ϕ is Higgs field, V is potential.
The conservation relation leads to

$$\frac{1}{\sqrt{-g}}\partial_i\left[\sqrt{-g}\partial^i\phi\right] = -\frac{dV}{d\phi} \tag{6}$$

The Einstein's field equation for the line-element (1) leads to the non-linear differential equations as follows

$$\frac{2A_{44}}{A} + \frac{{A_4}^2}{A^2} - \frac{1}{A^2} - \frac{3B^2}{4A^4} = -p - \frac{1}{2}{\phi_4}^2 - V(\phi) \tag{7}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4B_4}{AB} + \frac{B^2}{4A^4} = -p - \frac{1}{2}{\phi_4}^2 - V(\phi) \tag{8}$$

$$\frac{2A_4B_4}{AB} + \frac{{A_4}^2}{A^2} - \frac{1}{A^2} - \frac{B^2}{4A^4} = \rho + \frac{1}{2}{\phi_4}^2 - V(\phi) \tag{9}$$

Here the sub-induces 4 in A, B denotes differentiation with respect to time t.
From equation (6) for scalar field (ϕ) leads to

$$\phi_{44} + \left(\frac{2A_4}{A} + \frac{B_4}{B}\right)\phi_4 = -\frac{dV}{d\phi} \tag{10}$$

Where suffix 4 indicates derivative with respect to time *t*.

3 Solution of Field Equations :

The field equations (7)-(9) are the set of three equations with five unknown parameters-*A, B, p, ρ, ϕ*.
To solve these equations we assume the following conditions-

(i) In the presence of stiff fluid distribution

$$\rho = p \tag{11}$$

(ii) The expansion θ is proportional to the shear σ that leads to

$$B = A^n \tag{12}$$

Since, in this model we assume that flat potential so $V(\phi)$ is constant. Then let

$$V(\phi) = K \tag{13}$$

Where K is a constant.
From equations (10) and (13), we obtain

$$\phi_4 = \frac{l}{A^2 B} \tag{14}$$

where l is integration constant.
Scale factor (R) of line-element (1) is given by

$$R^3 = A^2 B = A^{n+2} \tag{15}$$

From equations (7),(9) and (11), we obtain

$$\frac{2A_{44}}{A} + \frac{2{A_4}^2}{A^2} + 2\frac{A_4 B_4}{AB} - \frac{2}{A^2} - \frac{B^2}{A^4} = -2V(\phi) \tag{16}$$

Equations (13) and (16) togerher lead to-

$$\frac{2A_{44}}{A} + \frac{2{A_4}^2}{A^2} + 2\frac{A_4 B_4}{AB} - \frac{2}{A^2} - \frac{B^2}{A^4} = -2K \tag{17}$$

From equations (12) and (17), we obtain

$$\frac{2A_{44}}{A} + \frac{2{A_4}^2}{A^2} + 2n\frac{{A_4}^2}{A^2} - \frac{2}{A^2} - \frac{A^{2n}}{A^4} = -2K \tag{18}$$

$$2A_{44} + (2n+2)\frac{A_4^2}{A} = \frac{2}{A} + A^{2n-3} - 2KA \tag{19}$$

Now we assume that $A_4 = f(A)$, $A_{44} = f\,\frac{df}{dA}$ then equation (19) leads to -

$$\frac{df}{dA}^2 + (2n+2)\frac{f^2}{A} = \frac{2}{A} + A^{2n-3} - 2KA \tag{20}$$

After integration of equation (20), we find

$$f^2 A^{(2n+2)} = \frac{A^{(2n+2)}}{n+1} + \frac{A^{4n}}{4n} - K\frac{A^{2n+4}}{n+2} + C_1 \tag{21}$$

where C_1 is integration constant.
Equation (21) leads to-

$$f^2 = \frac{1}{n+1} + \frac{A^{2n-2}}{4n} - K\frac{A^2}{n+2} + C_1 A^{-(2n+2)} \tag{22}$$

Equation (22) can be written as-

$$\frac{dA}{dt} = \sqrt{\frac{1}{n+1} + \frac{A^{2n-2}}{4n} - K\frac{A^2}{n+2} + C_1 A^{-(2n+2)}} \tag{23}$$

Equation (23) leads to-

$$\int \frac{dA}{\sqrt{\frac{1}{n+1} + \frac{A^{2n-2}}{4n} - K\frac{A^2}{n+2} + C_1 A^{-(2n+2)}}} = \int dt + C_2 = t + C_2 \tag{24}$$

where C_2 is constant of integration.
We can find the value of A from equation (24).
After acceptable transformation of coordinates, we find the line element (1) as-

$$\begin{aligned} ds^2 = &\left(\frac{1}{\frac{1}{n+1} + \frac{T^{2n-2}}{4n} - K\frac{T^2}{n+2} + C_1 T^{-(2n+2)}}\right) dT^2 - T^{2n} dX^2 - T^2 dY^2 \\ &- \left(T^2 \sinh^2 Y + T^{2n}\cosh^2 Y\right) dZ^2 - 2T^{2n}\cosh Y\, dX\, dZ \end{aligned} \tag{25}$$

where transformation of coordinates used as $x = X$, $y = Y$, $z = Z$, $A = T$

4 Special Model :

Take $n = 2$, and $C_1 = 0$
When we put $n = 2$, and $C_1 = 0$ in equation (22), we get-

$$f^2 = \frac{1}{3} + (1-2K)\frac{A^2}{8} \tag{26}$$

After rewriting the above equation (26), we get-

$$f = \frac{dA}{dt} = \sqrt{\frac{1}{3} + \frac{1}{8}(1-2K)A^2} \tag{27}$$

Then the equation (27) leads to-

$$A = \sqrt{\frac{8}{3(1-2K)}}\sinh\left[(t+C_4)\sqrt{\frac{1-2K}{8}}\right], K < \frac{1}{2} \tag{28}$$

Let, $\alpha = \sqrt{\frac{1-2K}{8}}, \gamma^2 = \frac{1}{3\alpha^2}, \delta = C_4\alpha$

$$A = \gamma \sinh(\alpha t + \delta),\ B = A^2 = \gamma^2 \sinh^2(\alpha t + \delta) \tag{29}$$

After suitable transformation of coordinates, the metric (1) leads to the form

$$ds^2 = \frac{d\tau^2}{\alpha^2} - \gamma^4 \sinh^4 \tau dX^2 - \gamma^2 \sinh^2 \tau dY^2 - (\gamma^2 \sinh^2 \tau \sinh^2 Y + \gamma^4 \sinh^4 \tau \cosh^2 Y)\, dZ^2 - 2\gamma^4 \sinh^4 \tau \cosh Y\, dX dZ \tag{30}$$

where $X = x,\ Y = y,\ Z = z,\ \alpha t + \delta = \tau$

5 Physical and Geometrical Aspects :

For the model (25) Higgs Field

$$\phi = l \int \frac{dT}{T^{(n+2)}\sqrt{\frac{1}{n+1} + \frac{T^{2n-2}}{4n} - K\frac{T^2}{n+2} + C_1 T^{-(2n+2)}}} + C_3 \tag{31}$$

where C_3 is integration constant.

The Spatial volume (V), Hubble directional parameters (H_x, H_y and H_z), Hubble parameter (H), the expansion (θ), shear (σ), energy density (ρ) isotropic pressure (p) and deceleration parameter (q) for the model (25) are given by

$$V = R^3 = T^{n+2} \tag{32}$$

$$H_x = H_y = \sqrt{\frac{1}{(n+1)T^2} + \frac{1}{4nT^{4-2n}} - \frac{K}{n+2} + \frac{C_1}{T^{(2n+4)}}} \tag{33}$$

$$H_z = n\sqrt{\frac{1}{(n+1)T^2} + \frac{1}{4nT^{4-2n}} - \frac{K}{n+2} + \frac{C_1}{T^{(2n+4)}}} \tag{34}$$

$$H = \frac{n+2}{3}\sqrt{\frac{1}{(n+1)T^2} + \frac{1}{4nT^{4-2n}} - \frac{K}{n+2} + \frac{C_1}{T^{(2n+4)}}} \tag{35}$$

$$\theta = (n+2)\sqrt{\frac{1}{(n+1)T^2} + \frac{1}{4nT^{4-2n}} - \frac{K}{n+2} + \frac{C_1}{T^{(2n+4)}}} \tag{36}$$

$$\sigma_1^1 = \frac{2(n-1)}{3}\sqrt{\frac{1}{(n+1)T^2} + \frac{1}{4nT^{4-2n}} - \frac{K}{n+2} + \frac{C_1}{T^{(2n+4)}}} \tag{37}$$

$$\sigma_2^2 = \sigma_3^3 = \frac{(1-n)}{3}\sqrt{\frac{1}{(n+1)T^2} + \frac{1}{4nT^{4-2n}} - \frac{K}{n+2} + \frac{C_1}{T^{(2n+4)}}} \tag{38}$$

$$\sigma_4^4 = 0 \tag{39}$$

$$\sigma = \frac{(n-1)}{\sqrt{3}}\sqrt{\frac{1}{(n+1)T^2} + \frac{1}{4nT^{4-2n}} - \frac{K}{n+2} + \frac{C_1}{T^{(2n+4)}}} \tag{40}$$

From equations (7) and (9), we get-

$$p=\rho=\frac{n}{(n+1)T^2}+\frac{(n+1)}{4nT^{4-2n}}-K\frac{(n-1)}{n+2}+\left(C_1(2n+1)-\frac{l^2}{2}\right)\frac{1}{T^{(2n+4)}} \quad (41)$$

$$q=-1-\frac{3}{(n+2)}\left(\frac{\frac{-1}{(n+1)T^2}+\frac{n-2}{4nT^{(4-2n)}}-C_1\frac{(n+2)}{T^{(2n+4)}}}{\frac{1}{(n+1)T^2}+\frac{1}{4nT^{4-2n}}-\frac{K}{n+2}+\frac{C_1}{T^{(2n+4)}}}\right) \quad (42)$$

From equations (36) and (40), we get

$$\frac{\sigma}{\theta}=\frac{(n-1)}{\sqrt{3}(n+2)};\ n\neq -2 \quad (43)$$

For the special model (30), Spatial volume (V), Hubble directional parameters (H_x, H_y and H_z), Hubble parameter (H), the expansion (θ), shear (σ), deceleration parameter (q) the energy density (ρ) and isotropic pressure (p), and Higgs Field (ϕ) are given by

$$V=\gamma 4\ \sinh^4\tau \quad (44)$$

$$H_x=H_y=\alpha\coth\tau \quad (45)$$

$$H_z=2\alpha\coth\tau \quad (46)$$

$$H=\frac{H_x+H_y+H_z}{3}=\frac{4\alpha}{3}\coth\tau \quad (47)$$

$$\theta=3H=4\alpha\coth\tau \quad (48)$$

$$\sigma_1^1=\frac{2\alpha}{3}\coth\tau \quad (49)$$

$$\sigma_2^2=\sigma_3^3=-\frac{\alpha}{3}\coth\tau \quad (50)$$

$$\sigma_4^4=0 \quad (51)$$

$$\sigma=\frac{\alpha}{\sqrt{3}}\coth\tau \quad (52)$$

$$q=-1+\frac{3}{4}sech^2\tau \quad (53)$$

$$\rho=p=\frac{1}{4}-\alpha^2+2\alpha^2\coth^2\tau-\frac{l^2}{2\gamma^8}\operatorname{csc h}^8(\tau) \quad (54)$$

$$\phi=-\frac{l}{3\alpha\gamma^4}\coth^3\tau+\frac{l}{\gamma^4\alpha}\coth\tau+C_3 \quad (55)$$

where C_3 is integration constant.

From equations (48) and (52), we get

$$\frac{\sigma}{\theta}=\frac{1}{4\sqrt{3}} \quad (56)$$

6 Graphical Representation :

Here we draw graphs of some cosmological parameters for n=2 and we take

$$C_1 = 0, K = \frac{1}{4}, \alpha = \frac{1}{4}, \gamma = \frac{4}{\sqrt{3}}, \delta = 1$$

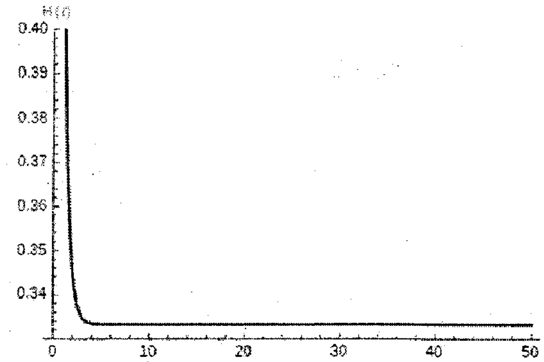

Figure 1: Hubble Parameter H[τ] verses τ

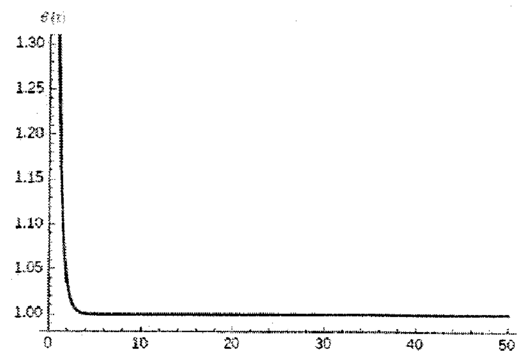

Figure 2: Expansion Parameter $\theta[\tau]$ verses τ

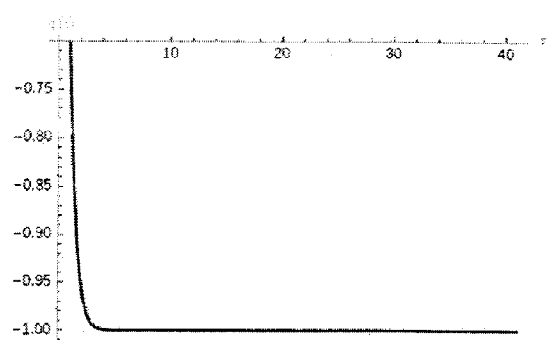

Figure 3: Decelaration Parameter $q[\tau]$ verses τ

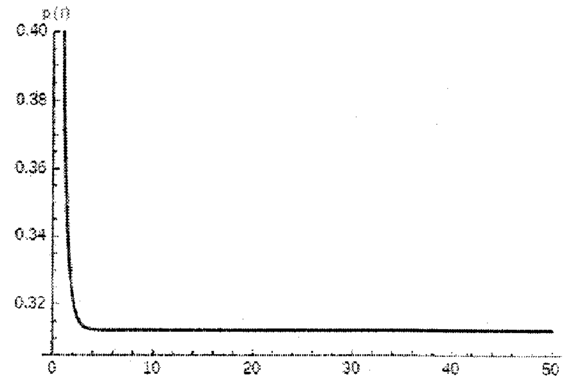

Figure 4: Pressure $p[\tau]$ verses τ

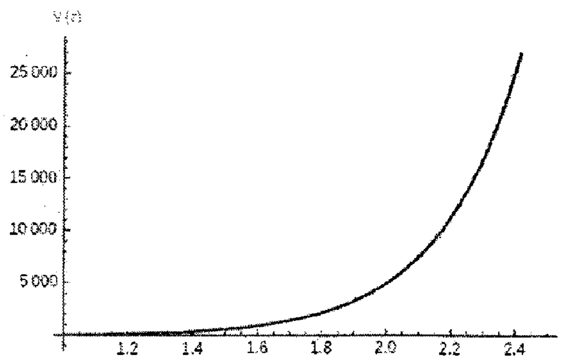

Figure 5: Volume $V[\tau]$ verses τ

7 Conclusion

The spatial volume (V) for the model (25) increases with time T for $n > -2$ which represents the inflationary behaviour of universe including massless scalar field with flat potential.

Hubble parameter is initially large but decreases with time T for $-2 < n < 2$. Pressure and energy density of the model is initially large for $-2 < n < 2$.

At starting, the rate of Higgs field is large but decreases as time increases for $n > -2$ and tends to a constant value as $T \to \infty$. Since, $\lim_{T \to \infty} \frac{\sigma}{\theta} \neq 0$ that represents anisotropic model in general. But for $n = 1$, $\lim_{T \to \infty} \frac{\sigma}{\theta} \neq 0$ that represents its isotropic behaviour.

The model (25) has point type singularity at $T = 0$.

We can see for special model (30), Hubble parameter (H) decreases with τ and tends to a constant value for $\tau \to \infty$.

Expansion parameter (θ) decreases as τ increases and tends to a constant value as time $\tau \to \infty$. Spatial volume (V) is increasing function of τ.

We observes for special model, as $\tau \to \infty$ deceleration parameter (q) tends to -1 that represents the accelerating phase of this model.

Pressure (p) is function of τ and decreases as τ increases. It tends to a constant value as $\tau \to \infty$.

For the model (30), $\lim_{T\to\infty} \frac{\sigma}{\theta} \neq 0$ that shows the anisotropic behaviour of its. The special model has point type singularity at $\tau = 0$.

We can observe the physical behaviour of special model from the graphical representation (1-5).

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