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Approximate solution of Improper Integrals

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Abstract

This paper proposes an efficient quadrature rule of precision eleven for the numerical evaluation of improper integrals with endpoint singularities. The proposed method combines Gaussian and Newton–Cotes quadrature rules of lower precision within a unified framework, incorporating cubic Newton divided differences and Romberg extrapolation to enhance computational efficiency and accuracy. An error bound is derived using a mixed quadrature approach, providing a theoretical basis for the performance of the proposed scheme. Numerical experiments and convergence analyses demonstrate its accuracy, stability, and efficiency. The proposed method is further validated through comparisons with existing quadrature techniques and available analytical solutions. Moreover, the Mellin transform is employed to accelerate the numerical computations and improve the accuracy of the resulting approximations. The results indicate that the proposed efficient quadrature scheme provides an effective and reliable approach for the numerical treatment of singular improper integrals.

Key words : efficient quadrature, Mellin transform, cubic Newton divided differences, Romberg extrapolation; Singular improper integrals.

MSC 2010: 65D30, 65D32.

1. Introduction

This section provides an overview of joint quadrature rules and product integration. Product integrals play an important role in several areas of applied mathematics, including survival analysis,

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Markov chains, and quantum theory. They can be regarded as multiplicative analogues of functions associated with Riemann integrals. In a product integral, the limiting process is based on a product rather than a sum. Numerical integration refers to the process of approximating a definite integral from a set of tabulated values of the integrand when an explicit analytical representation of the function is unavailable. In this work, we consider Newton–Cotes and Gaussian-type quadrature rules for functions of a single variable. The proposed approach is based on constructing a joint quadrature rule of higher precision by combining quadrature rules of lower precision. Several mixed quadrature rules have been developed in the literature for the numerical evaluation of real definite integrals of the form

$$\int_a^b f(x) dx = I(f). \quad (1)$$

The real definite integral in equation (1) can be approximated using various open and closed quadrature rules. A linear transformation of the integration variable,

$$x = \frac{1}{2}[(b-a)t + (b+a)], \quad (2)$$

maps the interval $[a, b]$ onto $[-1, 1]$. Although several techniques are available for transforming improper integrals into integrals over finite intervals, such transformations may introduce endpoint singularities. Consequently, the direct application of Romberg's rule to the resulting integral may not be appropriate. To overcome this difficulty and avoid the adverse effects of endpoint singularities in product integration, an appropriate transformation is employed to obtain an accurate approximation to equation (1).

In this study, an efficient quadrature rule of exactness eleven is developed by combining different Newton–Cotes and Gaussian-type quadrature rules of lower precision for the numerical evaluation of improper integrals. The proposed formulation is designed to improve both the accuracy and efficiency of the numerical approximation. In particular, a two-step transformation of the interval $[0, \infty)$ to $[-1, 1]$ through $[0, 1]$ is performed using equation (2), which facilitates the numerical treatment of the improper integral and the automatic extraction of endpoint singularities.

Several authors have investigated numerical approaches for obtaining approximate solutions of equation (1) [1-20]. Related developments in numerical integration method [21-74], provide a broad foundation for the present study.

The remainder of this paper is organized as follows. Section 2 presents the construction of the proposed efficient quadrature rules together with their precision, error estimates, and convergence analysis. Section 3 discusses the numerical results and compares the proposed method with existing approaches. Section 4 provides numerical verification of the theoretical findings through representative test problems. Finally, Section 5 summarizes the main conclusions and discusses the significance and applicability of the proposed approach and Section 6 contains scope of future work.

2. Methodology :

In this section, we construct an effective quadrature rule of precision 11 with linear combination

of lower degree precision rules as follows:

Gauss Legendre two point and Steffensen four-point rules are merged to obtain the joint quadrature rule of precision 5.

Steffensen four-point rule

$$I_{ST4}(f) = \frac{11}{12} \left[f\left(-\frac{3}{5}\right) + f\left(\frac{3}{5}\right) \right] + \frac{1}{12} \left[f\left(-\frac{1}{5}\right) + f\left(\frac{1}{5}\right) \right], \quad (3)$$

Gauss Legendre two point rule

$$I_{GL2}(f) = f\left(\frac{1}{\sqrt{3}}\right) + f\left(-\frac{1}{\sqrt{3}}\right), \quad (4)$$

$$E_{ST4}(f) = \frac{304}{375 \times 5!} f^{(iv)}(0) + \frac{13136}{9375 \times 7!} f^{(vi)}(0) + \frac{672992}{390625 \times 9!} f^{(viii)}(0) + \frac{11004256}{585937511 \times 11!} f^{(x)}(0) + \dots, \quad (5)$$

$$E_{GL2}(f) = \frac{8}{9 \times 5!} f^{(iv)}(0) + \frac{40}{27 \times 7!} f^{(vi)}(0) + \frac{16}{9 \times 9!} f^{(viii)}(0) + \frac{464}{243 \times 11!} f^{(x)}(0) + \dots, \quad (6)$$

Associating $\left(\frac{1}{3}\right)$ with equation (5) and $\left(-\frac{38}{125}\right)$ with equation (6) and the adding the resultant

$$I(f) = \frac{1}{11} [125I_{ST4}(f) - 114I_{GL2}(f)] + \frac{1}{11} [125E_{ST4}(f) - 114E_{GL2}(f)]. \quad (7)$$

$$\text{Where, } I(f) = I_A(f) + E_A(f), \quad (8)$$

$$\text{Where, } I_A(f) = \frac{1}{11} [125I_{ST4}(f) - 114I_{GL2}(f)].$$

Equation (9) is the desired joint quadrature rule with precision 5 for approximating $I(f)$.

Applying Maclaurin's expansion of functions in equation (1) and equation (9) and employing the result in equation (8), we obtain

$$E_A(f) = \frac{128}{225 \times 7!} f^{(vi)}(0) + \frac{10816}{9375 \times 9!} f^{(viii)}(0) + \frac{1964992}{1265625 \times 11!} f^{(x)}(0) + \dots, \quad (10)$$

and

$$E_{GL3}(f) = \frac{8}{25 \times 7!} f^{(vi)}(0) + \frac{88}{125 \times 9!} f^{(viii)}(0) + \frac{656}{625 \times 11!} f^{(x)}(0) + \dots, \quad (11)$$

Multiplying equation (11). by $-\left(\frac{16}{9}\right)$ and adding it to equation (10)

$$I(f) = \frac{1}{7} [16I_{GL3}(f) - 9I_A(f)] + \frac{1}{7} [16E_{GL3}(f) - 9E_A(f)]. \quad (12)$$

Where,

$$I(f) = I_B(f) + E_B(f), \quad (13)$$

Where,

$$I_B(f) = \frac{1}{7} [16I_{GL3}(f) - 9I_A(f)]. \quad (14)$$

This is the appropriate quadrature rule of precision 7 for the approximate evaluation of $I(f)$.

Further, Maclaurin's expansion is applied for equation (1) and equation (14) and substituting the results in equation (13)

$$E_B(f) = \frac{2752}{21875 \times 9!} f^{(viii)}(0) + \frac{3569472}{8859375 \times 11!} f^{(x)}(0) + \dots, \quad (15)$$

. Here, Gauss Legendre four-point rule and equation (14) are merged to expedite the required mixed rule. Gauss Legendre four-point rule

$$I_{GL4}(f) = \frac{1}{36} [(18 + \sqrt{30})[f(-m) + f(m)] + (18 - \sqrt{30})[f(-n) + f(n)]]. \quad (16)$$

Where

$$m = \sqrt{\frac{3-2\sqrt{\frac{6}{5}}}{7}}, \quad n = \sqrt{\frac{3+2\sqrt{\frac{6}{5}}}{7}}, \quad (17)$$

$$E_{GL4}(f) = \frac{128}{1225 \times 9!} f^{(viii)}(0) + \frac{2432}{2432 \times 11!} f^{(x)}(0) + \dots \quad (18)$$

Multiplying equation (18) and equation (15) by $(-\frac{2}{7})$ and $(\frac{43}{125})$ respectively and then adding the resulting equations, yields

$$I(f) = \frac{1}{51} [301I_{GL4}(f) - 250I_B(f)] + \frac{1}{51} [301E_{GL4}(f) - 250E_B(f)], \quad (19)$$

$$I(f) = I_C(f) + E_C(f), \quad (20)$$

Where

$$I_C(f) = \frac{1}{51} [301I_{GL4}(f) - 250I_B(f)]. \quad (21)$$

Equation (21) is the proper joint quadrature rule of precision 9 for approximating $I(f)$.

Using Maclaurin's expansion is applied for function in equation (1) and equation (21). and substituting the results in equation (19), we get

$$E_C(f) = \frac{7619325}{25302375 \times 11!} f^{(x)}(0) + \dots, \quad (22)$$

In this section, an effective quadrature rule of precision 11 is obtained.

In this proof equation (21) and Gauss Legendre five-point rule are combined. The Gauss Legendre five-point rule is expressed as

$$I_{GL5}(f) = \frac{1}{900} [(322 + 13\sqrt{70})[f(-p) + f(p)] + (322 - 13\sqrt{70})[f(-q) + f(q)] + 512f(0)], \quad (23)$$

Where

$$m = \sqrt{\frac{5-2\sqrt{\frac{10}{7}}}{9}}, \quad n = \sqrt{\frac{5+2\sqrt{\frac{10}{7}}}{9}}, \quad (24)$$

$$I(f) = I_{GL5}(f) + E_{GL5}(f), \quad (25)$$

and

$$E_{GL5}(f) = \frac{128}{43659 \times 10!} f^{(x)}(0) + \frac{2706048}{338527917 \times 12!} f^{(xii)}(0) + \dots, \quad (26)$$

Adding the two equations after multiplication of equation (26) by $\left(\frac{128}{7}\right)$ and equation (22) by $\left(\frac{14513}{85}\right)$

$$I(f) = \frac{1}{112471} [10880I_C(f) + 101591I_{GL5}(f)] + \frac{1}{11271} [10881E_C(f) + E_{GL5}(f)], \quad (27)$$

$$I(f) = I_D(f) + E_D(f), \quad (28)$$

Where,

$$I_D(f) = \frac{1}{112471} [10880I_C(f) + 101591I_{GL5}(f)]. \quad (29)$$

Equation (29) is the rule of precision 11.

Theorem 1: The absolute truncation error is developed by this approximation represented by $I_D(f)$ is

$$|E_D(f)| \cong \frac{274910122368}{38074573352907} |f^{(xii)}(0)|.$$

Proof: Maclaurin's expansion is applied for function in equation (1) and equation (29) and substituting the result in equation (28), we have

$$|E_D(f)| \cong \frac{274910122368}{38074573352907} |f^{(xii)}(0)|. \quad (30)$$

In this section the convergence analysis is executed.

3. Implementations

Two transformations are considered to facilitate the evaluation of the integrals. For I_1 , the transformation

$$x = \log(y)$$

is employed to transform the integral from the semi-infinite interval to the finite interval $[0, 1]$. Thus,

$$I_1 = \int_0^\infty \frac{10(x^2 + 1)}{e^{2x}} dx = \int_0^1 10 y [(\log y)^2 + 1] dy = 7.5.$$

For the evaluation of $x = \frac{1}{t}$ I_2 , the transformation

is employed. In conjunction with the Mellin transform of $\sin(t)$, evaluated at $s = 0.5$, and the Gamma function, this transformation facilitates the evaluation of the integral. Specifically,

$$I_2 = \int_0^\infty x^{-1.5} \sin(x^{-1}) dx = \int_0^\infty t^{s-1} \sin(t) dt = \sqrt{\frac{\pi}{2}} = 1.25331.$$

Furthermore,

the

integral

$$\int_1^\infty \frac{\sin(t)}{\sqrt{t}} dt = 0.6327775$$

is evaluated separately. Consequently,

$$\int_0^1 \frac{\sin(t)}{\sqrt{t}} dt + \int_1^\infty \frac{\sin(t)}{\sqrt{t}} dt = 1.25331.$$

Table 1: Comparative study of errors in problem I_1 with [13]

N	C	Exact Solution	Error 1	Error 2	Error 3($E_D(f)$)
16	1	7.5	0.070629121	0.032528748	0.010330126
16	2	7.5	0.026541351	0.013743017	0.010330126
32	1	7.5	0.023415684	0.010381343	0.010330126
32	2	7.5	0.007677673	0.003823071	0.010330126
64	1	7.5	0.007489323	0.003393561	0.010330126
64	2	7.5	0.002180337	0.001802767	0.010330126
256	1	7.5	0.000709294	0.000352698	0.010330126
256	2	7.5	0.000168680	0.000118525	0.010330126

Table 2: Comparison of the errors in problem I_2 with [13]

N	C	Exact Solution	Error 1	Error 2	Error 3($E_D(f)$)
16	1	1.25330	0.003208	0.001754	0.000143
16	2	1.25330	0.001267	0.001140	0.000143
32	1	1.25330	0.001137	0.000643	0.000143
32	2	1.25330	0.000447	0.000344	0.000143
64	1	1.25330	0.000402	0.000231	0.000143
64	2	1.25330	0.000157	0.000111	0.000143
256	1	1.25330	0.000051	0.000028	0.000143
256	2	1.25331	0.000018	0.000013	0.0001443

4. Numerical Results

The two integrals, I_1 and I_2 , are not evaluated analytically; instead, the proposed scheme provides their numerical approximations by appropriately extracting the endpoint singularities. To assess the accuracy and efficiency of the proposed approach, three error measures are considered: Error 1, representing the absolute error between the exact value and the Romberg approximation; Error 2, representing the absolute error between the exact value and the mixed-method approximation; and Error 3, representing the absolute error between the exact value and the proposed joint quadrature approximation. The corresponding numerical results, obtained using MATLAB, are presented in Tables 1 and 2. In the implementation of Romberg's method, N and C denote the number of quadrature points and the number of columns in the Romberg extrapolation, respectively.

5. Conclusions

A hybridized numerical technique for the approximation of improper integrals is developed by implementing a joint quadrature rule. The proposed effective quadrature technique achieves the same level of functional evaluation accuracy with only 17 function evaluations, compared with 32 and 64

function evaluations required by the Romberg and mixed methods, respectively. The numerical results demonstrate that the proposed quadrature rule is more efficient and computationally economical than the existing methods in terms of the number of functional evaluations required to obtain accurate numerical approximations. In addition to this computational advantage, the proposed scheme is applicable to hypersingular integrals through the extraction of the singularity at the boundary point. The high-precision joint quadrature rule is found to provide improved accuracy compared with the mixed quadrature rule, which is constructed by combining the cubic Newton divided-difference formula with Romberg's method. The effectiveness of the proposed approach is verified through a set of numerical test problems. The results indicate that the joint quadrature rule produces smaller errors while requiring fewer functional evaluations and yields numerical approximations that are in close agreement with the exact results. A detailed error analysis is also presented to further substantiate the accuracy, efficiency, and reliability of the proposed method.

6. Future Work :

Future research will focus on extending the proposed quadrature rule of precision eleven to practical problems in physics, engineering, and applied sciences. Improper integrals with endpoint singularities frequently arise in electromagnetic field analysis, heat transfer, fluid mechanics, quantum mechanics, and potential theory. The proposed method, combining Gaussian and Newton–Cotes quadrature rules with cubic Newton divided differences and Romberg extrapolation, may provide an efficient approach for evaluating such integrals accurately. Further investigations will explore its applications to singular integral equations, fractional differential equations, boundary element methods, and models involving weakly singular kernels. Numerical experiments will be conducted to assess the method's accuracy, convergence, stability, and computational efficiency compared with existing quadrature techniques. Special attention will be given to strongly singular, oscillatory, and multidimensional integrals encountered in scientific simulations. Appropriate modifications may also be developed to address complex physical problems requiring high computational precision. These investigations aim to establish the proposed quadrature rule as a reliable and efficient numerical tool for solving challenging problems in applied mathematics, physics, and engineering.

Conflicts of Interest: The authors declare no conflicts of interest.

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