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Hydromagnetic Convection Flow with Radiative Heat and Mass Transfer Past an Infinite Impulsively Moving Vertical Plate in the Presence of Heat Source/Sink

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Abstract

In this study, an analysis has been performed for heat and mass transfer with radiation effect in transient laminar boundary layer flow of a viscous fluid past an impulsively moving infinite vertical flat plate in a homogenous porous medium in the presence of thermal diffusion and heat source/sink. Exact solution of momentum, energy and diffusion equations, under Boussinesq approximation, is obtained in closed form by use of Laplace transform technique. The variations in fluid velocity, temperature and concentration distribution are shown graphically, whereas numerical values of skin-friction, Nusselt number and Sherwood number are presented in tabular form and discussed.

Key words: Radiative Heat and Mass Transfer, Homogeneous Porous Medium, Thermal diffusion, Heat Source/Sink.

Nomenclature

B_0 the magnetic field intensity,

C' concentration at the plate,

C'_∞ concentration far away from the plate,

C'_w concentration at the plate,

C_p specific heat at constant pressure,

D_T chemical molecular diffusivity,

q'_r radiative heat flux in y' - direction,

t' time,

g acceleration due to gravity,

K' permeability of the porous medium,

k^* mean absorption coefficient,

K_T thermal conductivity,

Q' heat source,

Q non- dimensional heat source,

t non-dimensional time,

T' temperature of the fluid in the boundary layer,

T'_w temperature of the plate,

T'_∞ temperature at equilibrium,

T non-dimensional temperature,

U uniform velocity of the plate,

u', v' velocity components in x' and y'-direction,

u, v non-dimensional velocity components in x and y-direction,

x', y' cartesian coordinate,

x, y non-dimensional cartesian coordinate,

Greek Symbols :

β' volumetric expansion for heat transfer,

β^* volumetric expansion for mass transfer,

ρ density of the fluid,

σ electrical conductivity of the fluid,

σ^* Stefan Boltzmann constant,

μ viscosity of the fluid,

υ kinematic viscosity of the fluid.

I. Introduction

The phenomenon of hydromagnetic (MHD) flow with heat transfer has been a subject of interest for many researchers due to its varied applications in thermal sciences and technology. In fact, the problems of magneto-hydrodynamic free and forced convection flow in porous and non-porous media are being investigated due to significant effect of magnetic field on the boundary layer control and on the performance of many engineering devices using electrically conducting fluids. Such fluid flows find application in MHD power generation, MHD pumps, flow meters, accelerators, nuclear reactors using liquid metal coolant and geothermal energy extraction. Unsteady MHD convection flow past a vertical plate is investigated by a number of researchers considering different sets of momentum and thermal boundary conditions at the bounding plate. Mention may be made of the research studies by [1, 2, 3, 4, 5, 6].

In the above mentioned investigations, the effects of thermal radiation is not taken into account. However, free and forced convection flow with thermal radiation finds numerous applications in engineering and technology viz. in glass manufacturing, furnace design, high temperature aerodynamics, thermo-nuclear fusion, casting and levitation, cosmical flight propulsion system, plasma physics and spacecraft reentry. Keeping this fact in view, investigated laminar free convection along a vertical isothermal plate with thermal radiation using Rosseland diffusion approximation[7]. Studied radiation effects on mixed convection boundary layer flow along a vertical plate with uniform surface temperature using Rosseland flux model[8]. Studied solar radiation assisted natural convection in a uniform porous medium supported by a vertical flat plate[9]. Investigated radiation effect on flow past an impulsively started vertical plate with variable temperature[10]. Considered the effects of radiation on transient free convection flow past an impulsively moving hot vertical infinite plate in a porous medium[11]. Investigated the effects of radiation on MHD free convection flow of a gas past a semi-infinite vertical plate[12]. Considered thermal radiation and buoyancy effects on hydromagnetic flow over an accelerating permeable surface with heat source or sink[13]. Considered radiation effects on MHD mixed free and forced convection flow past a semi-infinite moving vertical plate for high temperature differences[14]. Reported effect of the radiation on hydromagnetic convection flow of a gas past a semi-infinite vertical plate for high temperature differences taking into account variable thermo-physical properties[15]. Studied the influence of viscous dissipation and radiation on unsteady MHD free convection flow past an infinite heated vertical plate in a porous medium taking into account the variable suction[16]. Considered radiation effect on the convective flow of a viscous fluid under transversely applied uniform magnetic field[17]. Analyzed thermal radiation effects on unsteady MHD free convection flow of an electrically conducing fluid past an infinite vertical porous plate considering the effects of viscous dissipation[18]. Considered the effects of thermal radiation on an unsteady MHD free convection flow past a vertical plate with constant heat flux[19]. Reported magnetohydrodynamic radiative convection flow of viscous fluid past a vertical plate embedded in a non-uniform porous medium considering variable suction and temperature gradient dependent heat source/sink[20]. Investigated the influence of radiation on transient hydromagnetic natural convection flow past an impulsively moving vertical flat plate with ramped wall temperature and continuous temperature for a isothermal wall[21]. In above mentioned studies mass transfer effect in the presence of radiation and magnetic field is not taken into account.

Discussed free convection and mass transfer effects on hydromagnetic oscillatory flow past an infinite vertical porous plate with suction/injection under usual Boussinesq approximation[22]. Examined hydromagnetic flow of a viscous fluid with convective heat and mass transfer past a vertical porous plate subjected to oscillatory suction velocity[23]. Considered the effect of mass transfer and free convection heat transfer in a viscous conducting fluid flow past a vertical porous plate embedded in a porous medium in the presence of heat source subjected to time dependent suction velocity[24]. Analyzed the effect of uniform transverse magnetic field on steady free and forced convective flow with mass transfer past an infinite vertical porous plate[25]. Reported numerical solution of transient free convection and mass transfer in the flow of a viscous incompressible fluid under the influence of uniform magnetic field[26]. Investigated convective flow considering different sets of thermal and diffusive conditions[27-30].

In the studies on mass transfer effects mentioned above, the radiative heat transfer is not taken into account, whereas its consideration is essential particularly at high temperature differences. In view of this fact, the aim of the present investigation is to consider unsteady hydromagetic natural convection and mass transfer flow of a viscous, incompressible, electrically conducting fluid with radiative heat transfer past an impulsively moving vertical flat plate embedded in a porous medium, under usual Boussinesq approximation with concentration in the presence of heat source/sink and is extension of [21] for mass transfer. MHD natural convection flow with radiative heat and mass transfer resulting from such a plate in the presence of heat source/sink is likely to be of relevance in several engineering applications, especially where the initial temperature and concentration profiles assume importance in designing of electromagnetic devices.

II. Formulation of the problem :

We consider flow of a viscous, incompressible, electrically conducting fluid past an infinite vertical plate embedded in a uniform porous medium. In two-dimensional rectangular cartesian coordinate system (x', y'), let x'-axis be chosen along the plate in the upward direction and y'-axis normal to plane of the plate in the fluid. The fluid flows under the influence of uniform transverse magnetic field B_0 applied parallel to y'-axis. The physical configuration and coordinate system is shown in Fig.1. Initially, at time $t' \le 0$, both the fluid and plate are at rest and at uniform temperature T'_∞ and concentration C'_∞

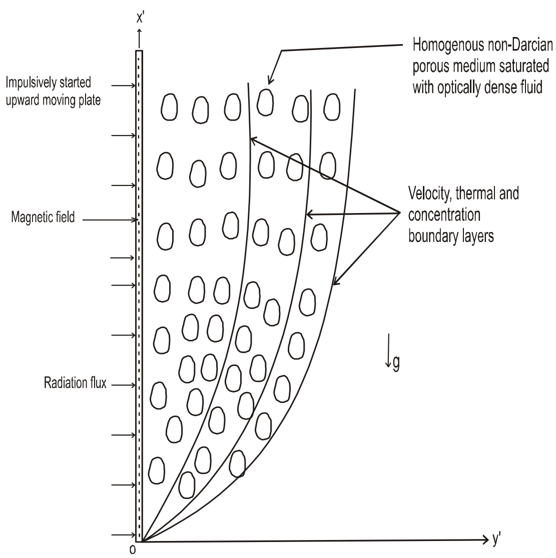

Fig. 1: Physical model and coordinate system.

At time $t' > 0$, the plate suddenly starts moving along x'-direction with uniform velocity U_0 and temperature of the plate as well as concentration at the plate raised to T'_W and C'_W respectively and maintained as such. Since the plate is of infinite extent along -direction and electrically non-conducting, all physical quantities except pressure, are functions of y' and t' only. Besides, the analysis is based on the following assumptions:

1. The induced magnetic field produced by the fluid motion is negligible in comparison to the applied magnetic field so that we consider magnetic field $\vec{B} = (0, B_0, 0)$. This assumption is justified because magnetic Reynolds number is very small for metallic liquids and partially ionized fluids[17].
2. [18]No external electric field is applied so that effect of polarization of fluid is negligible, so we assume $\vec{E} = (0,0,0)$.
3. All fluid properties, except the density in the buoyancy force term, are constant.
4. The viscous dissipation and Ohmic dissipation in the energy equation are negligible.
5. The concentration of the diffusing species in the binary mixture is considered to be very small in comparison with the other chemical species so that the Soret and Dufour effects are negligible.

Taking into account the assumptions mentioned above, the equations governing the flow, under Boussinesq approximation are:

Continuity Equation:

$$\frac{\partial v'}{\partial y'} = 0 \,. \tag{1}$$

Momentum Equation:

$$\frac{\partial u'}{\partial t'} = \upsilon \frac{\partial^2 u'}{\partial y'^2} + g\beta'(T' - T'_\infty)$$

$$+g\beta^{*}(C'-C'_{\infty})-\frac{\upsilon}{K'}u'-\frac{\sigma B_0^2}{\rho}u'. \qquad (2)$$

Energy Equation:

$$\frac{\partial T'}{\partial t'}=\frac{K_T}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2}-\frac{1}{\rho C_p}\frac{\partial q'_r}{\partial y'}-\frac{Q'}{\rho C_p}(T'-T'_{\infty}). \qquad (3)$$

Diffusion Equation:

$$\frac{\partial C'}{\partial t'}=D_T\frac{\partial^2 C'}{\partial y'^2}. \qquad (4)$$

The initial and boundary conditions relevant to the problem are:

$$t'\le 0:\ u'=0,\ T'=T'_{\infty},\ C'=C'_{\infty}\ \text{ for all }\ y'.$$
$$t'>0:\ u'=U_0,\ T'=T'_W,\ C'=C'_W\ \text{ at }\ y'=0.$$
$$u'\to 0,\ T'\to T'_{\infty}\ \ C'\to C'_{\infty}\ \text{ as }\ y'\to\infty. \qquad (5)$$

Pointed out that for an optically thick fluid, in addition to emission, there is also self-absorption[14]. Usually, the absorption coefficient is dependent on wave length and is very large[31]. Therefore, we can adopt Rosseland approximation for radiative flux vector q'_r. The radiative heat flux vector q'_r, under Rosseland approximation, is simplified[32] as follows:

$$q'_r=-\frac{4\sigma^{*}}{3k^{*}}\frac{\partial T'^4}{\partial y'}, \qquad (6)$$

where k^* is the mean absorption coefficient and σ^* is the Stefan Boltzmann constant. For small temperature difference between fluid temperature T' and free stream temperature T'_{∞}, the temperature T' can be expanded in Taylor series about the free stream temperature T'_{∞}. We now expand T'^4 in Taylor series about T'_{∞} as follows:

$$T'^4=T'^4_{\infty}+4T'^3_{\infty}(T'-T'_{\infty})+6T'^2_{\infty}(T'-T'_{\infty})^2+\ldots\ldots\ldots\ldots.$$

Neglecting higher order terms in the above series beyond the first degree in $(T'-T'_{\infty})$, we get:

$$T'^4\cong 3T'^3_{\infty}+4T'^3_{\infty}T'. \qquad (7)$$

Introducing (6) and (7) in equation (3), we obtain:

$$\frac{\partial T'}{\partial t'}=\frac{K_T}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2}+\frac{16\sigma^{*}T'^3_{\infty}}{\rho C_p 3k^{*}}\frac{\partial^2 T'}{\partial y'^2}-\frac{Q'}{\rho C_p}(T'-T'_{\infty}). \qquad (8)$$

We introduce the following non-dimensional quantities:

$$y=\frac{v_0 y'}{\upsilon},\quad t=\frac{v_0^2 t'}{\upsilon},\quad u=\frac{u'}{U_0},\quad T=\frac{T'-T'_{\infty}}{T'_W-T'_{\infty}},$$

$$C=\frac{C'-C'_{\infty}}{C'_W-C'_{\infty}}.$$

Introducing above mentioned quantities in (2), (8) and (4), we obtain:

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial y^2}+GrT+GmC-\left(M^2+K^{-1}\right)u. \qquad (9)$$

$$\frac{\partial T}{\partial t}=\left(\frac{1+N}{Pr}\right)\frac{\partial^2 T}{\partial y^2}-QT. \qquad (10)$$

$$\frac{\partial C}{\partial t}=\frac{1}{Sc}\frac{\partial^2 C}{\partial y^2}. \qquad (11)$$

where $Gr=\frac{g\beta'(T'_W-T'_{\infty})\upsilon}{U_0 v_0^2}$ (Grashof number), $Gm=\frac{g\beta^{*}(C'_W-C'_{\infty})\upsilon}{U_0 v_0^2}$ (Modified number), $M=\sqrt{\frac{\sigma\upsilon}{\rho}}\frac{B_0}{v_0}$ (Magnetic parameter), $K=\frac{K'v_0^2}{\upsilon^2}$ (Permeability parameter), $Sc=\frac{\upsilon}{D_T}$ (Schmidt number),

$Pr = \dfrac{\mu C_p}{K_T}$ (Prandtl number), $N = \dfrac{16\sigma^* T_\infty'^3}{3K_T k^*}$ (Radiation parameter) and $Q = \dfrac{Q'\upsilon}{\rho C_p v_0^2}$ (Heat source/sink parameter).

The initial and boundary conditions (4) in non-dimensional form reduce to:

$$t \le 0: \quad u = 0, \quad T = 0, \quad C = 0 \text{ for all } y.$$

$$t' > 0: \quad u = 1, \quad T = 1, \quad C = 1 \quad \text{at } y = 0.$$

$$u \to 0, \quad T \to 0 \quad C \to 0 \text{ as } y \to \infty, \quad (12)$$

III. Solution of the problem :

It is evident from equation (9)-(11) that the energy equation (10) and the diffusion equation (11) are uncoupled with the momentum equation (9). Therefore, we can obtain the solution for fluid temperature $T(y,t)$ and concentration distribution $C(y,t)$ directly by solving the equation (10) and (11). Thereafter, using $T(y,t)$ and $C(y,t)$ in the momentum equation (9), the solution for fluid velocity $u(y,t)$ can be obtained.

Applying Laplace transform technique[27,33] to equations (9)-(11) exact solutions for fluid concentration $C(y,t)$, fluid temperature $T(y,t)$ and fluid velocity $u(y,t)$ (for $Sc \neq 1$ (13)

$$T(y,t) = H_2(y, a, Q, t). \tag{14}$$

$$u(y,t) = \left(1 - \frac{Gr}{\alpha(a-1)} - \frac{Gm}{\beta(Sc-1)}\right) H_3(y, 1.0, M_1, t)$$
$$+ \frac{Gr}{\alpha(a-1)} e^{\alpha t} H_4(y, 1.0, M_1 + \alpha, t) + \frac{Gr}{\alpha(a-1)} H_2(y, a, Q, t)$$
$$- \frac{Gr}{\alpha(a-1)} H_5(y, a, Q + \alpha, t) + \frac{Gm}{\beta(Sc-1)} e^{\beta t} H_6(y, 1.0, M_1 + \beta, t)$$
$$+ \frac{Gm}{\beta(Sc-1)} H_7(y, Sc, 0.0, t) - \frac{Gm}{\beta(Sc-1)} e^{\beta t} H_8(y, Sc, \beta, t), \tag{15}$$

where $a = \dfrac{Pr}{N+1}$, $\chi = \dfrac{(N+1)\left(M + K^{-1} - aQ\right)}{Pr - N - 1}$, $\beta = \dfrac{M_1}{Sc - 1}$

and

$$F_1(G_1, G_2, G_3, G_4) = \frac{1}{2}\left[\exp\left(-G_1\sqrt{G_2 G_3}\right) erfc\left(\frac{G_1\sqrt{G_2}}{2\sqrt{G_4}} - \sqrt{G_3 G_4}\right)\right.$$
$$\left. + \exp\left(G_1\sqrt{G_2 G_3}\right) erfc\left(\frac{G_1 G_2}{2\sqrt{G_4}} + \sqrt{G_3 G_4}\right)\right].$$

IV. Skin-friction, rate of heat and mass transfer :

The skin-friction (τ), rate of heat transfer in terms of Nusselt number (Nu) and rate of mass transfer in terms of Sherwood number (Sh) at the plate y=0 for y=0 are obtained as follows:

$$Nu = -\left(\frac{\partial T}{\partial y}\right)_{y=0} = D_1(0.0, 1.0, Q, t). \tag{16}$$

$$Sh = -\left(\frac{\partial C}{\partial y}\right)_{y=0} = D_2(0.0, Sc, 0.0, t). \tag{17}$$

$$\tau = -\left(\frac{\partial u}{\partial y}\right)_{y=0} = \left[1 - \frac{Gr}{\alpha(a-1)} - \frac{Gm}{\beta(Sc-1)}\right] D_3(0.0, 1.0, M_1, t)$$
$$+ \frac{Gr}{\alpha(a-1)} e^{\alpha t} D_4(0.0, 1.0, M_1 + \alpha, t)$$
$$- \frac{\sqrt{Sc}\,Gm}{\beta(Sc-1)} e^{\beta t} D_7(0.0, 1.0, \beta, t)$$
$$- \frac{\sqrt{a}\,Gr}{\alpha(a-1)}\left[e^{\alpha t} D_5(0.0, 1.0, Q + \alpha, t) - D_1(0.0, 1.0, Q, t)\right]$$
$$+ \frac{Gm}{\beta(Sc-1)}\left[e^{\beta t} D_6(0.0, 1.0, M_1 + \beta, t) + D_2(0.0, Sc, 0.0, t)\right], \tag{18}$$

where

$$F_2(G_1, G_2, G_3, G_4) = \sqrt{G_2 G_3}\exp\left(G_1\sqrt{G_3 G_2}\right) erfc\left(\frac{G_1\sqrt{G_2}}{2\sqrt{G_4}} + \sqrt{G_3 G_4}\right)$$
$$- \sqrt{G_3 G_2}\exp\left(-G_1\sqrt{G_3 G_2}\right) erfc\left(\frac{G_1\sqrt{G_2}}{2\sqrt{G_4}} - \sqrt{G_3 G_4}\right)$$

$$-2\sqrt{\frac{G_2}{\pi G_4}}\exp\left(-\frac{G_2 G_1^2}{4G_4}-G_3 G_4\right).$$

V. Particular case for Sc=1 :

When Sc=1, the concentration field $C(y,t)$, temperature distribution $T(y,t)$ and velocity $u(y,t)$ are obtained as follows:

$$C(y,t)=J_1(y,1.0,0.0,t)\,. \tag{19}$$

$$T(y,t)=J_2(y,\,a,\,Q,\,t)\,. \tag{20}$$

$$u(y,t)=\left(1-\frac{Gr}{\alpha(a-1)}-\frac{Gm}{M_1}\right)J_3(y,\,1.0,\,M_1,\,t)$$

$$+\frac{Gr}{\alpha(a-1)}\Big[e^{\alpha t}\{J_4(y,1.0,M_1+\alpha,\,t)-J_5(y,a,Q+\alpha,\,t)\}$$

$$+J_2(y,a,Q,\,t)\Big]+\frac{Gm}{M_1}J_1(y,1.0,0.0,\,t)\,. \tag{21}$$

The rate of heat transfer (Nu), the rate of mass transfer (Sh) and the skin-friction (τ) at the plate, y=0, for Sc=1 are as follows:

$$Nu=I_1(0.0,1.0,Q,t)\,. \tag{22}$$

$$Sh=I_2(0.0,\,1.0,\,0.0,\,t)\,. \tag{23}$$

$$\tau=I_3(0.0,\,1.0,\,M_1,\,t)$$

$$+\frac{Gr}{\alpha(a-1)}\Big[e^{\alpha t}I_4(0.0,1.0,M_1+\alpha,\,t)-I_3(0.0,1.0,M_1,\,t)\Big]$$

$$-\frac{Gm}{M_1}I_3(0.0,1.0,M_1,\,t)-\frac{\sqrt{a}Gr}{\alpha(a-1)}\Big[e^{\alpha t}I_5(0.0,1.0,Q+\alpha,\,t)$$

$$-I_1(0.0,1.0,Q,\,t)\Big]+I_2(0.0,1.0,0.0,\,t)\,. \tag{24}$$

VI. Results and Discussion

An analysis is performed to study the effects of magnetic field, permeability of the medium, thermal buoyancy force, solutal buoyancy force, radiation flux, heat source/ sink and time on flow field in the boundary layer region past an impulsively moving vertical plate. The non-dimensional equations of momentum, energy and diffusion are expressed in (9)-(11) and solutions are shown in (13)-(15) under the boundary conditions (12). The numerical values of fluid velocity versus y, computed from the analytical solution, are displayed graphically for various values of the magnetic parameter (M), Grashof number (Gr), radiation parameter (N), permeability parameter (K), heat source/sink (Q) and time (t). In the presence of modified Grashof number (Gm), the numerical values of temperature versus boundary layer coordinate y are displayed graphically for various values of Prandtl number (Pr), radiatio parameter (N), heat source/sink parameter (Q) and time (t); whereas concentration versus y is displayed for different Schmidt number (Sc) and time (t). To be realistic, the values of Prandtl number (Pr) are chosen to be Pr=0.71, Pr=1.0 and Pr=7.0, which correspond to air, electrolyte solutions and water respectively; whereas the values of Schmidt number (Sc) are chosen to be Sc=0.22, Sc=0.30, Sc=0.60, Sc=0.66, Sc=0.78 and Sc=0.94, which correspond to hydrogen, helium, water-vapour, oxygen, ammonia and carbon-dioxide respectively, the species of most common interest present in air. The values of heat source (Q<0) and heat sink (Q>0) are chosen arbitrarily, but the numerical values of the remaining parameters are chosen following[21].

Figures 2-7 show effects of different parameters on fluid velocity (u) versus non-dimensional y. It is observed that the fluid velocity attains distinctive maximum value in the vicinity of the plate surface and then decreases with increasing boundary layer coordinate y to approach the free stream value.

Besides, it is noted that when M=0, Q=0 and Gm=0, the results of present study are the same as obtained by[21].

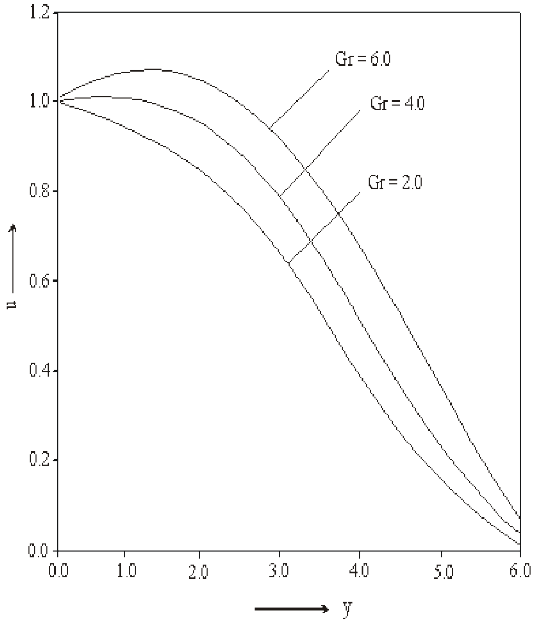

Fig.2: Effect of Grashof number on velocity field when M = 0.5, Gm = 2.0, N = 1.0, Q = 2.0, K = 0.5 and t = 0.5

Figure-2 demonstrates the influence of Grashof number (Gr) on fluid velocity (u) against boundary layer coordinate y, when M=0.5, Gm=2.0, N=1.0, Q=2.0, K=0.5 and t=0.5. It is observed from the figure that an increase in Grashof number (Gr) leads to an increase in fluid velocity (u) in the boundary layer region. This is due to the fact that Grashof number (Gr) signifies the relative effect of thermal buoyancy force to viscous hydrodynamic force in the boundary layer region, this implies that increase in thermal buoyancy force increases the Grashof number (Gr), so that significant increase in velocity is noted.

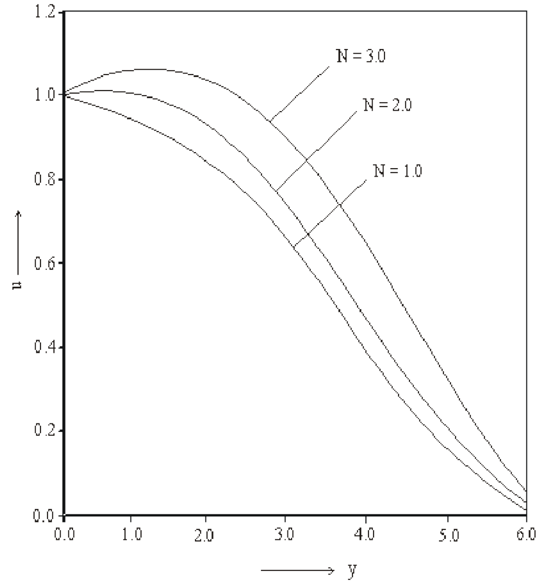

Fig.3: Effect of radiation parameter on velocity field when M = 0.5, Gm = 2.0, Gr = 2.0, Q = 2.0, K = 0.5 and t = 0.5

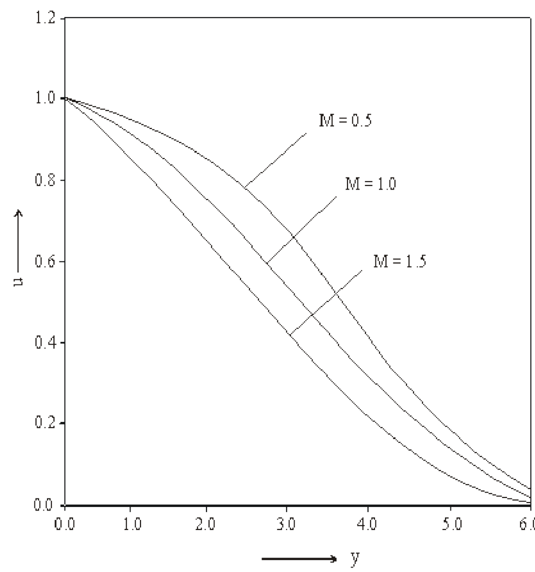

Fig. 4: Effect of magnetic parameter on fluid velocity when Gm = 2.0, Gr = 2.0,N = 1.0, Q = 2.0, K = 0.5 and t = 0.5

Figure-3 displays the influence of radiation parameter (N) on fluid velocity against boundary layer coordinate y, when M=0.5, Gr=2.0, Gm=2.0, Q=2.0, K=0.5 and t=0.5. It is noted that the fluid velocity increases on increasing radiation parameter (N) in the boundary layer region. This implies that radiation has an accelerating influence on the fluid flow. This is due to the fact that the radiation parameter is chosen in such a way that non-dimensional energy equation (11) embodies the radiation parameter (N) in numerator. Thus, the presence of the term of temperature in momentum equation has positive effect on velocity.

Figure-4 is intended to illustrate fluid velocity (u) versus boundary layer coordinate y for different values of the magnetic parameter (M), when Gr=2.0, Gm=2.0, N=1.0, Q=2.0, K=0.5 and t=0.5. The figure reveals that the fluid velocity (u) decreases on increasing magnetic parameter (M) in the boundary layer region. This implies that the magnetic field decelerates fluid velocity. This is due to the fact that the application of the transverse magnetic field to the electrically conducting fluid generates resistive force, which is known as Lorentz force. This force acts as resistive force and has tendency to decelerate fluid flow in the boundary layer region. Due to this property, the magnetic field is applied to control velocity in naval energy systems.

Figure-5 shows the effects of permeability parameter (K) on fluid velocity (u) with respect to boundary layer coordinate y, when M=0, Gr=2.0, Gm=2.0, Q=2.0, N=1.0 and t=0.5. It is found that the fluid velocity (u) increases with increase in the permeability parameter (K) in the boundary layer region. The physics behind this phenomenon is that an increase in permeability parameter (K) implies that there is a decrease in the resistance of porous medium, which tends to enhance the fluid flow.

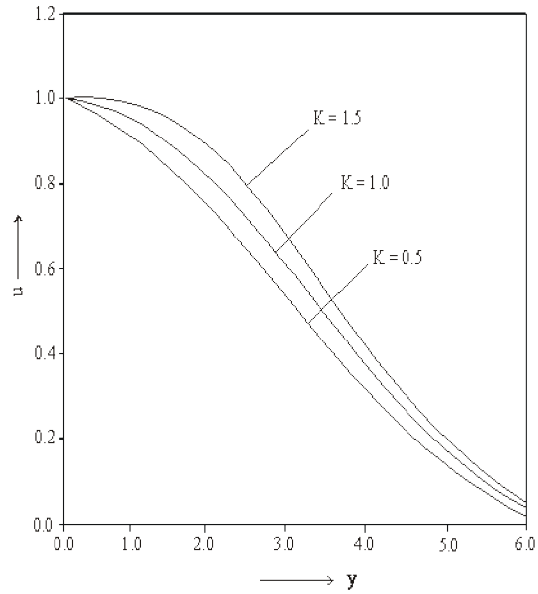

Fig.5: Effect of permeability on velocity field when M = 0.5, Gm = 2.0, Gr = 2.0, Q = 2.0, N = 1.0 and t = 0.5

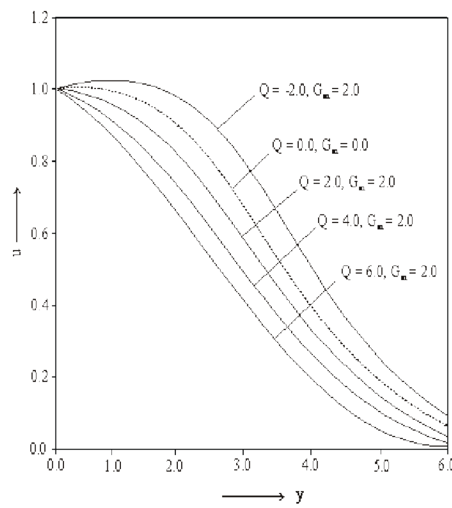

Fig.6:Effect of heat source/sink on fluid velocity when M = 0.5, Gr = 2.0, K = 0.5, N = 1.0 and t = 0.5

Figure-6 is intended to illustrate the effects of heat source/sink on fluid velocity (u) against y, when M=0, Gr=2.0, Gm=2.0, N=1.0, K=0.5 and t=0.5 It is observed that an increase in numerical values of $-Q$ increases the fluid velocity, but an increase in numerical values of $+Q$ decreases the fluid velocity. This implies that heat source enhances the fluid velocity whereas appositive effect is noted for heat sink. In fact, the presence of heat source increases the thermal boundary layer, which increases the fluid velocity but heat sink reduces the thermal boundary layer, which decreases the fluid velocity. Besides, it is noticed that when Gm=0 and Q=0, the results of the present study for velocity distribution are in agreement with the existing results obtained by[21].

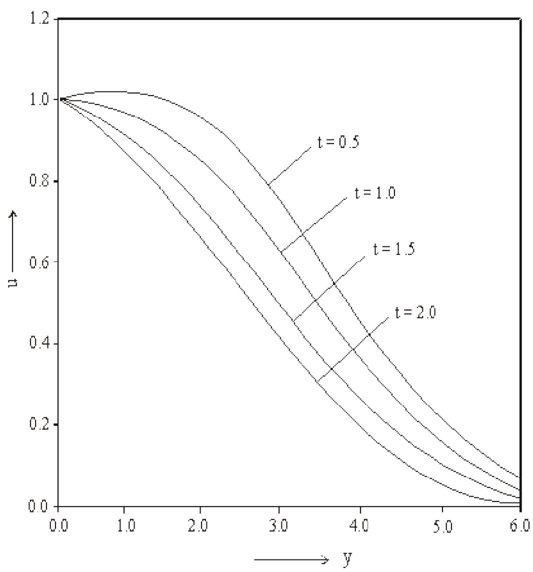

Fig.7: Effect of time on fluid velocity when M = 0.5, Gr = 2.0, Gm = 2.0, K = 0.5, N = 1.0 and Q = 2.0

Figure-7 represents the effects of time on fluid velocity (u) versus y, when M=0, Gr=2.0, Gm=2.0, N=1.0, Q=2.0 and K=0.5. It is noticed that the fluid velocity (u) decreases in the boundary layer region with increase in time (t). This implies that there is retardation in fluid velocity (u) as time progresses. In fact, as time progresses the fluid velocity tends to uniform velocity.

The numerical values of fluid temperature, computed from the analytical solution (14) are presented graphically in Figures-8, 9 and 10 for various values of Prandtl number (Pr), radiation parameter (N), heat source/sink parameter (Q) and time (t). It is noticed from Figures-8, 9 and 10 that the fluid temperature is maximum at the surface of the plate and it decreases on increasing y to approach free stream temperature. Besides, it is observed that when Q=0, the results for temperature field of the present study are exactly the same to those obtained by[21].

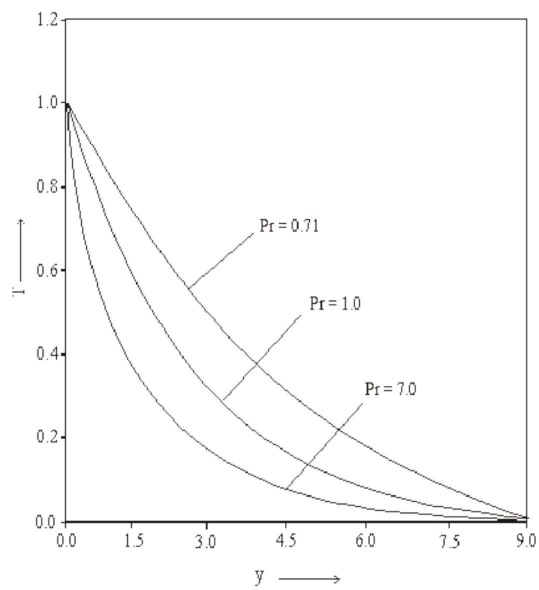

Fig.8: Effect of Prandtl number on fluid temperature when N = 1.0, Q = 2.0 and t = 0.5

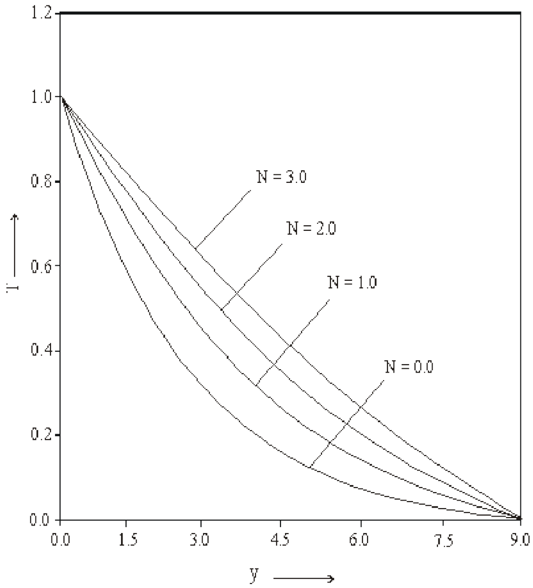

Fig. 9: Effect of radiation parameter on fluid temperature when Pr = 0.7, Q = 2.0 and t = 0.5.

Figure-8 displays the influence of Prandtl number (*Pr*) on fluid temperature (*T*) versus non-diemsional *y*, when *N*=1.0, *Q*=2.0 and *t*=0.5. It is observed that in the boundary layer region, the fluid temperature (*T*) decreases with increase in Prandtl number (*Pr*). The Prandtl number mathematically defines the ratio of the momentum diffusivity to the thermal diffusivity. As such, lower *Pr*-value fluids transfer heat more effectively than higher *Pr*-value fluids; consequently lower temperatures are observed for higher *Pr*-value fluids. Besides, Prandtl number signifies the relative effects of viscosity to thermal conductivity. This implies that thermal diffusion tends to increase fluid temperature. Hence, increase in Prandtl number decreases the temperature.

Figure-9 is intended to demonstrate the effects of radiation parameter (*N*) on fluid temperature (*T*) with respect to *y*, when *Pr*=0.71, *Q*=2.0 and *t*=0.5. It is noted that the fluid temperature (*T*) increases with increase in the radiation parameter (*N*) in the boundary layer region. In fact, in the energy equation radiation parameter exists in numerator, which implies that increase in radiation parameter results in an increase in temperature. Hence, increase in radiation tends to enhance fluid temperature.

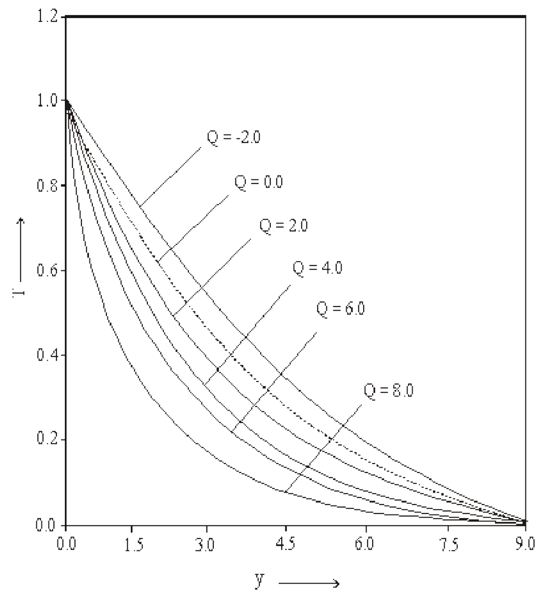

Fig. 10: Effect of heat source/sink on fluid temperature when Pr = 0.7, N = 1.0 and t = 0.5.

Figure-10 represents the effects of heat source/sink parameter (*Q*) on temperature distribution (*T*) against non-dimensional *y*, when *Pr*=0.71, *N*=1.0 and *t*=0.5. It is found that in the boundary layer region, the fluid temperature (*T*) increases with increase in the heat source (*Q*<0), whereas decreases with increase in the heat sink (*Q*>0). This implies that the heat source is favourable to temperature and boosts it but heat sink shown apposite effect. Physically, heat source increases the thermal boundary layer, which increases the temperature and vice-versa for heat sink. Besides, it is observed that when , the results of the present study for temperature field are exactly similar to those obtained by[21].

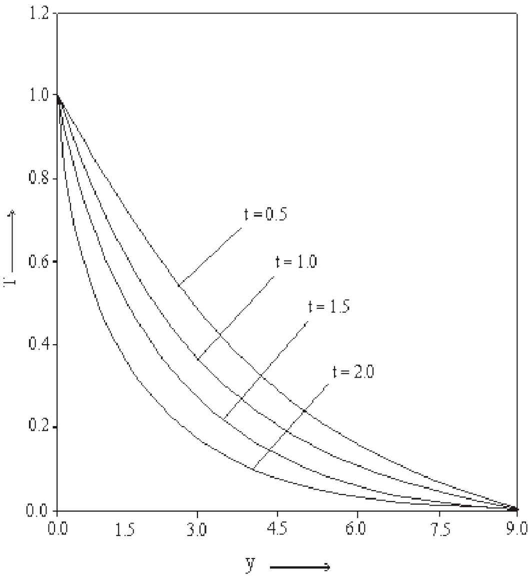

Fig.11: Effect of time on fluid temperature when Pr = 0.7, N = 1.0 and Q = 2.0.

Figure-11 illustrates the effects of time (t) on fluid temperature (T) against non-dimensional y, when Pr=0.71, N=1.0 and Q=2.0. It is noted that in the boundary layer region, the fluid temperature (T) decreases with increase in time (t), which implies that there is decrement in the fluid temperature as time progresses. The physics behind this phenomenon lies in the fact that as time progresses the fluid temperature tends to the equilibrium temperature.

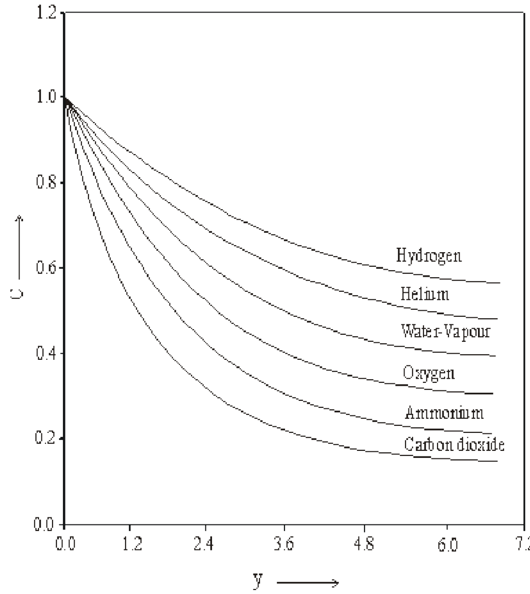

Fig.12: Effect of Schmidt number (Sc) on species concentration (C).

Figure-12 is intended to display the effects of Schmidt number (Sc) on species concentration (C) with respect to non-dimensional y, choosing Sc=0.22 (hydrogen), Sc=30 (helium), Sc=0.60 (water-vapour), Sc=0.66 (oxygen), Sc=0.78 (ammonia) and Sc=0.94 (carbon dioxide). It is observed that as Schmidt number (Sc), increases the species concentration at the plate decreases, which decreases smoothly with the non-dimensional y. Besides, it is noted that the curves with low Schmidt number values fall more rapidly in comparison with-high Schmidt number values. In fact, concentration field with higher Schmidt number is less stable in comparison with the lower Schmidt number gases.

Table-1 represents the numerical values of non-dimensional skin-friction (τ) computed from the analytical expression (18)

Table-1. Skin-friction (τ) at the plate for different *M, Q, Gr, N* and time
(*Pr*=0.71, *Gm*=2.0, *K*=0.2)

M	*Q*	*Gr*	*N*	time (*t*)				
				0.2	0.4	0.6	0.8	1.0
2.0	2.0	2.0	1.0	1.8436	1.7253	1.5967	1.4689	1.3796
4.0	2.0	2.0	1.0	2.3782	2.2468	2.1359	2.0198	1.9836
2.0	-2.0	2.0	1.0	1.9834	1.8275	1.7012	1.5819	1.4237
2.0	2.0	4.0	1.0	0.9968	0.8254	0.6937	0.5213	0.3854
2.0	2.0	2.0	2.0	1.2563	1.1193	0.9874	0.7951	0.6296
2.0	4.0	2.0	1.0	1.7384	1.6285	1.5264	1.4198	1.2398

Table-2**.** Rate of heat transfer (*Nu*) at the plate for different
Pr, N, Q and time

Pr	*N*	*Q*	time (*t*)				
			0.2	0.4	0.6	0.8	1.0
0.71	1.0	2.0	0.8493	0.7689	0.6437	0.5308	0.4467
1.00	1.0	2.0	0.9756	0.8395	0.7205	0.5938	0.4899
0.71	2.0	2.0	0.7098	0.6627	0.5839	0.4821	0.3906
0.71	1.0	4.0	0.7685	0.6984	0.6194	0.5037	0.4218
0.71	1.0	“2.0	0.9468	0.8175	0.7269	0.6859	0.5437

for different values of *M*, *K*, *Gr*, *N* and *t* choosing *Pr*=0.71, *Gm*=2.0 and *Q*=2.0. It is evident from the table, that the skin-friction (τ) increases on increasing magnetic parameter (*M*) or heat source parameter ($Q<0$), whereas it decreases with increase in Grashof number (*Gr*), heat sink parameter ($Q>0$), radiation parameter (*N*) or time (*t*). These results imply that magnetic field and heat source have tendency to enhance the skin-friction whereas thermal buoyancy force, radiation and heat sink have reverse effect. Besides, as time progresses the skin-friction decreases.

Table-2 depicts, in tabular form, the numerical values of non-dimensional rate of heat transfer (*Nu*) computed from the analytical expression (16). It is observed that the Nusselt number (*Nu*) decreases with increase in radiation parameter (*N*) or heat sink, whereas it increases with increase in Prandtl number (*Pr*) or heat source. Also, it is noticed that as time progresses the rate of heat transfer at the plate decreases, which implies that time tends to reduce the rate of heat transfer.

Table-3**.** Rate of mass transfer (*Sc*) at the isothermal plate for different *Sc* and time

Sc \ *t*	0.2	0.4	0.6	0.8	1.0
0.22	0.9873	1.0738	1.2654	1.3719	1.4535
0.30	0.7644	0.8957	1.5386	1.2564	1.3587
0.60	0.6819	0.7936	0.9169	1.1386	1.2678
0.66	0.6257	0.7195	0.8034	0.9984	1.0086
0.78	0.5736	0.6634	0.7116	0.8235	0.9979
0.94	0.4293	0.5768	0.6384	0.7096	0.8435

Table-3 presents the numerical values of the non-dimensional rate of mass transfer (*Sh*) for different values of Schmidt number (*Sc*) and time (*t*). It is observed that rate of mass transfer in terms of Sherwood number (*Sh*) decreases with increase in Schmidt number, which implies that the gases with high valued Schmidt number transfer mass less effectively compared to the gases with low Schmidt number.

VII. Conclusions

The study presents a theoretical investigation of unsteady hydromagnetic natural convection and mass transfer flow of a viscous, incompressible, electrically conducting fluid with radiative heat transfer near an impulsively moving vertical isothermal flat plate embedded in a porous medium in the presence of heat source/sink. The significant findings are summarized below:

1. The magnetic field, Prandtl number and heat sink decelerates fluid flow, whereas thermal buoyancy force, radiation, heat source and permeability of the medium have reverse effect on it. Besides, the fluid velocity attains distinctive maximum value in the vicinity of the plate and then decreases with increase in the boundary layer coordinate. Also, the velocity decreases as time progresses.
2. The thermal buoyancy force, radiation and heat source enhance fluid temperature, whereas Prandtl number decelerates it. As time progresses, there is deceleration in temperature.
3. The species concentration is highest in case of hydrogen and lowest for the case of carbon dioxide gas.
4. An increase in the magnetic field or heat source enhances skin-friction, whereas buoyancy force, Prandtl number, radiation and heat sink have reverse effect on it. Also, the skin-friction decelerates as time progress.
5. An increase in Prandtl number or heat sink decreases the rate of heat transfer, whereas the radiation parameter and heat source increase rate of heat transfer.
6. As Schmidt number increase, the rate of mass transfer decreases.

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