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On *g-Closed Sets in Bitopological Spaces

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Abstract

In this paper, we introduce *g-closed sets[13] in bitopological spaces. Properties of these sets are investigated and we introduce the two new bitopoligical spaces (i, j)-$*gT*_{1/2}$ and (i, j)-$*g*T_{1/2}$ spaces as applications. Further we introduce and study *g-continuity[13] in bitopological spaces.

Key words : (i, j)-*g closed sets; (i, j)-$*gT*_{1/2}$ spaces; (i, j)-$*g*T_{1/2}$ spaces and *D(i, j)-continuity.

1. Introduction

A triple (X, τ_1, τ_2) where X is non empty set and τ_1, τ_2 are two topologies on X is called a bitopological space. Kelly[3] initiated and study such spaces in 1985. Fukutaka[6] introduced the concept of g-closed sets[2] in bitopological spaces and after that many authors turned their attention towards generalizations of various concepts of topology by considering bitopological spaces. Recently Veera Kumar introduced and studied the concepts of *g-closed sets and *g-continuity in topological spaces. *g-closed sets lies between closed sets and g-closed sets.

In this paper we introduce the concepts of *g-closed sets[13] and *g-continuity[13] for bitopological spaces and then investigate two new bitopological spaces (i, j)-$^*gT^*_{1/2}$ space and (i, j)-$^*g^*T_{1/2}$ space.

2. Preliminaries :

If A is a subset of X with a topology τ, then the closure of A is denoted by τ-cl(A) or cl(A), the interior of A is denoted by τ-int(A) or int(A) and the complement of A is denoted by A^C.

Definition 2.01 : (i) A subset A of a topological space (X, τ) is called semi-open[1]. (resp. regular open[4], pre-open[2]) if $A \subseteq cl(int(A))$ (resp. $A = int(Cl(A))$, $A \subseteq int(cl(A))$).

(ii) A subset A of a topological space

(X, τ) is called a generalized closed set[2] (briefly g-closed set) (resp. $\hat{g}$-closed set[14], *g-closed set) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open (resp. semi-open, $\hat{g}$-open) in X.

Definition 2.02 : The intersection of all pre-closed sets containing A is called the pre-closure of A and it is denoted by τ-pcl(A) or pcl(A).

Throughout this paper, X and Y represent non-empty bitopological spaces (X, τ_1, τ_2) and (Y, σ_1, σ_2) on which no separation axioms are assumed unless otherwise explicitly mentioned and integers $i, j, k \in \{1, 2\}$. For a subset A of X, τ_i-cl(A) (resp. τ_i-int(A), τ_i-pcl(A)) denote the closure (resp. interior, preclosure) of A w.r.t. topology τ_i. We denote the family of all $\hat{g}$-open subsets of X w.r.t. topology τ_i by $\hat{G}O(X, \tau_i)$. and the family of all τ_j-closed sets is denoted by F_j. By (i, j) we mean the pair of topologies (τ_i, τ_j).

Definition 2.03 : A subset A of a bitopological space (X, τ_1, τ_2) is called :

(i) (i, j)-g-closed[6] if τ_j-cl(A) $\subseteq U$ whenever $A \subseteq U$ and $U \in \tau_i$.

(ii) (i, j)-rg-closed[8] if τ_j-cl(A) $\subseteq U$ whenever $A \subseteq U$ and U is regular open in τ_i.

(iii) (i, j)-gpr-closed[12] if τ_j-cl(A) $\subseteq U$ whenever $A \subseteq U$ and U is regular open in τ_i.

(iv) (i, j)-Wg-closed[10] if τ_j-cl(τ_i-int(A)) $\subseteq U$ whenever $A \subseteq U$ and $U \in \tau_i$.

(v) (i, j)-ω-closed[12] if τ_j-cl(A) $\subseteq U$ whenever $A \subseteq U$ and U is semi-open in τ_i.

(vi) (i, j)-g*-closed[15] if τ_j-cl(A) $\subseteq U$ whenever $A \subseteq U$ and U is generalized-open in τ_i.

The family of all (i, j)-g-closed (resp. (i, j)-rg-closed, (i, j)-gpr-closed, (i, j)-Wg-closed, (i, j)-ω-closed and (i, j)-g*-closed) subsets of a bitopological space (X, τ_1, τ_2) is denoted by D(i, j) (resp. Dr(i, j), ζ(i, j), W(i, j), C(i, j) and D*(i, j)).

Definition 2.04 : (i) A bitopological space (X, τ_1, τ_2) is said to be (i, j)-$T_{1/2}$[6] (resp. (i, j)-$T^*_{1/2}$[15], (i, j)-$^*T_{1/2}$[15]) if every (i, j)-g-closed (resp. (i, j)-g*-closed, (i, j)-g-closed) sets is τ_j-closed (resp. τ_j-closed, (i, j)-g*-closed).

(ii) A bitopological space (X, τ_1, τ_2) is said to be strongly pairwise-$T_{1/2}$[6] (resp. strongly pairwise $T^*_{1/2}$[15]) space if it is (1, 2)-$T_{1/2}$ and (2, 1)-$T_{1/2}$ (resp. (1, 2)-$T^*_{1/2}$ and (2, 1)-$T^*_{1/2}$) space.

Definition 2.05 : A map $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called :

(i) τ_j-σ_k-continuous[7] if $f^{-1}(V) \in \tau_j$ for every $V \in \sigma_k$.

(ii) D (i, j)-σ_k-continuous[7] (resp. Dr(i, j)-σ_k-continuous[8], ζ(i, j)-σ_k-continuous[12], W(i, j)-σ_k-continuous[10], C(i, j)-σ_k-continuous[12] and D*(i, j)-σ_k-continuous[15]) if the inverse image of every σ_k-closed set is (i, j)-g-closed (resp. (i, j)-rg-closed, (i, j)-gpr-closed, (i, j)-Wg-closed, (i, j)-ω-closed and (i, j)-g*-closed) set in (X, τ_1, τ_2).

*3. (i, j)-*g-Closed sets :*

In this section we introduce the

concepts of (i, j)-*g-closed sets in bitopological spaces.

Definition 3.01 : A subset A of a bitopological space (X, τ_1, τ_2) is said to be an (i, j)-*g-closed set if $\tau_j\text{-cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U \in \hat{G}\, O(X, \tau_i)$.

We denote the family of all (i, j)-*g-closed sets in (X, τ_1, τ_2) by *D(i, j).

Remark 3.02 : By setting $\tau_1 = \tau_2$ in definition (3.01), an (i, j)-*g-closed set is a *g-closed set.

Proposition 3.03 : If A is τ_j-closed subset of (X, τ_1, τ_2), then A is (i, j)-*g-closed[5].

The converse of the above proposition is not necessarily true as seen from following example.

Example 3.04 : Let X = {a, b, c}, τ_1 = {ϕ, {a}, X} and τ_2 = {ϕ, {b}, {a, c}, X} then the subset {b, c} is (1, 2)-*g-closed but not τ_2-closed in (X, τ_1, τ_2).

Proposition 3.05 : In a bitopological space every (i, j)-*g-closed set is (i) (i, j)-g-closed (ii) (i, j)-rg-closed (iii) (i, j)-gpr-closed (iv) (i, j)-Wg-closed.

The following examples show that the reverse implications of the above proposition are not true.

Example 3.06 : Let X = {a, b, c}, τ_1 = {ϕ, {a}, {b}, {a, b}, X} and τ_2 = {ϕ, {a}, X} then the subset {a, c} is (1, 2)-rg-closed but not (1, 2)-*g-closed.

Example 3.07 : Let X= {a, b, c}, τ_1= {ϕ, {a}, {c}, {a, c}, X} and τ_2 = {ϕ, {a}, X} then the subset {a, c} is (1, 2)-rg-closed but not (1, 2)-*g-closed.

Example 3.08 : Let X = {a, b, c}, τ_1 = {ϕ, {b}, {c}, {b, c}, {a, c}, X} and τ_2 = {ϕ, {a, b}, X} then the subset {a} is (1, 2)-gpr-closed but not (1, 2)-*g-closed.

Example 3.09 : In example (3.08), the subset {a} is (1, 2)-Wg-closed but not (1, 2)-*g-closed.

Remark 3.10 : The following examples show that (i, j)-*g-closed sets and (i, j)-ω-closed sets are independent.

Example 3.11 : Let X = {a, b, c}, τ_1 = {ϕ, {a}, {b, c}, X} and τ_2 = {ϕ, {a}, {a, b}, X} then the subset {b} is (1, 2)-ω-closed but not (1, 2)-*g-closed[9].

Example 3.12 : In example (3.07), the subset {b} is (1, 2)-*g-closed but not (1, 2)-ω-closed.

Remark 3.13: The following examples show that (i, j)-*g-closed sets and τ_j-g-closed sets are independent.

Example 3.14 : Let X = {a, b, c}, τ_1 = {ϕ, {a}, X} and τ_2 = {ϕ, {a}, {c}, {a, c}, X} then the subset {b, c} is (1, 2)-*g-closed but not τ_2-g-closed[11].

Example 3.15 : In example (3.07),

the subset {a, c} is τ_2-g-closed but not (1, 2)-*g-closed.

Proposition 3.16 : In a bitopological space (X, τ_1, τ_2), every (i, j)-g*-closed set is (i, j)-*g-closed set.

The converse of the above proposition is not necessarily true as seen from following example.

Example 3.17 : In example (3.07), the subset {c} is (2, 1)-*g-closed but not (2, 1)-g*-closed.

The following diagram summarizes the above discussions.

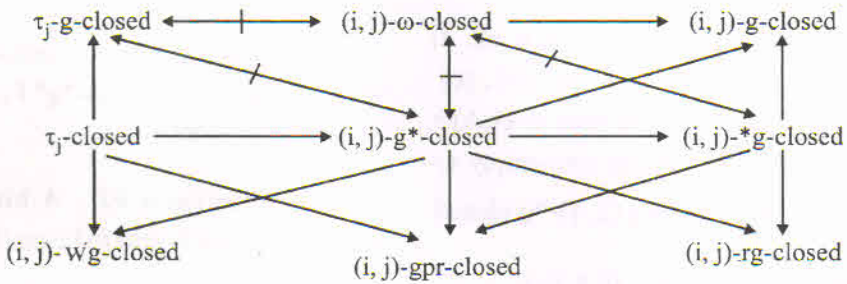

Diagram (3.18)

Where A⟶B (resp. A⟷B) represents A implies B but converse is not necessary true (resp. A and B are independent).

Proposition 3.19 : If A, B $\in$*D(i, j), then A $\cup$ B $\in$ *D(i, j).

Remark 3.20 : The intersection of two (i, j)-*g-closed sets need not be (i, j)-*g-closed set as seen from the following example.

Example 3.21 : Let X = {a, b, c, d}, τ_1={ϕ, {a, b}, X} and τ_2={ϕ, {a}, {a, b, c},X} then {a, c} and {a, d} are (1, 2)-*g-closed sets but their intersection {a} is not a (1, 2)-*g-closed set.

Remark 3.22 : *D(1, 2) is generally not equal to *D(2, 1), For example *D(1, 2) $\neq$ *D(2, 1) in example (3.11).

Proposition 3.23 : If $\tau_1 \subseteq \tau_2$ in (X, τ_1, τ_2) then *D(2, 1) $\subseteq$ *D(1, 2).

The converse of the above proposition is not necessarily true as seen from following example.

Example 3.24 : Let X = {a, b, c}, τ_1 = {ϕ, {b}, X} and τ_2 = {ϕ , {b, c}, X} then *D(2, 1) $\subseteq$ *D(1, 2) but τ_1 is not contained in τ_2.

Proposition 3.25 : For each element x of (X, τ_1, τ_2), {x} is τ_i-$\hat{g}$ -closed or $\{x\}^C$ is (i, j)-*g-closed.

Propositon 3.26 : If A is (i, j)-*g-closed then τ_j-cl(A) - A contains no non-empty τ_i-$\hat{g}$ -closed set.

Proof : Let A be an (i, j)-*g-closed set and B be τ_i- $\hat{g}$ -closed set such that B $\subseteq$ τ_j-cl(A) = A. Since A $\in$ *D(i, j), we have τ_j-cl(A) $\subseteq B^C$. Thus B $\subseteq$ τ_j-cl(A) $\cap$ (τ_j-cl(A))^C = ϕ.

The converse of the above proposition is not true as it can be seen from the following example.

Example 3.27 : Let X = {a, b, c}, τ_1 = {ϕ, {a}, {b, c}, X} and τ_2 = {ϕ , {b}, {c}, {b, c}, {a, c}, X}. If A = {b} then τ_1-cl(A) = A = {c} does not contain any non-empty τ_2- $\hat{g}$ -closed set. But A is not a (2, 1)-*g-closed.

Proposition 3.28 : If A is (i, j)-*g-closed set in (X, τ_1, τ_2) then A is τ_j-closed iff τ_j-cl(A) = A is τ_i- $\hat{g}$ -closed.

Proposition 3.29 : If A is an (i, j)-*g-closed set of (X, τ_1, τ_2) such that A $\subseteq$ B $\subseteq$ τ_j-cl(A) then B is also an (i, j)-*g-closed set of (X, τ_1, τ_2).

Proposition 3.30 : Let A $\subseteq$ Y $\subseteq$ X and A is (i, j)-*g-closed in X. Then A is (i, j)-*g-closed related to Y.

Theorem 3.31 : In a bitopological space (X, τ_1, τ_2), $\hat{G}O$ (X, τ_i) $\subseteq$ F_j (Family of all closed sets in τ_j) iff every subset of X is an (i, j)-*g-closed set.

Proof : Suppose that $\hat{G}O$ (X, τ_i) $\subseteq$ F_j. Let A be a subset of X such that A $\subseteq$ U where U $\in$ $\hat{G}O$ (X, τ_i) then τ_j-cl(A) $\subseteq$ τ_j-cl(U) =U and hence A is (i, j)-*g-closed.

Conversely let every subset of X is (i, j)-*g-closed. Let U $\in$ $\hat{G}O$ (X, τ_i). Since U is (i, j)-*g-closed, we have τ_j-cl(U) $\subseteq$ U. So U $\in$ F_j and hence $\hat{G}O$ (X, τ_i) $\subseteq$ F_j.

*4. (i, j)-*gT*$_{1/2}$ Spaces and (i, j)-*g*T$_{1/2}$ spaces:*

In this section we introduce (i, j)-*gT*$_{1/2}$ spaces and (i, j)-*g*T$_{1/2}$ spaces in bitopological spaces.

Definition 4.01 : A bitopological space (X, τ_1, τ_2) is said to be an (i, j)-*gT*$_{1/2}$ space if every (i, j)-*g-closed set is τ_j-closed.

Proposition 4.02 : If (X, τ_1, τ_2) is (i, j)-T$_{1/2}$ space then it is (i, j)-*gT*$_{1/2}$ space but not conversely.

Example 4.03 : In example (3.11), (X, τ_1, τ_2) is (1, 2)-*gT*$_{1/2}$ space but not (1, 2)-T$_{1/2}$ space.

Theorem 4.04 : A bitopological space (X, τ_1, τ_2) is an (i, j)-*gT*$_{1/2}$ space iff {x} is τ_j-open or τ_i- $\hat{g}$ -closed for each x $\in$ X.

Proof: Suppose that {x} is not τ_i- $\hat{g}$ -closed then {x}^{C} is (i, j)-*g-closed by proposition (3.25). Since (X, τ_1, τ_2) is an (i, j)-*gT*$_{1/2}$ space, {x}^{C} is τ_j-closed *i.e.* {x}^{C} is τ_j-open.

Conversely let F be an (i, j)-*g-closed set, By assumption {x} is τ_j-open or τ_i- $\hat{g}$ -

closed for any $x \in \tau_j$-cl(F). Now consider two cases.

Case-I : Suppose $\{x\}$ is τ_j-open. Since $\{x\} \cap F \neq \phi$ we have $x \in F$

Case-II : Suppose $\{x\}$ is τ_j- $\hat{g}$ -closed. If $x \notin F$ then $\{x\} \subseteq \tau_j$-cl(F)–F, which is a contradiction to proposition (3.28) so $x \in F$.

Thus in both cases we conclude that F is τ_j-closed. Hence (X, τ_1, τ_2) is (i, j)-$*gT*_{1/2}$ space.

Remark 4.05 : (X, τ_1, τ_2) is not generally (1, 2)-$*gT*_{1/2}$ space even if both (X, τ_1) and (X, τ_2) are $*gT*_{1/2}$-spaces. It can be shown by the following example.

Example 4.06 : In example (3.06), both (X, τ_1) and (X, τ_2) are $*gT*_{1/2}$-space but (X, τ_1, τ_2) is not (1, 2)-$*gT*_{1/2}$ space.

Remark 4.07 : (X, τ_1, τ_2) is not generally (1, 2)-$*gT*_{1/2}$ space even if (X, τ_1, τ_2) is (2, 1)-$*gT*_{1/2}$-space. It can be shown by the following example.

Example 4.08 : In example (3.04), (X, τ_1) is not $*gT*_{1/2}$-space but (X, τ_1, τ_2) is (2, 1)-$*gT*_{1/2}$ space.

Definition 4.09 : A bitopological space (X, τ_1, τ_2) is said to be an (i, j)-$*g*T_{1/2}$ space if every (i, j)-g-closed set is (i, j)-*g-closed set.

Definition 4.10 : A bitopological space (X, τ_1, τ_2) is said to be strongly pairwise $*g*T*_{1/2}$ space if it is both (1, 2)-$*gT*_{1/2}$ space and (2, 1)-$*gT*_{1/2}$ space.

Proposition 4.11. : If (X, τ_1, τ_2) is strongly pairwise $T_{1/2}$-space then it is strongly pairwise $*gT*_{1/2}$-space.

Proposition 4.12 : Every (i, j)-$*gT*_{1/2}$ space is (i, j)-$T*_{1/2}$ space but not conversely as seen from the following example.

Example 4.13 : Let X = {a, b, c}, τ_1 = {ϕ, {a}, X} and τ_2 = {ϕ , {a}, {b, c}, X}. Then (X, τ_1, τ_2) is (1, 2)-$T*_{1/2}$ space but not a (1, 2)-$*gT*_{1/2}$ space.

Proposition 4.14 : If (X, τ_1, τ_2) is strongly pairwise $*gT*_{1/2}$-space then it is strongly pairwise $T*_{1/2}$-space.

Proposition 4.15 : Every (i, j)-$T_{1/2}$ space is (i, j)-$*g*T_{1/2}$ space but not conversely.

Example 4.16 : Let X = {a, b, c}, τ_1 = {ϕ, {a}, {a, c}, X} and τ_2 = {ϕ, {a, b}, X}. Then (X, τ_1, τ_2) is a (1, 2)-$*g*T_{1/2}$ space but not a (1, 2)-$T_{1/2}$ space.

Remark 4.17 : (i, j)-$*gT*_{1/2}$ space and (i, j)-$*g*T_{1/2}$ space are independent as seen from the following examples.

Example 4.18 : In example (3.11), (X, τ_1, τ_2) is (1, 2)-$*gT*_{1/2}$ space but not (1, 2)-$*g*T_{1/2}$ space.

Example 4.19 : In example (3.21), (X, τ_1, τ_2) is (2, 1)-$*g*T_{1/2}$ space but not (2, 1)-$*gT*_{1/2}$ space.

Theorem 4.20 : A bitopological space (X, τ_1, τ_2) is an (i, j)-$T_{1/2}$ space iff it is both (i, j)-$*g*T_{1/2}$ space and (i, j)-$*gT*_{1/2}$ space.

Proposition 4.21 : Every (i, j)-$*T_{1/2}$ space is (i, j)-$*g*T_{1/2}$ space but not conversely as seen from the following example.

Example 4.22 : Let X = {a, b, c}, τ_1 = {ϕ, {a}, {b}, {a, b}, X} and τ_2 = {ϕ , {c}, {a, c}, X}, then (X, τ_1, τ_2) is (2, 1)-$*g*T_{1/2}$ space but not (2, 1)-$*T_{1/2}$ space.

The following diagram summarizes the above discussions.

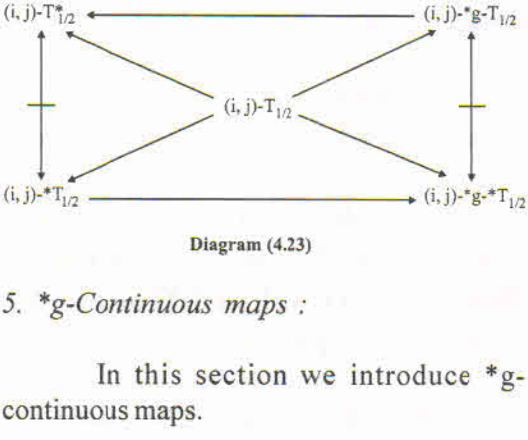

Diagram (4.23)

5. **g-Continuous maps :*

In this section we introduce *g-continuous maps.

Definition 5.01 : A map. f : $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called *D(i, j)-σ_k-continuous if the inverse image of every σ_k-closed set is an (i, j)-*g-closed set.

Proposition 5.02 : If f : $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is τ_j-σ_k-continuous then it is *D(i, j)-σ_k-continuous but not conversely.

Example 5.03 : Let X = {a, b, c}, τ_1 = {ϕ, {a}, X}, τ_2 = (ϕ, {b}, {a, c}, X } and Y = {p, q}, σ_1 = {ϕ, {p}, Y}, σ_2 = {ϕ, {q}, Y}. Define f : $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ by f(a) = q, f(b) = q, f(c) = p, then map f is *D(1, 2)-σ_2-continuous but not τ_2–σ_2-continuous.

Proposition 5.04 : If f : $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is *D(i, j)-σ_k-continuous then it is (i) D(i, j)-σ_k-continuous (ii) D_r(i, j)-σ_k-continuous (iii) ζ(i, j)-σ_k-continuous (iii)W(i, j)-σ_k-continous.

However the reverse implications of the above proposition are not true in general as seen from the following examples.

Example 5.05 : Let X = Y ={a, b, c}, τ_1 = {ϕ, {a}, {b}, {a, b}, X}, τ_2 = (ϕ, {a}, {b, c}, X } and σ_1 = {ϕ, {a}, {b, c}, Y}, σ_2 = {ϕ, {a}, {a, b}, {a, c}, Y}. Define f : $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ by identity mapping, then f is D(2, 1)-σ_2-continuous but not *D(2, 1)-σ_2-continuous.

Example 5.06 : Let X = Y ={a, b, c}, τ_1 = {ϕ, {a}, {a, c}, X}, τ_2 = (ϕ, {a, b}, X } and σ_1 = {ϕ, {a}, Y}, σ_2 = {ϕ, {a}, {a, b}, {a, c}, Y}. Define f : $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ by f(a) = b, f(b) = c, f(c) = a then map f is D_r(2, 1)-σ_2-continuous but not *D(2, 1)-σ_2-continuous.

Example 5.07 : Let X = Y ={a, b, c},

$\tau_1 = \{\phi, \{b\}, \{c\}, \{b, c\}, \{a, c\}, X\}$, $\tau_2 = (\phi, \{a, b\}, X\}$ and $\sigma_1 = \{\phi, \{a\}, \{a, b\}, Y\}$, $\sigma_2 = \{\phi, \{b, c\}, Y\}$ Define $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ by $f(a) = b$, $f(b) = c$, $f(c) = a$ then map f is $\zeta(1, 2)$-σ_1-continuous but not $^*D(1, 2)$-σ_1-continuous.

Example 5.08 : Let $X = Y = \{a, b, c\}$, $\tau_1 = \{\phi, \{b\}, \{c\}, \{b, c\}, \{a, c\}, X\}$, $\tau_2 = (\phi, \{a, b\}, X\}$ and $\sigma_1 = \{\phi, \{a\}, \{b\}, \{a, b\}, Y\}$, $\sigma_2 = \{\phi, \{c\}, \{b, c\}, Y\}$. Define $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ by identity mapping, then f is $W(1, 2)$-σ_2-continuous but not $^*D(1, 2)$-σ_2-continuous.

Proposition 5.09 : If $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is $D^*(i, j)$-σ_k-continuous then it is $^*D(i, j)$-σ_k-continuous.

The converse of the above proposition is not true as seen from the following example.

Example 5.10 : Let $X = Y = \{a, b, c\}$, $\tau_1 = \{\phi, \{a\}, \{c\}, \{a, c\}, X\}$, $\tau_2 = (\phi, \{a\}, X\}$ and $\sigma_1 = \{\phi, \{a\}, \{a, b\}, \{a, c\}, Y\}$, $\sigma_2 = \{\phi, \{c\}, \{b, c\}, Y\}$. Define $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ by $f(a) = a$, $f(b) = c$, $f(c) = b$ then f is $^*D(2, 1)$-σ_1-continuous but not $D^*(2, 1)$-σ_1-continuous.

Remark 5.11 : $C(i, j)$-σ_k-continuity and $^*D(i, j)$-σ_k-continuity are independent as it can be seen from the following examples.

Example 5.12 : Let $X = Y = \{a, b, c\}$, $\tau_1 = \{\phi, \{a\}, \{b, c\}, X\}$, $\tau_2 = (\phi, \{a\}, \{a, b\}, X\}$ and $\sigma_1 = \{\phi, \{a\}, \{c\}, \{a, c\}, Y\}$, $\sigma_2 = \{\phi, \{a\}, \{a, b\}, \{a, c\}, Y\}$. Define $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ by $f(a) = a$, $f(b) = c$, $f(c) = b$ then map f is $C(1, 2)$-σ_2-continuous but not $^*D(1, 2)$-σ_2-continuous.

Example 5.13 : Let $X = Y = \{a, b, c\}$, $\tau_1 = \{\phi, \{a\}, \{c\}, \{a, c\}, X\}$, $\tau_2 = (\phi, \{a\}, X\}$ and $\sigma_1 = \{\phi, \{a\}, \{a, c\}, Y\}$, $\sigma_2 = \{\phi, \{a\}, \{b\}, \{a, b\}, Y\}$. Define $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ by $f(a) = a$, $f(b) = b$, $f(c) = c$ then map f is $^*D(1, 2)$-σ_1-continuous but not $C(1, 2)$-σ_1-continuous.

The following diagram summarizes the above discussions.

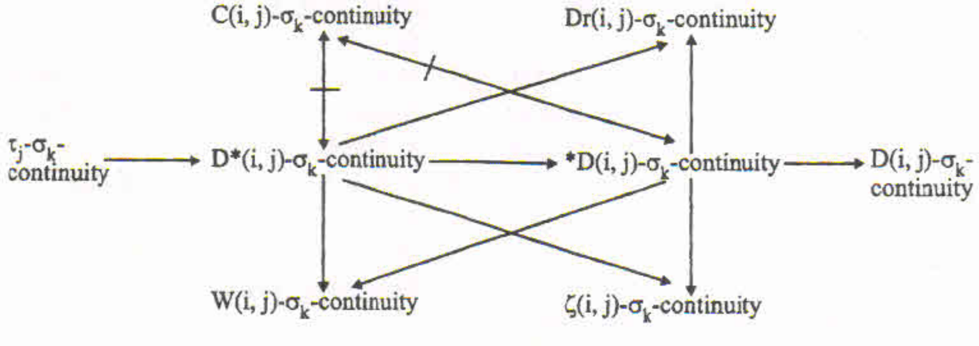

Diagram (5.14)

Theorem 5.15 : Let f : (X, τ_1, τ_2) $\to$ (Y, σ_1, σ_2) be a map:

(i) if (X, τ_1, τ_2) is an (i, j)-$T_{1/2}$ space then f is D(i, j)-σ_k-continuous iff it is *D(i, j)-σ_k-continuous.

(ii) If (X, τ_1, τ_2) is an (i, j)-*g$T^*_{1/2}$-space then f is τ_j-σ_k-continuous iff it is *D(i, j)-σ_k-continuous.

(iii) If (X, τ_1, τ_2) is an (i, j)-*g$T^*_{1/2}$-space then f is *D(i, j)-σ_k-continuous iff it is D*(i, j)-σ_k-continuous.

Definition 5.16 : A map. f : (X, τ_1, τ_2) $\to$ (Y, σ_1, σ_2) is called *D(i, j)-σ_k-irresolute if the inverse image of every σ_k-*g-closed set in Y is (i, j)-*g-closed set in X.

Proposition 5.17 : If f : (X, τ_1, τ_2) $\to$ (Y, σ_1, σ_2) is *D(i, j)-σ_k-irresolute then it is *D(i, j)-σ_k-continuous but not conversely.

Example 5.18 : Let X = Y = {a, b, c}, τ_1 = {ϕ, {a}, X}, τ_2 = (ϕ, {b}, {a, c}, X } and σ_1 = {ϕ, {a}, {c}, {a, c}, Y}, σ_2 = {ϕ, {b}, Y}. Define f : (X, τ_1, τ_2) $\to$ (Y, σ_1, σ_2) by identity mapping then f is *D(1, 2)-σ_2-continuous but not *D(1, 2)-σ_2-irresolute.

Proposition 5.19 : If f : (X, τ_1, τ_2) $\to$ (Y, σ_1, σ_2) is *D(i, j)-σ_k-continuous and in (Y, σ_k) every σ_k-*g-closed set is σ_k-closed then f is *D(i, j)-σ_k-irresolute.

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