Ultra Scientist Vol. 26(1)A, 111-113 (2014).

SHORT COMMUNICATIONS

Reverse Derivations On Semiprime Rings

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(Acceptance Date 5th February, 2014)

Abstract

In this paper some results concerning to reverse derivations on semiprime rings are presented. If R be a semi prime ring with a reverse derivation d and S be the left ideal of R then $[S,R]d(R)=0$. Also if S be a right ideal of R then $[R,S]d(S)=0$ is proved by using reverse derivation.

Key words : Center, Semi prime ring, derivation, reverse derivation.

Bresar and Vukman[1] have introduced the notion of a reverse derivation. The reverse derivations on semi prime rings have been studied by Samman and Alyamani[2].

Preliminaries :

Through out, R will represent an associative ring with center $Z(R)$ defined as $Z=\{z \in R/[z,R]=0\}$. We write $[x,y]$ for $xy-yx$. Recall that a ring R is called prime if $aRb=0$ implies $a=0$ or $b=0$; and it is called semi prime if $aRa=0$ implies $a=0$. An additive mapping d from R into itself is called a derivation if $d(xy)=d(x)y+xd(y)$ for all $x,y\in R$ and is called a reverse derivation if $d(xy)=d(y)x+yd(x)$ for all $x,y\in R$.

Main Results :

Theorem 1: Let be a semi prime ring with a reverse derivation d and let S be a left ideal of R. Suppose that $d(rx)\in Z\ \forall\, x\in S$, $r\in R$ where Z denotes the center of R. Then $[S,R]d(R)=0$.

Proof:

Given $[d(rx),y]=0\ \forall\, x\in S, y,r\in R$ (1)

i.e., $[d(x)r+xd(r),y]=0$

$\Rightarrow (d(x)r+xd(r))y-y(d(x)r+xd(r))=0$

$\Rightarrow d(x)ry+xd(r)y-yd(x)r-yxd(r)=0$

Adding and subtracting, $d(x)yr, xyd(r)$

$$\Rightarrow d(x)ry+d(x)yr-d(x)yr+xd(r)y- yd(x)r-yxd(r)+xyd(r)-xyd(r)=0$$

$$\Rightarrow d(x)ry-d(x)yr+xd(r)y-xyd(r)+xyd(r)- yxd(r)+d(x)yr-yd(x)r=0$$

$$\Rightarrow d(x)(ry-yr)+x(d(r)y-yd(r))+ (xy-yx)d(r)+(d(x)y-yd(x))r=0$$

$$\Rightarrow d(x)[r,y]+x[d(r),y]+[x,y]d(r)+[d(x),y]r=0$$

$$\Rightarrow d(x)[r,y]+x[d(r),y]+[x,y]d(r)=0$$

(since by (1))

Put y = r in the above equation, then we get,

$$\Rightarrow d(x)[r,r]+x[d(r),r]+[x,r]d(r)=0$$

$$\Rightarrow x[d(r),r]+[x,r]d(r)=0$$

By expanding this equation, we conclude that,

$$\Rightarrow x(d(r)r-rd(r))+(xr-rx)d(r)=0$$

$$\Rightarrow xd(r)r-xrd(r)+xrd(r)-rxd(r)=0$$

$$\Rightarrow xd(r)-rxd(r)=0$$

$$\Rightarrow xd(r)r=rxd(r) \quad (2)$$

We write xz instead of x in (2) and using this equality, we get,

$$\Rightarrow xzd(r)r=rxzd(r)$$

$$\Rightarrow xzd(r)r-rxzd(r)=0$$

$$\Rightarrow xzd(r)r-rxzd(r)=0$$

$$\Rightarrow xrzd(r)-rxzd(r)=0$$

$$\Rightarrow [xr-rx]zd(r)=0$$

$$\Rightarrow [x,r]zd(r)=0, \ \forall x \in S \ and \ z,r \in R \quad (3)$$

Put $z=d(r)r[x,r]$ in (3), then we get,

$\Rightarrow [x,r]d(r)r[x,r]d(r)=0$ which implies

$$\Rightarrow [x,r]d(r)R[x,r]d(r)=0 \qquad \forall r \in R$$

Since is semi prime, we have,

$[x,r]d(r)=0 \ , \forall x \in S, r \in R$.

$\therefore [S,R]d(R)=0.$ ∎

Theorem 2: Let R be a semi prime ring with a reverse derivation d and let S be a right ideal of R. Suppose that $d(xr) \in Z$, $\forall x \in S, r \in R$ where Z denotes the center of R. Then $[R,S]d(S)=0$.

Proof :

Given $[d(xr),y]=0$, $\forall x \in S \ and \ y,r \in R$

i.e., $[d(r)x+rd(x),y]=0$

$$\Rightarrow [d(r)x+rd(x)]y-y[d(r)x+rd(x)]=0$$

$$\Rightarrow d(r)xy+rd(x)y-yd(r)x-yrd(x)=0$$

Adding and subtracting $ryd(x), d(r)yx$, then we get,

$$\Rightarrow d(r)xy+rd(x)y-ryd(x)+ryd(x)- yd(r)x+d(r)yx-d(r)yx-yrd(x)=0$$

$$\Rightarrow d(r)xy+r[d(x)y-yd(x)]+[ry-yr]d(x)+ [d(r)y-yd(r)]x-d(r)yx=0$$

$$\Rightarrow [d(r)xy-d(r)yx]+r[d(x)y-yd(x)]+ [ry-yr]d(x)+[d(r)y-yd(r)]x=0$$

$$\Rightarrow d(r)[xy-yx]+r[d(x),y]+ [r,y]d(x)+[d(r),y]x=0$$

$$\Rightarrow d(r)[x,y]+r[d(x),y]+ [r,y]d(x)+[d(r),y]x=0$$

$$\Rightarrow d(r)[x,y]+r[d(x),y]+[r,y]d(x)=0$$

Put y = x in the above equation, then we get,

$$\Rightarrow d(r)[x,x]+r[d(x),x]+[r,x]d(x)=0$$

$$\Rightarrow r[d(x),x]+[r,x]d(x)=0$$

By expanding this equation, we conclude that,

$$\Rightarrow r(d(x)x-xd(x))+(rx-xr)d(x)=0$$

$$\Rightarrow rd(x)x-rxd(x)+rxd(x)-xrd(x)=0$$

$$\Rightarrow rd(x)x-xrd(x)=0$$

$$\Rightarrow rd(x)x=xrd(x) \qquad (4)$$

We write zr instead of r in (4) and using this equality, we get,

$$\Rightarrow zrd(x)x=xzrd(x)$$

$$\Rightarrow zxrd(x)=xzrd(x)$$

$$\Rightarrow zxrd(x)-xzrd(x)=0$$

$$\Rightarrow [zx-xz]rd(x)=0$$

$$\Rightarrow [z,x]rd(x)=0, \qquad \forall x\in S \text{ and } z,r\in R$$

By interchanging r and z in the above equation, we get,

$$\Rightarrow [r,x]zd(x)=0 \quad \forall x\in S \text{ and } z,r\in R \quad (5)$$

Put $z=[r,x]d(x)r[r,x]d(x)$ in (2), then we get,

$\Rightarrow [r,x]d(x)r[r,x]d(x)=0$ which implies

$$\Rightarrow [r,x]d(x)R[r,x]d(x)=0, \ \forall r\in R \text{ and } x\in S$$

By using the semi primeness of R,

$$[r,x]d(x)=0, \quad \forall x\in S, r\in R$$

$$\therefore [R,S]d(S)=0.$$

■

References

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