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Reliability Analysis of a Two-Unit Cold Standby System with Arrival Time of the Server Subject to MOT

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Abstract

This paper analyses the reliability of a cold standby system consisting of two repairable units and a server. At any time, one of the two units is operating while the other is on cold standby. The server takes some time to visit the system to do preventive maintenance and repair of the system. The preventive maintenance of the system is done by the server after a maximum operation time. The unit fails directly from the normal mode. The failure time of the unit follows negative exponential distribution while the distribution of preventive maintenance and repair times are taken as arbitrary. Repair and maintenance are perfect. The semi-Markov process and regenerative point technique is used to obtain expressions for various measures of system effectiveness. The behaviour of some important reliability measures has been observed graphically giving particular values to various costs and parameters.

Key words: Cold Standby System, Preventive Maintenance, Maximum Operation Time and Arrival Time of the Server.

2000 Mathematics Subject Classification: 90B25 and 60K10

Notations

No : The unit is operative and in normal mode

Cs : The unit is cold standby

λ : Constant failure rate

α_0 : Constant rate of Maximum Operation Time.

Pm/PM : The unit is under preventive Maintenance/ under preventive maintenance continuously from previous state

WPm/WPM : The unit is waiting for preventive Maintenance/waiting for preventive maintenance continuously from previous state

FUr/FUR : The unit is failed and is under repair/ under repair continuously from previous state

FWr / FWR: The unit is failed and is waiting for repair/ waiting for repair continuously from previous state

w(t) / W(t) : pdf/cdf of arrival time of the server

g(t) / G(t) : pdf / cdf of repair time of the unit

f(t) / F(t) : pdf / cdf of the time for preventive maintenance of the unit

q_{ij} (t)/ Q_{ij}(t): pdf / cdf of passage time from regenerative state i to a regenerative state j or to a failed state j without visiting any other regenerative state in (0, t]

pdf / cdf : Probability density function/ Cumulative density function

$q_{ij.kr}$ (t)/$Q_{ij.kr}$(t): pdf/cdf of direct transition time from regenerative state i to a regenerative state j or to a failed state j visiting state k, r once in (0, t]

μ_i(t) : Probability that the system up initially in state $S_i \in E$ is up at time t without visiting to any regenerative state

W_i(t) : Probability that the server is busy in the state S_i up to time 't' without making any transition to any other regenerative state or returning to the same state via one or more non-regenerative states.

m_{ij} : Contribution to mean sojourn time (μ_i) in state S_i when system transit directly to state S_j so that $\mu_i = \sum_j m_{ij}$ and $m_{ij} = \int t dQ_{ij}(t) = -q_{ij}^{*\prime}(0)$

□/© : Symbol for Laplace-Stieltjes convolution/Laplace convolution

~ / * : Symbol for Laplace -Steiltjes Transform (LST)/Laplace Transform (LT)

'(desh) : Used to represent alternative result

Introduction

Since the breakdowns of a system increase costs and inconvenience or sometimes gravely threaten of public safety, the demand for systems that performs better and costs less increases. It is well known in the reliability field that providing redundancy improves the performance of the system. The advantages of redundant systems include a reduction of the system down time and an enhancement of the reliability, within the technological constraints. For this reason much research has been reported on the analysis of redundant systems. The analysis of standby redundant systems has an extensive literature. Goel and Sharma[1] and Singh and Singh[2] studied stochastically the two unit standby system under different repair policies of the server. Mokaddis and Elias[3] developed a reliability model for a two-unit system with a cold standby and a single service facility for the performance of preventive maintenance and repair. In most of these studies, it is assumed that repair facility becomes available immediately as and when required. But, in practice, this assumption is not always true may because of the pre occupations of the repair facility and in such a situation service facility may take some time to arrive at the system. Chander[4] has suggested reliability models of a standby system with arrival time of the server. Further, it is proved that preventive maintenance can slow the deterioration process of operating system and restore them in a younger age or state. Thus, the method of preventive maintenance can be used to improve the performance of these systems. Recently, Malik and Kumar[5] investigated a reliability model for a computer system conducting preventive maintenance after a maximum operation time.

Keeping this above concepts in view, reliability analysis of a two-unit cold standby system is analyzed here. The concepts of arrival time of the server, preventive maintenance and maximum operation time are used. For this purpose, a stochastic model of two-units is developed in which one unit is operative and other is kept as cold standby. There is a single server who takes some time to visit the system to do preventive maintenance and repair. The preventive maintenance of the system is done by the server after a maximum operation time. The failure time of the unit follows negative exponential distribution while the distribution of preventive maintenance and repair are taken as arbitrary. The semi-Markov process and regenerative point technique is adopted to derive expressions for various measures of system effectiveness such as mean time to system failure, availability, busy period of the repairman due to preventive maintenance, busy period of the server due to repair, expected number of preventive maintenances and expected number of repairs. The behaviour of some important reliability measures has been observed graphically giving particular values to various costs and parameters.

Methodology :

The system has been analyzed using well known semi-Markov process and regenerative point technique which are briefly described as:

Markov Process: If $\{X(t), t\in T\}$ is a stochastic process such that, given the value of X(s), the value of X(t), t>s do not depend on the values of X(u), u<s Then the process $\{X(t), t\in T\}$ is a Markov process.

Semi-Markov Process: A semi-Markov process is a stochastic process in which changes of state occur according to a Markov chain and in which the time interval between two successive transitions is a random variable, whose distribution may depend on the state from which the transition take place as well as on the state to which the next transition take place.

Regenerative Process : Regenerative stochastic process was defined by Smith (1955) and has been crucial in the analysis of complex system. In this, we take time points at which the system history prior to the time points is irrelevant to the system conditions. These points are called regenerative points. Let X(t) be the state of the system of epoch. If $t_1, t_2, \ldots$ are the epochs at which the process probabilistically restarts, then these epochs are called regenerative epochs and the process $\{X(t), t = t_1, t_2 \ldots\ldots\ldots\}$ is called regenerative process. The state in which regenerative points occur is known as regenerative state.

Transition Probabilities and Mean Sojourn Times :

Simple probabilistic considerations yield the following expressions for the non-zero elements

$$p_{ij} = Q_{ij}(\infty) = \int_0^\infty q_{ij}(t)dt \quad \text{as} \qquad (1)$$

$$p_{01}=\frac{\alpha_0}{A},\ p_{02}=\frac{\lambda}{A}, p_{13}=w^*(A),\ p_{1.7}=\frac{\lambda}{A}[1-w^*(A)]$$

$$= p_{14,7,8}\ ,\ p_{1.19}= \frac{\alpha_0}{A}[1-\ w^*(A)]=\ p_{13.9.10,}$$

$p_{24} = w^*(A)$, $p_{2.13} = \frac{\alpha_0}{A}[1- w^*(A)] = p_{23,13,14}$, $p_{2.11} = \frac{\lambda}{A}[1- w^*(A)] = p_{24,11,12}$, $p_{30} = f^*(A)$, $p_{3.5} = \frac{\alpha_0}{A}[1- f^*(A)] = p_{3.3,5}$, $p_{3.6} = \frac{\lambda}{A}[1- f^*(A)] = p_{3,4,6}$, $p_{40} = g^*(A)$, $p_{4.16} = \frac{\lambda}{A}[1- g^*(A)] = p_{4,4.16}$, $p_{4.17} = \frac{\alpha_0}{A}[1- g^*(A)] = p_{4,3,15}$, $p_{53} = p_{84} = p_{10.3} = f^*(0) = p_{64}$, $p_{78} = w^*(0) = p_{9.10} = p_{11.12} = p_{13.14}$, $p_{14.3} = g^*(0) = p_{16.4} = p_{15.3} = p_{12.4}$ (2)

where $A = \lambda + \alpha_0$

It can be easily verified that $p_{01}+p_{02} = p_{13}+p_{1.17}+p_{1.19} = p_{24}+p_{2.11}+p_{2.13} = p_{30}+p_{35}+p_{36} = p_{4.16}+p_{4.15}+p_{40} = p_{64} = p_{78} = p_{84} = p_{9.10} = p_{10.3} = p_{11.12} = p_{12.4} = p_{13.14} = p_{14.3} = p_{15.3} = p_{16.4} = p_{13} + p_{13.9,10} + p_{14,7,8} = p_{24} + p_{24,11,12} + p_{23,13,14} = p_{30} + p_{33,5} + p_{34.6} = p_{40} + p_{44.16} + p_{43,15} = 1$ (3)

The mean sojourn times (μ_i) is the state S_i are

$$\mu_0 = \frac{1}{\lambda+\alpha_0},\quad \mu_1 = \frac{1}{\beta+\alpha_0+\lambda},\quad \mu_2 = \frac{1}{\beta+\alpha_0+\lambda},$$

$$\mu_3 = \frac{1}{\lambda+\alpha_0+\alpha},\quad \mu_4 = \frac{1}{\theta+\alpha_0+\lambda},\quad \mu_3' = \frac{1}{\alpha},\quad \mu_4' = \frac{1}{\theta},$$

$$\mu_1' = \frac{\alpha(\alpha_0+\lambda)+\beta(\alpha_0+\lambda)+\beta\alpha}{\alpha\beta(\beta+\alpha_0+\lambda)},\quad \mu_2' = \frac{\alpha(\alpha_0+\lambda)+\beta(\alpha_0+\lambda)+\beta\alpha}{\alpha\beta(\beta+\alpha_0+\lambda)} \tag{4}$$

The states S0, S1, S2, S3 and S4 are regenerative states while S5, S6, S7, S8, S9, S10, S11, S12, S13, S14, S15 and S16 are non-regenerative states. Thus E={S0, S1, S2,S3,S4}. The possible transition between states along with transition rates for the model is shown in fig. 1.

Reliability and Mean Time to System Failure (MTSF):

Let $\phi_i(t)$ be the cdf of first passage time from the regenerative state i to a failed state. Regarding the failed state as absorbing state, we have the following recursive relations for ϕ_i (t):

$$\varphi_i(t) = \sum_j Q_{i,j}(t) \circledR \varphi_j(t) + \sum_k Q_{i,k}(t) \tag{5}$$

where *j* is an un-failed regenerative state to which the given regenerative state *i* can transit and *k* is a failed state to which the state *i* can transit directly. Taking LST of above relation (5) and solving for $\tilde{\phi}_0(s)$. We have

$$R^*(s) = \frac{1-\tilde{\phi}_0(s)}{s} \tag{6}$$

The reliability of the system model can be obtained by taking Laplace inverse transform of (6). The mean time to system failure (MTSF) is given by

$$\text{MTSF} = \lim_{s\to o}\frac{1-\tilde{\phi}_0(s)}{s} = \frac{N_1}{D_1} \text{ where} \tag{7}$$

$N_1 = \mu_0 + p_{01}\mu_1 + p_{02}\mu_2 + p_{24}p_{02}\mu_4 + p_{01}p_{13}\mu_3$

and $D_1 = 1 - p_{01}p_{30}p_{13} - p_{02}p_{40}p_{24}$

Steady State Availability :

Let $A_i(t)$ be the probability that the system is in up-state at instant 't' given that the system entered regenerative state i at t = 0. The recursive relations for A_i (t) are given as

$$A_i(t) = M_i(t) + \sum_j q_{i,j}^{(n)}(t) © A_j(t) \quad (8)$$

where *j* is any successive regenerative state to which the regenerative state *i* can transit through n transitions. $M_i(t)$ is the probability that the system is up initially in state $S_i \in E$ up at time t without visiting to any other regenerative state, we have is

$$M_0(t) = e^{-(\lambda+\alpha_0)t},\ M_1(t) = e^{-(\lambda+\alpha_0)t}\overline{W(t)}$$

$$= M_2(t),\ M_3(t) = e^{-(\lambda+\alpha_0)t}\overline{F(t)},$$

$$M_4(t) = e^{-(\lambda+\alpha_0)t}\overline{G(t)} \quad (9)$$

Taking LT of above relations (8) and solving for $A_0^*(s)$. The steady state availability is given by $A_0(\infty) = \lim_{s\to 0} sA_0^*(s) = \frac{N_2}{D_2}$, where (10)

$N_2 = (\mu_0 + \mu_1 p_{01} + \mu_2 p_{02})\,[(1-p_{33.5})(1-p_{44.16}) - p_{34.6}\,p_{43.15}] + (\mu_3)\,[(1-p_{44.16})(p_{01}p_{13} + p_{13.9,10}p_{01} + p_{23.13,14}p_{02}) + p_{43.15}(p_{01}p_{14,7,8} + p_{24}p_{02} + p_{24.11,12}p_{02})] + \mu_4[(1-p_{33.5})\,(p_{01}p_{14,7,8} + p_{24}p_{02} + p_{24.11,12}p_{02}) + (p_{34.6})\,(p_{01}p_{13} + p_{13.9,10}p_{01} + p_{23,13,14}p_{02})]$

and

$D_2 = (\mu_0 + \mu_1' p_{01} + \mu_2' p_{02})\,[(1-p_{33.5})(1-p_{44.16}) - p_{34.6}\,p_{43.15}] + \mu_3'\,[(1-p_{44.16})(p_{01}p_{13} + p_{13.9,10}p_{01} + p_{23.13,14}p_{02}) + p_{43.15}(p_{01}p_{14,7,8} + p_{24}p_{02} + p_{24.11,12}p_{02})] + \mu_4'\,[(1-p_{33.5})\,(p_{01}p_{14,7,8} + p_{24}p_{02} + p_{24.11,12}p_{02}) + (p_{34.6})\,(p_{01}p_{13} + p_{13.9,10}p_{01} + p_{23,13,14}p_{02})]$

Busy Period Analysis of the Server :

(a) Due to Preventive Maintenance (PM) :

Let $B_i^P(t)$ be the probability that the server is busy in preventive maintenance of the system (unit) at an instant'‘t’ given that the system entered state i at t = 0. The recursive relations for $B_i^P(t)$ are as follows:

$$B_i^P(t) = W_i(t) + \sum_j q_{i,j}^{(n)}(t) © B_j^P(t) \quad (11)$$

Where *j* is any successive regenerative state to which the regenerative state *i* can transit through n transitions. $W_i(t)$ be the probability that the server is busy in state S_i due to preventive maintenance up to time t without making any transition to any other regenerative state or returning to the same via one or more non-regenerative states and so

$$W_3(t) = e^{-(\lambda+\alpha_0)t}\overline{F}(t) + (\alpha_0 e^{-(\lambda+\alpha_0)t} © 1)\overline{F}(t) + (\lambda e^{-(\lambda+\alpha_0)t} © 1)\overline{F}(t)$$

(b) Due to Repair :

Let $B_i^R(t)$ be the probability that the server is busy in repairing the unit due to failure at an instant ‘t’ given that the system entered state i at t = 0. The recursive relations for $B_i^R(t)$ are as follows:

$$B_i^R(t) = W_i(t) + \sum_j q_{i,j}^{(n)}(t) © B_j^R(t) \quad (12)$$

where *j* is any successive regenerative state to which the regenerative state *i* can transit through n transitions. $W_4(t)$ be the probability that the server is busy in state S_i due to failure up to time t without making any transition to any other regenerative state or returning to the same via one or more non-regenerative states and so

$$W_4(t)=e^{-(\lambda+\alpha_0+\beta_0)t}\overline{G}(t)+(\alpha_0 e^{-(\lambda+\alpha_0+\beta_0)t} © 1)\overline{G}(t)$$
$$+(\lambda e^{-(\lambda+\alpha_0+\beta_0)t} © 1)\overline{G}(t)$$

Taking LT of above relations (11) to (12) and solving for B_0^{*P} (s) and B_0^{*R} (s). The time for which server is busy due to preventive maintenance and repair respectively is given by

$$B_0^P = \lim_{s\to 0} sB_0^{*P}(s) = \frac{N_3^P}{D_2}, \text{ and}$$

$$B_0^R = \lim_{s\to 0} sB_0^{*R}(s) = \frac{N_3^R}{D_2} \quad (13)$$

$N_3^P = W_3^*(0)$ $[(1-p_{44.16})(p_{01}p_{13}+ p_{13.9,10}p_{01}+ p_{23.13,14}p_{02})+p_{43.15}(p_{01}p_{14,7,8}+p_{24}p_{02}+p_{24.11,12}p_{02})]$

$N_3^R =[(1-p_{33.5})$ $(p_{01}p_{14,7,8}+ p_{24}p_{02}+p_{24.11,12}p_{02})+ (p_{34.6})$ $(p_{01}p_{13}+ p_{13.9,10}p_{01}+ p_{23,13,14}p_{02})]$ $W_4^*(0)$

Expected Number of Preventive Maintenances:

Let $R_i^p(t)$ be the expected number of preventive maintenances by the server in (0, t] given that the system entered the regenerative state i at t = 0. The recursive relations for $R_i^p(t)$ are given as

$$R_i^p(t) = \sum_j Q_{i,j}^{(n)}(t) ® \left[\delta_j + R_j^p(t)\right] \quad (14)$$

where j is any regenerative state to which the given regenerative state *i* transits and δj =1, if *j* is the regenerative state where the server does job afresh, otherwise δj = 0.

Taking LST of relations (14) and solving for $\tilde{R}_0^p(s)$. The expected numbers of preventive maintenances per unit time are given by

$$R_0^p(\infty) = \lim_{s\to 0} s\tilde{R}_0^p(s) = \frac{N_4^p}{D_2} \quad (15)$$

where D_2 is already mentioned.

N_4^p = $[(1-p_{44.16})(p_{01}p_{13}+p_{13.9,10}p_{01}+ p_{23.13,14}p_{02})+ p_{43.15}(p_{01}p_{14,7,8}+ p_{24}p_{02}+ p_{24.11,12}p_{02})]$

Expected Number of Repairs :

Let $R_i^r(t)$ be the expected number of repairs by the server in (0, t] given that the system entered the regenerative state i at t=0. The recursive relations for $R_i^r(t)$ are given as

$$R_i^r(t) = \sum_j Q_{i,j}^{(n)}(t) ® \left[\delta_j + R_j^r(t)\right] \quad (16)$$

Where j is any regenerative state to which the given regenerative state *i* transits and δj =1, if *j* is the regenerative state where the server does job afresh, otherwise δj = 0.

Taking LST of relations (16) and solving for $\tilde{R}_0^r(s)$. The expected numbers of repairs per unit time are given by

$$R_0^r(\infty) = \lim_{s\to 0} s\tilde{R}_0^r(s) = \frac{N_4^r}{D_2} \quad (17)$$

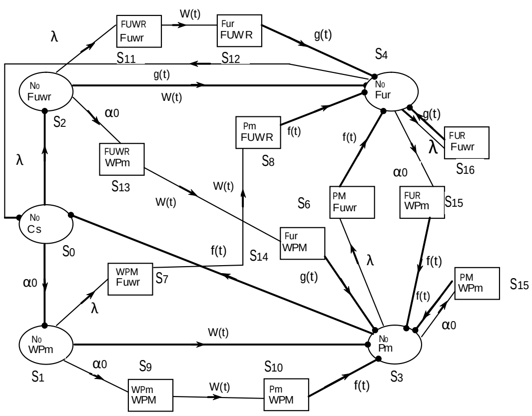

Fig. 1. State Transition Diagram

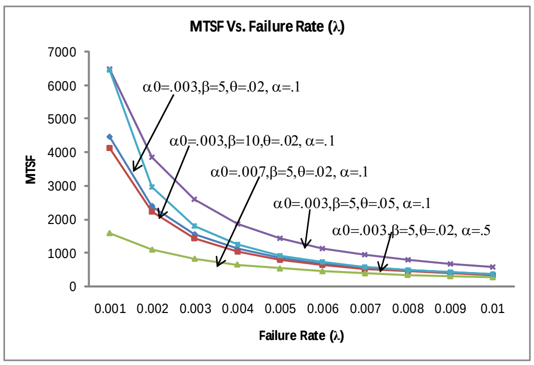

Fig. 2

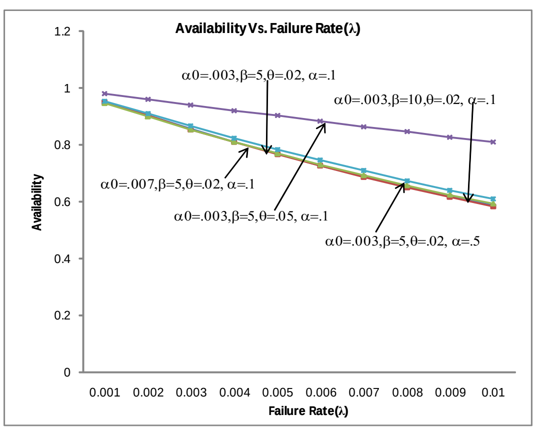

Fig. 3

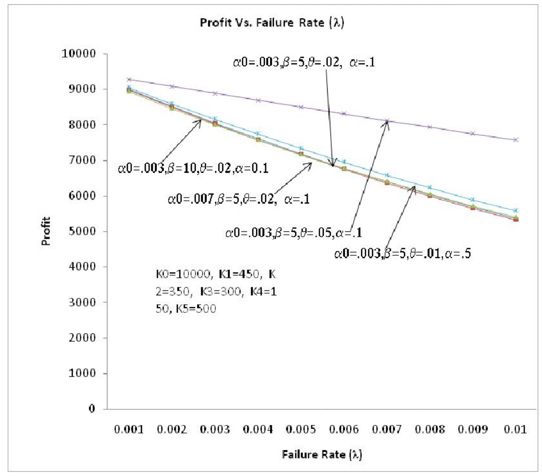

Fig. 4

where D_2 is already mentioned.

N_4^r = [(1-$p_{33.5}$) ($p_{01}p_{14,7,8}$+ $p_{24}p_{02}$+ $p_{24.11,12}p_{02}$)+ ($p_{34.6}$) ($p_{01}p_{13}$+ $p_{13.9,10}p_{01}$+ $p_{23,13,14}p_{02}$)])]

Cost-Benefit Analysis :

The profit incurred to the system model in steady state can be obtained as

$$P = K_0A_0 - K_1B_0^P - K_2B_0^R - K_4R_0^P - K_5R_0^r - K_7 \quad (18)$$

K_0 = Revenue per unit up-time of the system

K_1 = Cost per unit time for which server is busy due preventive maintenance

K_2 = Cost per unit time for which server is busy due to repair

K_3 = Cost per unit time for preventive maintenance

K_4 = Cost per unit time for repair

K_5 = Cost per unit time for replacement

K_6 = Total cost for per visit by the server

Conclusion

By considering a particular case $g(t) = \theta e^{-\theta t}$, $w(t)=\beta e^{-\beta t}$ and $f(t) = \alpha e^{-\alpha t}$, the numerical results some reliability measures are obtained for the system under study. The graphs for mean time to system failure (MTSF), availability and profit are drawn with respect to failure rate (λ) for fixed values of parameters as shown respectively in figs.2, 3 and 4. It is revealed that MTSF, Availability and profit decrease with the increase of failure rate (λ), arrival rate (β) and maximum operation time (α_0). But the value of these measures increase with the increase of preventive maintenance rate (α) and repair rate (θ). Thus finally it is concluded that a system in which server takes some time to arrive can be made reliable and economical to use

(i) By taking one more unit in cold standby.

(ii) By conducting PM of the system after a specific period of time.

References

1. Goel, L. R. and Sharma, S. C., Stochastic analysis of a two-unit standby system with two failure modes and slow switch, *Microelectronics and Reliability,* Vol. *29(4)*, pp. 493-498 (1989).
2. Singh, S.K. and Singh, R.P., Profit evaluation of a two-unit cold standby system with a provision of rest. *Microelectronics and Reliability*, Vol. *29(5)*, pp. 705-709 (1989).
3. G.S. Mokaddis, S.S. Elias, The reliability function and the availability of a duplication redundant system with a single service facility for preventive maintenance and repair. *Microelectronics and Reliability,* Vol. *29(5)*, pp. 751-756 (1989).
4. Chander, S., Reliability models with priority for operation and repair with arrival time of server, *Pure and Applied Mathematika Sciences,* Vol. LXI, No. 1–2, pp. 9–22 (2005).
5. Malik, S.C. and Kumar, A., Reliability modeling of a computer system with priority to s/w replacement over h/w replacement subject to MOT and MRT, International *Journal of Pure and Applied Mathematics,* Vol. *80(5)*, pp. 693-709 (2012).