Ultra Scientist Vol. 25(3)A, 431-434 (2013).

The Semientire Edge Dominating Graph

V.R. KULLI*

(Acceptance Date 5th November, 2013)

Abstract

The *semientire edge dominating graph* $E_e d(G)$ of a graph $G=(V, E)$ is a graph with the vertex set E S where S is the set of all minimal edge dominating sets G and with two vertices u,v E S adjacent if u,v F where F is a minimal edge dominating set in S or u E and $v=F$ is a minimal edge dominating set of G containing u. In this paper, a characterization is given for graphs G for which $E_e d(G)$ is connected. Further, some new results are established relating to this new graph.

Key words: edge dominating graph, semientire edge dominating graph.

Mathematics Subject Classification: 05*C*.

1. Introduction

By a graph $G=(V, E)$, we mean a finite, undirected without loops, multiple edges or isolated vertices. Any undefined term in this paper may be found in Harary[1]. A set F of edges in a graph $G=(V, E)$ is an *edge dominating set* if every edge in $E - F$ is adjacent to at least one edge in F. The *edge domination number* $'(G)$ of G is the minimum cardinality of an edge dominating set of G. An edge dominating set F of G is a *minimal edge dominating set* if for every e F, $F-e$ is not an edge dominating set of G. A set D of vertices in a graph G is a *dominating set* if every vertex in $V - D$ is adjacent to at least one vertex in D. The *domination number* (G) of G is the minimum cardinality of a dominating set of G. A recent survey of (G) can be found in Kulli[2].

The *common minimal edge dominating graph* $CD_e(G)$ of a graph G is the graph with the vertex set E where E is the set of all minimal edges of G and with two vertices u, v in E adjacent in $CD_e(G)$ if there exists a minimal edge dominating set of G containing them. This concept was introduced[6] by Naik.

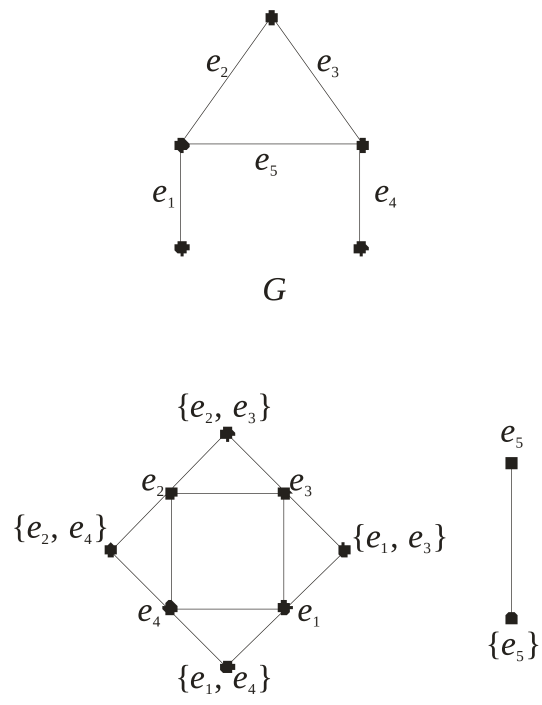
The edge dominating graph $D_e(G)$ of a graph G is the graph with the vertex set E S where S is the set of all minimal edge dominating sets of G and two vertices u, v in E S adjacent if u E and v=F is a minimal edge dominating set of G containing u. This concept was introduced by Kulli[3].

The semientire dominating graph $Ed(G)$ of a graph G is the graph with the vertex set V S where S is the set of all minimal edge dominating sets of G and with two vertices u,v in V S adjacent if u, v D where D is a minimal dominating set of G or u V and v = D is a minimal dominating set of G containing u. This concept was introduced by Kulli[4].

In this paper, we introduce a new class of intersection graphs in the field of domination theory as follows:

The *semientire edge dominating graph* $E_ed(G)$ of a graph G=(V, E) is a graph with the vertex set E S where S is the set of all minimal edge dominating sets of G and with two vertices u,v E S adjacent if u, v F where F is a minimal edge dominating set in G or u E and v=F is a minimal edge dominating set G containing u.

In Figure 1, a graph G and its semientire edge dominating graph E_ed(G) are shown.



$E_ed(G)$ Figure 1

2. Results

Remark 1. For any graph G without isolated vertices, $CD_e(G)$ and $D_e(G)$ are edge disjoint subgraphs of $E_ed(G)$.

Remark 2. For a graph G without isolated vertices, $CD_e(G)$ is an induced subgraph of $E_ed(G)$.

Theorem 3. $E_ed(G) = K_{p+1}$ if and only if G=pK_2, p 1.

Proof: Suppose G=pK_2, p 1. Then there exists exactly one minimal edge dominating set F containing all edges of G. Let u be the corresponding vertex of F in $E_ed(G)$. Then $V(E_ed(G))$= $E(G)$ $\{u\}$. Since F contains all edges of G, every two vertices of $E_ed(G)$ are adjacent in $E_ed(G)$. Since $V(E_ed(G))$ has

p+1 vertices, $E_ed(G)$ is a complete graph with p+1 vertices.

Conversely suppose $E_ed(G) = K_{p+1}$. Assume $G\ \ pK_2$. Then there exist minimal edge dominating sets F_1 and F_2 in G. Hence the corresponding vertices of F_1 and F_2 in G. Hence the corresponding vertices of F_1 and F_2 are not adjacent in $E_ed(G)$, which is a contradiction. Thus G=pK_2.

Theorem 3 can be stated as given below.

Theorem 4. The semientire edge dominating graph of a graph G is complete if and only if G=pK_2,$p\ \ 1$.

Theorem 5. $E_ed(G) = pK_2$ if and only if G=$K_{1,p}$, $p\ \ 1$ or K_3.

Proof : Suppose $E_ed(G)$=pK_2. On the contrary, assume $G\ \ K_{1,p}\ p\ \ 1$ and $G\ \ K_3$. Then there exists at least one minimal edge dominating set F containing at least two edges e_1 and e_2 of G. By definition, the corresponding vertices of F, e_1, e_2 form a subgraph P_3 in $E_ed(G)$, a contradiction. Hence G=$K_{1,p}$, or K_3.

Conversely, suppose G=$K_{1,p}$. Then each minimal edge dominating set contains exactly one edge $e_i\ 1\ \ i\ \ p$. By definition, it follows that $E_ed(G)$=pK_2. Now suppose G=K_3. Then each minimal edge dominating set contains exactly one edge. Thus $E_ed(G)$=$3K_2$.

Theorem 6. For a graph G without isolated vertices,

$$D_e(G) \quad E_ed(G). \tag{1}$$

Furthermore, equality is attained if and only if every minimal edge dominating set of G contains exactly one edge.

Proof: By Remark 1,

$D_e(G) \quad E_ed(G)$.

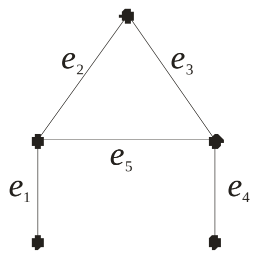
We now prove the second part.

The equality in (1) is attained. Let e_1 and e_2 be any two edges of G. Then the corresponding vertices of e_1 and e_2 in $D_e(G)$ are not adjacent. Thus two edges of G are not in the same minimal edge dominating set. Hence every edge dominating set of G contains exactly one edge.

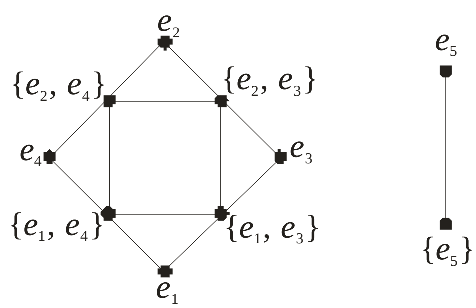
Conversely suppose every minimal edge dominating set of G contains exactly one edge. Then any two edges of G in $E_ed(G)$ are not adjacent. Thus $E_ed(G) \quad D_e(G)$ and since $D_e(G) \quad E_ed(G)$, it implies that the equality in (1) is attained.

The middle edge dominating graph $M_{ed}(G)$ of a graph G is a graph with the vertex set $E\ \ S$ adjacent if $u\ \ E$ and v=F is a minimal edge dominating set of G containing u or u, v are not disjoint minimal edge dominating sets in S, see[5].

In Figure 2, a graph G and its middle edge dominating graph $M_{ed}(G)$ are shown.



G



$M_{ed}(G)$

Figure 2.

From Figure 1 and Figure 2, we have

Remark 7. If G is a graph as shown in Figure 1, then $E_ed(G) = M_{ed}(G)$.

Problem 1. Characterize graphs G for which $E_ed(G) = M_{ed}(G)$.

References

1. F. Harary, *Graph Theory*, Addison Wesley, Reading Mass. (1969).
2. V.R. Kulli, *Theory of Domination in Graphs*, Vishwa International Publications, Gulbarga, India (2010).
3. V.R. Kulli, The edge dominating graph of a graph. In V.R.Kulli, ed., *Advances in Domination Theory I*, Vishwa International Publications, Gulbarga, India, 127-131 (2012).
4. V.R. Kulli, The semientire dominating graph. In V.R. Kulli, ed., *Advances in Domination Theory I*, Vishwa International Publications, Gulbarga, India, 63-70 (2012).
5. V.R. Kulli, The middle edge dominating graph. In V.R. Kulli, editor, *Advances in Domination Theory I*, Vishwa International Publications, Gulbarga, India 153 (2012).
6. Vidya S. Naik, Some Results on Domination in Graph Theory, Ph.D. Thesis, Bangalore University, Bangalore, India (2003).