Ultra Scientist Vol. 25(3)A, 445-452 (2013).

A Common Fixed Point Theorem for five Weakly Compatible Maps Satisfying E.A. Property in Intuitionistic Fuzzy Metric Spaces

GEETA MODI* MONA VERMA** and SUSHMA CHANDEL***

(Acceptance Date 23rd November, 2013)

Abstract

In this paper, we use the notion of E.A. property in intuitionistic fuzzy metric space and prove a common fixed point theorem for five weakly compatible mappings using this property.

Key words: Intuitionistic fuzzy metric space, E.A property, weakly compatible maps.

AMS Subject Classification Codes: 47H10, 54H25

1. Introduction

A fuzzy set was defined by Zadeh [10]. After this during the last few decades many authors have established the existence of a lot of fixed point theorems in fuzzy space. Atanassove[3] introduced and studied the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets. In 2004, Park[7] defined the notion of intuitionistic fuzzy metric space with the help of continuous t-norms and continuous t-conorms. Recently, in 2006, Alaca *et al*.[2] using the idea of Intuitionistic fuzzy sets, defined the notion of intuitionistic fuzzy metric space with the help of continuous t-norm and continuous t- conorms as a generalization of fuzzy metric space due to Kramosil and Michalek[6]. In 2006, Turkoglu[8] proved Jungck's[5] common fixed point theorem in the setting of intuitionistic fuzzy metric spaces for commuting mappings. In this paper, we use the notion of E.A. property in intuitionistic fuzzy metric space and prove a common fixed point theorem for five weakly compatible mappings using this property.

2 Preliminary Notes :

Definition 1.[10] A fuzzy set A in X is a function with domain X and values in $[0, 1]$.

Definition 2.[8] A binary operation $*$: $[0, 1] \times [0, 1]$ $[0, 1]$ is a continuous t-norms if $*$ is satisfying conditions:

(i) $*$ is an commutative and associative;

(ii) $*$ is continuous;
(iii) $a * 1 = a$ for all a [0, 1];
(iv) $a * b$ $c*d$ whenever a c and b d, and a, b, c, d [0, 1].

Definition[8] *3.* A binary operation :[0, 1] × [0, 1] [0, 1] is a continuous t-conorm if it satisfies the following conditions:
(a) is commutative and associative;
(b) is continuous;
(c) a 0 = a for all a [0, 1]
(d) a b = c d whenever a c and b d, for each a, b, c, d [0, 1].

Definition 4.[8] A 5-tuple (X, M, N, *,) is said to be an intuitionistic fuzzy metric space (shortly IFM-Space) if X is an arbitrary set, $*$ is a continuous t-norm is a continuous t-conorm and M, N are fuzzy sets on $X^2 \times (0,\ )$ satisfying the following conditions: for all x, y , z X and s, t > 0;

(IFM-1) M(x, y, t) + N (x, y, t) 1;
(IFM-2) M(x, y, 0) = 0; for all x, y in X
(IFM-3) M(x, y, t) = 1 for all x, y in X and t > 0 if and only if x = y;
(IFM-4) M(x, y, t) = M(y, x, t), for all x, y in X and t > 0
(IFM-5) M(x, y, t) $*$ M (y, z, s) M(x, z, t + s);
(IFM-6) M(x, y, .): [0,) [0, 1] is left continuous;
(IFM-7) $\lim_{t}$ M (x, y, t) =1, for all x, y in X and t > 0,
(IFM-8) N(x, y, 0) = 1; for all x, y in X
(IFM-9) N(x, y, t) = 0 for all x, y in X and t > 0 if and only if x = y;
(IFM-10) N(x, y, t) = N(y, x, t), for all x, y in X and t > 0
(IFM-11) N(x, y, t) N (y, z, s) N (x, z, t + s)
(IFM-12) N(x, y, .): [0,) [0, 1] is right continuous;
(IFM-13) $\lim_{t}$ N(x, y, t) = 0, for all x, y in X and t > 0

Then (M, N) is called an intuitionistic fuzzy metric on X. The functions M (x, y, t) and N(x, y, t) denote the degree of nearness and degree of non-nearness between x and y with respect to t, respectively.

Remark 1. Every fuzzy metric space (X, M, *) is an intuitionistic fuzzy metric space of the form (X, M, 1-M, *,) such that t- norm * and t-conorm are associated that is, x y =1- ((1 - x) * (1 - y)) for any x, y X.

Remark 2.[2] In intuitionistic fuzzy metric space (X, M, N, *,), M(x, y, *) is non-decreasing and N(x, y, is non-increasing for all x, y in X.

Definition 5.[2] Let (X, M, N, *,) be an intuitionistic fuzzy metric space. Then (a) a sequence $\{x_n\}$ in X is said to be Cauchy sequence if, for all t > 0 and p > 0,

$\lim_{n} M(x_{n+p}, x_n, t)=1$ and $\lim_{n} N(x_{n+p}, x_n, t) = 0$:

(b) a sequence $\{x_n\}$ in X is said to be convergent to a point x X if, for all t > 0,

$\lim_{n} M(x_n, x, t) = 1$ and $\lim_{n} N(x_n, x, t) = 0$

(c) (X, M, N, *,) is said to be complete if and only if every Cauchy sequence in X is convergent.

Example 1. Let X = {1/n: n = 1, 2, 3 …} {0} and let * be the continuous t-norm and be the continuous t-conorm defined by a*b = a b and a b = min. {1, a+b} respectively, for all a, b [0, 1]. For each t (0,) and x, y in X, define (M, N) by M(x, y, t) = $\frac{t}{t \ \ |x \ \ y|}$ if t > 0, M(x, y, t)= 0 if t = 0, and N(x, y, t) = $\frac{|x \ \ y|}{t \ \ |x \ \ y|}$ if t > 0, N(x, y, t)= 1 if t = 0. Clearly, (X, M, N, *,) is complete intuitionistic fuzzy metric space.

Definition 6.[9] A pair of self mappings (f, g) of a intuitionistic fuzzy metric space (X, M, N, *,) is said to be commuting if

M(fgx , gfx , t) = 1 and N(fgx , gfx , t) = 0 for all x in X .

Recently, Amari and Moutawakil[1] introduced a generalization of non compatible maps as E.A. property.

Definition 7.[1] Let A and S be two self-maps of a metric space (X, d). The pair (A, S) is Said to satisfy E.A. property, if there exists a sequence {x_n} in X such that
Ax_n = Sx_n = t, for some t in X.

Example 2. Let X = [0, +). Define S, T: X X by Tx = $\frac{x}{4}$ and Sx = $\frac{3x}{4}$, for all x in X. Consider the sequence {x_n} = {1/n}. Clearly $\lim_{n} Sx_n = \lim_{n} Tx_n = 0$. Then S and T satisfy E.A. property.

Example 3. Let X = [2, +). Define S, T: X X by Tx = x+1 and Sx = 2x+1 for all x in X. Suppose that the E.A. property holds. Then, there exists in X, a sequence {x_n} satisfying $\lim_{n} Sx_n = \lim_{n} Tx_n$ = z for some z in X. Therefore, $\lim_{n} x_n$ = z -1 and $\lim_{n} x_n = \frac{z \ \ 1}{2}$. Thus, z = 1, which is a contradiction, is said to satisfy the E.A. property if there exist a sequence {x_n}in X such that since 1 is not contained in X. Hence S and T do not satisfy E.A. property.

Definition 8. A pair of self mappings (S, T) of a intuitionistic fuzzy metric space (X, M, N, *,) is said to satisfy the E.A. property if there exist a sequence {x_n} in X such that $\lim_{n}$ M (Sx_n, Tx_n, t) = 1 and $\lim_{n}$ N (Sx_n, Tx_n, t) = 0.

Example 4. Let X = [0,). Let us consider (X, M, N, *,) be an intuitionistic fuzzy metric space as in example 1. Define T, S: X [0,) by Tx = $\frac{x}{5}$ and Sx = $\frac{2x}{5}$.

Now, $\lim_{n}$ M (Sx_n, Tx_n, t) = 1 and $\lim_{n}$ N (Sx_n, Tx_n, t) = 0

Clearly S and T satisfy E.A. property.

Definition 9. A pair of self mappings (S, T) of a intuitionistic fuzzy metric space (X, M, N, *,) is said to be weakly compatible if they commute at the coincidence points i.e. Su = Tu for some u in X , then STu = TSu. Alaca[2] proved the following result:

Lemma 1. Let (X, M, N, *,) be an intuitionistic fuzzy metric space.If there exists a constant k (0, 1) such that
M(x, y, kt) M (x, y, t), N (x, y, kt) N (x, y, t), for x, y X. Then x = y.

3 Main Results

Theorem 3.1 Let A, B, S, T and P be self mappings of an intuitionistic fuzzy metric space (X, M, N, *,) satisfying[3,4],

(3.1) P(X) AB(X), P(X) ST(X).
(3.2) Pair (P, ST) and (P, AB) are weak compatible
(3.3) Pair (P, ST) or (P, AB) satisfy the property (E.A)
(3.4) for all x, y X and t > 0,
M (Px, Py, qt) min. {M (Px, STx, t), M (Px, ABy, t), M (STx, ABy, t), M(Py, STx, t), M (Py, ABy, t)}
N (Px, Py, qt) max. {N (Px, STx, t), N (Px, ABy, t), N (STx, ABy, t), N (Py, STx, t), N (Py, ABy, t)}

(3.5) PB = BP, PT = TP, AB = BA, ST = TS

Then A, B, S, T, and P have a unique common fixed point in X.

Proof: Case I: Suppose that pair (P, ST) satisfy the property (E.A),
Therefore there exist a sequence $\{x_n\}$ in X such that,

$\lim_{n} Px_n = \lim_{n} STx_n = z$, for some z X … (I)
From (3.1) P(X) AB(X) therefore z AB(X) hence there exist u X such that
z = ABu

Step I: By putting x = x_n and y = u in (3.4) we obtain that

M(Px_n , Pu, qt) min. {M(Px_n, STx_n, t), M(Px_n, ABu, t), M(STx_n , ABu, t), M (Pu, STx_n, t), M (Pu, ABu, t)}
Taking limit as n and using (I) we get that,
M(z , Pu, qt) min. {M(z , z , t), M(z , z, t), M(z, z, t), M(Pu, z , t), M(Pu, z, t)}
M (z, Pu, qt) M (Pu, z, t)...................... (1)

N (Px_n, Pu, qt) max. {N (Px_n, STx_n , t), N(Px_n, ABu, t), N(STx_n , ABu, t), N (Pu, STx_n, t), N (Pu, ABu, t)}
Taking limit as n and using (I) we get that,
N (z, Pu, qt) max. {N(z , z , t), N(z , z, t), N(z , z, t), N(Pu, z , t), N(Pu, z, t)}
N (z, Pu, qt) N (Pu, z, t)................... (2)

From (1) and (2)
Pu = z
Pu = z = ABu, as (P, AB) is weak compatible we get that,
AB (Pu) = P (ABu), i.e ABz = Pz .... (3.6)

Step II: By putting x = x_n and y = z in (3.4) we obtain that
M(Px_n , Pz, qt) min. {M(Px_n, STx_n, t), M(Px_n, ABz, t), M(STx_n , ABz, t), M (Pz, STx_n, t), M (Pz, ABz, t)}
Taking limit as n and using (I) and (3.5) we get that,
M(z , Pz, qt) min. {M(z , z , t), M(z ,Pz, t), M(z , Pz, t), M(Pz, z , t), M(Pz, Pz, t)}
M (z, Pz, qt) M (Pz, z, t)................. (3)

N (Px_n, Pz, qt) max. {N (Px_n, STx_n, t), N(Px_n, ABz, t), N(STx_n , ABz, t), N (Pz, STx_n, t), N (Pz, ABz, t)}
Taking limit as n and using (I) and (3.5) we get that,
N (z, Pz, qt) max. {N(z , z , t), N(z ,Pz, t),

N(z , Pz, t), N(Pz, z , t), N(Pz, Pz, t)}
N (z, Pz, qt)   N (Pz, z, t)................ (4)

From (3) and (4)
Pz = z
Pz = z = ABz .... (3.7)

Step III: As P(X)   ST(X) there exist v   X such that z = Pz = STv.
By putting x = v, y = z in (3.4) we get,
M(Pv, Pz, qt) min.{M(Pv, STv, t), M(Pv, ABz, t), M(STv , ABz, t),
M (Pz, STv, t), M (Pz, ABz, t)}
M(Pv, z, qt)   min. {M(Pv , z , t), M(Pv ,z, t), M(z, z, t), M(z, z, t), M(z, z, t)} M (Pv , z, qt)
M(Pv, z , t) .... (5)

N (Pv, Pz, qt)   max. {N (Pv, STv, t), N (Pv, ABz, t), N (STv, ABz, t),
N (Pz, STv, t), N(Pz, ABz, t)}
N (Pv, z, qt)   max. {N(Pv, z , t), N(Pv, z, t), N(z , z, t), N(z, z , t), N(z, z, t)} N (Pv , z, qt)
N(Pv, z, t) .... (6)
From (5) and (6)
Pv = z
Pv = z = STv .... (3.8)

Now we shall show that Bz = z by (3.4), we have
M(z, Bz, qt) = M (Pz, B (Pz), qt),
= M (Pz, P (Bz), qt)
min. {M (Pz, STz, t), M (Pz, AB (Bz), t), M (STz, AB (Bz), t),
M (P (Bz), STz, t), M (P (Bz), AB (Bz), t)}
min. {M(z, z, t), M(z, Bz, t), M(z, Bz, t), M(Bz, z, t), M(Bz, Bz, t)}
M (z, Bz, qt)   M (z, Bz, t)................ (7)

N (z, Bz, qt) = N (Pz, B (Pz), qt),
= N (Pz, P (Bz), qt)
max. {N (Pz, STz, t), N (Pz, AB (Bz), t), N (STz, AB (Bz), t),
N (P (Bz), STz, t), N (P (Bz), AB (Bz), t)}
max. {N(z, z, t), N(z, Bz, t), N(z, Bz, t), N(Bz, z, t), N(Bz, Bz, t)}
N (z, Bz, qt)   N (z, Bz, t)................ (8)
From (7) and (8)
Bz = z
Then ABz = z = Az
i.e. ABz = z = Az = Bz .... (3.9)

Similarly we can show that Tz = z
M (z, Tz, qt) = M (Pz, T (Pz), qt),
= M (Pz, P (Tz), qt)
min. {M (Pz, STz, t), M (Pz, AB (Tz), t), M (STz, AB (Tz), t),
M (P(Tz), STz, t), M(P(Tz), AB(Tz),t)}
min. {M (z, z, t), M (z, Tz, t), M (z, Tz, t), M (Tz, z, t), M (Tz, Tz, t)}
M (z, Tz, qt)   M (z, Tz, t)................... (9)

N (z, Tz, qt) = N (Pz, T (Pz), qt),
= N (Pz, P (Tz), qt)
max.{N (Pz, STz, t), N (Pz, AB(Tz), t), N (STz, AB (Tz), t),
N(P(Tz), STz, t), N(P (Tz), AB (Tz), t)}
max. {N (z, z, t), N(z, Tz, t), N(z, Tz, t), N (Tz, z, t), N (Tz, Tz, t)}
N (z, Tz, qt)   N (z, Tz, t)................... (10)

From (IX) and (X)
Tz = z
Since STz=z, we have z=STz=Sz=Tz ....(3.10)

Therefore combining (3.8), (3.9), and (3.10) we have,
z = Az = Bz = Sz = Tz = Pz that is z is common fixed point of A, B, S, T, and P.

Case II: Suppose that pair (P, AB)

satisfy the property (E.A),

Therefore There exist a sequence $\{x_n\}$ in X such that,

$\lim_{n} Px_n = \lim_{n} ABx_n = z$, for some z   X …(II)

From (3.1) P(X)   ST(X) therefore z   ST(X) hence there exists u   X such that

z = STu

Step I: By putting x = u, y = x_n in (3.4), we obtain that

M (Pu, Px_n, qt)   min. {M (Pu, STu, t), M (Pu, ABx_n, t), M (STu, ABx_n, t), M (Px_n, STu, t), M (Px_n, ABx_n, t)}

Taking limit as n   and using (II) we get that,

M (Pu, z, qt)   min. {M (Pu, z, t), M (Pu, z, t), M (z, z, t), M (z, z, t), M (z, z, t)} M (Pu, z, qt)   M (Pu, z, t)................ (11)

N (Pu, Px_n, qt)   max. {N (Pu, STu, t), N (Pu, ABx_n, t), N (STu, ABx_n, t), N (Px_n, STu, t), N (Px_n, ABx_n, t)}

Taking limit as n   and using (II) we get that,

N (Pu, z, qt)   max. {N (Pu, z, t), N (Pu, z, t), N (z, z, t), N (z, z, t), N (z, z, t)}N (Pu, z, qt)   N (Pu, z, t)................. (12)

From (11) and (12)

Which gives z = Pu = STu. As (P, ST) is weak compatible we get that

ST (Pu) = P (STu), i.e STz = Pz............ (3.11)

Step II: By putting x = z and y = x_n in (3.4) we obtain that

M(Pz, Px_n, qt)   min. {M(Pz, STz, t), M(Pz, ABx_n, t), M(STz, ABx_n, t), M (Px_n, STz, t), M (Px_n, ABx_n , t)}

Taking limit as n   and using (II) and (3.5) we get that,

M(Pz, z, qt)   min. {M(Pz ,Pz , t), M(Pz ,z, t), M(Pz, z, t), M(z, Pz, t), M(z, z, t)} M (Pz, z, qt)   M (Pz, z, t)................. (13)

N (Pz, Px_n , qt)   max. {N (Pz, STz, t), N (Pz, ABx_n, t), N(STz, ABx_n, t), N (Px_n, STz, t), N (Px_n, ABx_n, t)}

Taking limit as n   and using (II) and (3.5) we get that,

N (Pz, z, qt)   max. {N(Pz ,Pz , t), N(Pz ,z, t), N(Pz , z, t), N(z, Pz , t), N(z, z, t)} N (Pz, z, qt)   N (Pz, z, t).................. (14)

From (13) and (14)

Pz = z

STz = Pz = z .... (3.12)

Step III: As P(X)   AB(X) there exist v   X such that z = Pz = ABv.

By putting x = z, y = v in (3.4) we get,

M(Pz , Pv, qt)   min. {M(Pz , STz , t), M(Pz , ABv, t), M(STz , ABv, t), M (Pv, STz, t), M (Pv, ABv, t)}

M(z, Pv, qt)   min. {M(z, z , t), M(z ,z, t), M(z, z, t), M(Pv, z , t), M(Pv, z, t)} M (z, Pv, qt)   M (Pv, z, t)................. (15)

N(Pz, Pv, qt)   max.{N(Pz , STz , t), N(Pz , ABv, t), N(STz, ABv, t), N (Pv, STz , t), N(Pv, ABv, t)}

N(z, Pv, qt)   max.{N(z, z , t), N(z ,z, t), N(z, z, t), N(Pv, z , t), N(Pv, z, t)} N (z, Pv, qt)   N (Pv, z, t)............... (16)

From (15) and (16)

Pv = z = ABv and weak compatibility of (P, AB) we have

P (ABv) = AB (Pv) i.e Pz = ABz = z......(3.13)

Now we shall show that Bz = z by (3.4) and (3.5), we have

M (z, Bz, qt) = M (Pz, B (Pz), qt),
= M (Pz, P (Bz), qt)
min. {M (Pz, STz, t), M (Pz, AB(Bz), t), M (STz, AB (Bz), t),
M (P (Bz), STz, t), M (P (Bz), AB (Bz), t)}
min. {M(z, z, t), M(z, Bz, t), M(z,Bz, t),M(Bz, z, t), M(Bz, Bz, t)}
M (z, Bz, qt) M (Bz, z, t)............... (17)

N (z, Bz, qt) = N (Pz, B(Pz), qt),
= N (Pz, P(Bz), qt)
max. {N (Pz, STz, t), N (Pz, AB(Bz), t), N (STz, AB (Bz), t),
N (P (Bz), STz, t), N (P (Bz), AB (Bz), t)}
max. {N(z, z, t), N(z, Bz, t), N(z,Bz, t),N(Bz, z, t), N(Bz, Bz, t)}
N (z, Bz, qt) N (Bz, z, t)................. (18)
From (17) and (18)
Bz = z
Then ABz = z = Az
i..e. z = ABz = Az = Bz ....(3.14)

Similarly we can show that Tz = z by (3.4) and (3.5) we have
M (z, Tz, qt) = M (Pz, T (Pz), qt),
= M (Pz, P (Tz), qt)
min. {M (Pz, STz, t), M (Pz, AB (Tz), t), M (STz, AB (Tz), t),
M (P(Tz), STz, t), M(P (Tz), AB (Tz), t)}
min. {M (z, z, t), M (z, Tz, t), M (z, Tz, t), M (Tz, z, t), M (Tz, Tz, t)}
M (z, Tz, qt) M (z, Tz, t)

N (z, Tz, qt) = N (Pz, T (Pz), qt),
= N (Pz, P (Tz), qt)
max. {N (Pz, STz, t), N (Pz, AB(Tz), t), N (STz, AB (Tz), t),
N (P (Tz), STz, t), N(P(Tz), AB(Tz), t)}
max.{N(z, z, t), N(z, Tz, t),N(z, Tz, t), N(Tz, z, t), N(Tz, Tz, t)}
N (z, Tz, qt) N (z, Tz, t)
Tz = z
Since STz = z, we have z = STz=Sz =Tz. (3.15)

Therefore combining (3.13), (3.14), and (3.15) we have,
z = Az = Bz = Sz = Tz = Pz that is z is common fixed point of A, B, S, T, and P.

Uniqueness :

Let z and w be two common fixed point of the maps A, B, S, T and P then z = Az = Sz = Tz = Pz. And w = Aw = Bw = Sw = Tw = Pw. On assuming z w and using (3.4) we get
M (z, w,qt) = M(Pz, Pw, qt)
min. {M (Pz, STw, t), M (Pz, ABw, t), M (STz, ABw, t), M (Pw, STz, t), M (Pw, ABw, t)}
min. {M (z, w, t), M (z, w, t), M (z, w, t), M (w, z, t), M (w, w, t)}
M (z, w, t)

N (z, w,qt) = N(Pz, Pw, qt)
max. {N (Pz, STw, t), N (Pz, ABw, t), N (STz, ABw, t), N (Pw, STz, t), N (Pw, ABw, t)}
max. {N (z, w, t), N (z, w, t), N (z, w, t), N (w, z, t), N (w, w, t)}
N (z, w, t)

By lemma 1 z = w which is contradict our assumption z w. Hence we get z = w and z is unique common fixed point of the maps A, B, S, T and P.

Theorem 3.2 Let A, B, S, T and P be self mappings of an intuitionistic fuzzy metric space (X, M, N, *,) Satisfying,
(3.1) P(X) AB(X), P(X) ST(X).

(3.2) Pair (P, ST) and (P, AB) are weak compatible

(3.3) Pair (P, ST) or (P, AB) satisfy the property (E.A)

(3.4) For all x, y   X and t > 0,

M (Px, Py, qt)   r min. {M (Px, STx, t), M (Px, ABy, t), M (STx, ABy, t), M (Py, STx, t),
M (Py, ABy, t)}

N (Px, Py, qt)   r max. {N (Px, STx, t), N (Px, ABy, t), N(STx, ABy, t), N(Py, STx, t), N (Py, ABy, t)}

(3.5) PB = BP, PT = TP, AB = BA, ST = TS

Where r: [0, 1]   [0, 1] is some continuous function such that r (t) > t, for each
0< t <1.Then A, B, S, T, and P have a unique common fixed point in X.

Proof: The proof follows from Theorem 3.1

Acknowledgements

The authors are thankful to Dr. Bijendra Singh (Professor and Head, School of Studies in Mathematics, Ujjain) for his useful suggestions and valuable comments in modifying the ûrst version of this paper.

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