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On $\hat{\mu}$-Irresolute functions in topological spaces

S. PIOUS MISSIER[1] and E. SUCILA[2]

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Abstract

In this paper, we introduce the concept of $\hat{\mu}$-irresolute maps, strongly $\hat{\mu}$-continuous maps, perfectly $\hat{\mu}$-continuous maps, totally $\hat{\mu}$-continuous maps and contra $\hat{\mu}$-continuous maps in topological spaces and their properties are studied.

Key words : $\hat{\mu}$-irresolute map, strongly $\hat{\mu}$-continuous map, perfectly $\hat{\mu}$-continuous map, totally $\hat{\mu}$-continuous map, contra $\hat{\mu}$-continuous map.

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1. Introduction

Irresolute maps are introduce and studied by Crossley and Hildebrand[1]. Dontchev[2] introduced the notion of contra continuity in 1996. Dontchev and Noiri[3] introduced contra-semicontinuous functions in 1999. Jafari and Noiri[4] introduced contra α-continuous functions between topological spaces.

M.K.R.S.Veerakumar[13] introduced μ-closed sets in topological spaces. Recently author[11,12] introduced $\hat{\mu}$-closed sets and $\hat{\mu}$-continuity in topological spaces. In this paper we introduce $\hat{\mu}$-irresolute maps, strongly $\hat{\mu}$-continuous maps, perfectly $\hat{\mu}$-continuous maps, totally $\hat{\mu}$-continuous maps and contra $\hat{\mu}$-continuous maps in topological spaces and their properties.

2. Preliminaries :

Throughout this paper, we consider spaces on which no separations are assumed unless explicity stated. For $A \subset X$, the closure and interior of A is denoted by cl(A) and int(A) respectively. The complement of A is denoted by A^C, the power set of X is denoted by P(X).

Definition : 2.1

A subset A of a topological space (X,τ) is called:

1. a preopen set[8] if $A \subseteq$ int(cl(A)) and preclosed if cl(int(A)) $\subseteq$ A.
2. a semiopen set[6] if $A \subseteq$ cl(int(A)) and a semiclosed set if int(cl(A)) $\subseteq$ A.
3. an α-open set[9] if $A \subseteq$ int(cl(int(A))) and α-closed set if cl(int(cl(A))) $\subseteq$ A.

The intersection of all α-closed (resp. semiclosed) subsets of (X,τ) containing a subset A of X is called α-closure (resp. semiclosure) of A is denoted by αcl(A) (resp.scl(A)).

Definition : 2.2

A subset A of a topological space (X, τ) is called

1. a $g\alpha^*$-closed set[7] if αcl(A) $\subseteq$ int(U) whenever $A \subseteq U$ and U is α-open in (X,τ). The complement of $g\alpha^*$-closed set is called $g\alpha^*$-open set.
2. a μ-closed set[13] if cl(A)$\subseteq$ U whenever A $\subseteq$ U and U is $g\alpha^*$-open in (X, τ). The complement of μ-closed is called μ-open.
3. a $\hat{\mu}$-closed set[11] if scl(A) U whenever $A \subseteq U$ and U is μ-open in (X,τ). The complement of $\hat{\mu}$-closed set is called $\hat{\mu}$-openset.

Notation : 2.3

1. The class of $\hat{\mu}$-closed subsets of (X, τ) is denoted by $c(X, \tau)$[11].
2. The class of $\hat{\mu}$-open subsets of (X, τ) is denoted by $\hat{\mu}\,o(X,\tau)$[11].
3. For every set $E \subset X$, $\hat{\mu}$-closure of E to be the intersection of all $\hat{\mu}$-closed sets containing E, In symbols, $\hat{\mu}\,cl(E) = \cap \{A : E \subset A, A \in \hat{\mu}\,c(X, \tau)\}$[12].
4. For every set $E \subset X$, $\hat{\mu}$-interior of E to be the union of all $\hat{\mu}$-open sets contained in E. In symbols, $\hat{\mu}\,int(E) = \cup \{A : A \subset E$ and $A \in \hat{\mu}\,o(X, \tau)\}$[12].

Definition : 2.4

A map $f : (X, \tau) \to (Y, \sigma)$ is

(i) strongly continuous[5] if $f^{-1}(V)$ is both open and closed in (X, τ) for each subset V of (Y, σ).

(ii) perfectly continuous[10] if $f^{-1}(V)$ is both open and closed in (X, τ) for each open set V of (Y, σ).

(iii) contra continuous[2] if $f^{-1}(V)$ is closed in (X,τ) for each open subset V of (Y, σ).

(iv)contra α-continuous[4] if $f^{-1}(V)$is α- closed in (X, τ) for each open subset V of (Y, σ).

(v) contra semi continuous[3] if $f^{-1}(V)$ is semiclosed in (X, τ) for each open subset V of (Y,σ).

(vi) μ-continuous[13] if $f^{-1}(V)$ is μ-closed in (X, τ) for every closed set V of (Y, σ).

(vii) $\hat{\mu}$-continuous[12] if $f^{-1}(V)$ if $\hat{\mu}$-closed in (X, τ) for every closed set V of (Y,σ).

Definition 2.5:

A topological space (X, τ) is a

(i) $T\hat{\mu}$ space[11] if every $\hat{\mu}$-closed subset of (X,τ) is closed in (X,τ).

(ii) $\mu T\hat{\mu}$ space[11] if every $\hat{\mu}$-closed subset of (X,τ) is μ-closed in (X,τ).

3. $\hat{\mu}$- *Irresolute maps :*

We introduce the following definition.

Definition 3.1 :

A map $f : X \to Y$ is called $\hat{\mu}$-irresolute if $f^{-1}(V)$ is $\hat{\mu}$-closed in X for every $\hat{\mu}$-closed set V in Y.

Example 3.2 :

Let $X = Y = \{a, b, c\}$, $\tau = \{x, \phi, \{a\}, \{b, c\}\}$ and $\sigma = \{Y, \phi, \{a\}, \{a, c\}\}$. Clearly the identity map $f : X \to Y$ is $\hat{\mu}$- irresolute, since every subset of X is $\hat{\mu}$-closed in X.

Proposition 3.3 :

If $f : X \to Y$ is $\hat{\mu}$-irresolute, then f is $\hat{\mu}$-continuous but not conversely.

Proof :

Since every closed set is $\hat{\mu}$-closed, we get the proof.

Example 3.4 :

Let $X = Y = \{a, b, c\}$, $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$ and $\sigma = \{Y, \phi, \{a\}, \{b, c\}\}$.

Here $\hat{\mu}c(X, \tau) = P(X) - \{a, b\}$ and $\hat{\mu}c(X, \tau) = P(X)$. Clearly the identity map $f : X \to Y$ is $\hat{\mu}$-continuous but not $\hat{\mu}$-irresolute, since $\{a, b\}$ is $\hat{\mu}$-closed in Y but $f^{-1}(\{a, b\}) = \{a, b\}$ is not $\hat{\mu}$-closed in X.

Proposition 3.5 :

A map $f : X \to Y$ is $\hat{\mu}$-irresolute if and only if $f^{-1}(V)$ is $\hat{\mu}$-open in X for every $\hat{\mu}$-open set V of Y.

Proof :

Let $f : X \to Y$ be $\hat{\mu}$-irresolute and V be an $\hat{\mu}$-open set in Y. Then $f^{-1}(V^C)$ is $\hat{\mu}$-closed in X. But $f^{-1}(V^C) = (f^{-1}(V))^C$ and so $f^{-1}(V)$ is $\hat{\mu}$-open in X. Converse is similar.

Proposition 3.6 :

Let $f : X \to Y$ and $g : Y \to Z$ be any two maps. Then

a) $g \circ f$ is $\hat{\mu}$-irresolute if both f and g are $\hat{\mu}$-irresolute.

b) $g \circ f$ is $\hat{\mu}$-continuous if g is $\hat{\mu}$-continuous and f is $\hat{\mu}$-irresolute.

Proof (a) :

Let F be $\hat{\mu}$-closed set in Z. Since g is $\hat{\mu}$-irresolute, $g^{-1}(F)$ is $\hat{\mu}$-closed in Y. Since f is $\hat{\mu}$-irresolute, $f^{-1}(g^{-1}(F)) = (g \circ f)^{-1}(F)$ is $\hat{\mu}$-closed in X. Thus $g \circ f$ is $\hat{\mu}$-irresolute.

Proof (b) :

Let F be closed set in Z. Since g is $\hat{\mu}$-continuous, $g^{-1}(F)$ is $\hat{\mu}$-closed in Y. Since f is $\hat{\mu}$-irresolute, $f^{-1}(g^{-1}(F)) = (g \circ f)^{-1}(F)$ is $\hat{\mu}$-closed in X. Thus $g \circ f$ is $\hat{\mu}$-continuous.

Proposition 3.7 :

Let X be a topological space, Y be a $T\hat{\mu}$-space and $f : X \to Y$ be a map. Then the following are equivalent.

(i) f is $\hat{\mu}$-irresolute.

(ii) f is $\hat{\mu}$-continuous.

Proof :

(i) $\Rightarrow$ (ii) : Follows from Proposition 3.3

(ii) $\Rightarrow$ (i) : Let F be a $\hat{\mu}$-closed in Y. Since Y is T$\hat{\mu}$-space, F is a closed set in Y and by hypothesis, $f^{-1}(F)$ is $\hat{\mu}$-closed in X. Therefore f is $\hat{\mu}$-irresolute.

Theorem 3.8 :

Let $f : X \rightarrow Y$ be $\hat{\mu}$-irresolute. Then f is μ-continuous if X is a μT$\hat{\mu}$-space.

Proof :

Let F be a closed subset of Y. Since every closed set is $\hat{\mu}$-closed, F is $\hat{\mu}$-closed in Y. Since f : X Y is $\hat{\mu}$-irresolute, $f^{-1}(F)$ is $\hat{\mu}$-closed in X. Also X is a T$\hat{\mu}$-space, we get $f^{-1}(F)$ is μ-closed in X.

4. Strongly $\hat{\mu}$- Continuous, Perfectly $\hat{\mu}$-Continuous and Totally $\hat{\mu}$-Continuous :

Definition 4.1 :

A map $f : X \rightarrow Y$ is said to be strongly $\hat{\mu}$-continuous if the inverse image of every $\hat{\mu}$-open set of Y is open in X.

Proposition 4.2 :

If a map $f : X \rightarrow Y$ is strongly $\hat{\mu}$-continuous, then f is continuous but not conversely.

Proof :

Since every open set is -open, we get the proof.

Example 4.3 :

Let X = Y = {a, b, c}, τ = {X, ϕ, {c}, {b, c}} and σ = {Y, ϕ, {b,c}}. Let $f : x \rightarrow Y$ be an identity map. Then f is continuous but not strongly $\hat{\mu}$-continuous, since for the $\hat{\mu}$-open set {b} in Y, $f^{-1}(\{b\}) = \{b\}$ is not open in X.

Proposition 4.4 :

Let X be a topological space, Y be a T$\hat{\mu}$-space and $f : X \rightarrow Y$ be a map. Then the following are equivalent.

(i) f is strongly $\hat{\mu}$-continuous.

(ii) f is continuous.

Proof : (i) $\Rightarrow$ (ii) Follows from Proposition 4.2

(ii) $\Rightarrow$ (i). Let V be $\hat{\mu}$-open in Y. Since Y is a T$\hat{\mu}$-space, V is open in Y. By (ii) $f^{-1}(V)$ is open in X. Therefore f is strongly $\hat{\mu}$-continuous.

Proposition 4.5 :

If a map $f: X \rightarrow Y$ is strongly continuous, then f is strongly $\hat{\mu}$-continuous.

Proof : Follows from definitions.

The following example supports that the converse of the Proposition 4.5 is not true.

Example 4.6 :

Let X = Y = {a, b, c}, τ = {X, ϕ, {c}, {a, c}, {b, c}} and σ = {Y, ϕ, {c}, {a, c}}. Let f be an identity map. Clearly f is strongly $\hat{\mu}$-continuous but not strongly continuous. Since $f^{-1}(\{c\}) = \{c\}$ which is open but not closed in X.

Definition 4.7 :

A map $f : X \rightarrow Y$ is called perfectly $\hat{\mu}$-continuous if the inverse image of every $\hat{\mu}$-open set in Y is both open and closed in X.

Proposition 4.8 :

If a map $f : X \to Y$ is perfectly $\hat{\mu}$-continuous then f is perfectly continuous (resp. continuous) but not conversely.

Proof :

Let V be an open set in Y. Then V is $\hat{\mu}$-open in Y. Since f is perfectly $\hat{\mu}$-continuous, $f^{-1}(V)$ is both open and closed in X. Thus f is perfectly continuous and also continuous.

Example 4.9 :

Let $X = Y = \{a, b, c\}$, $\tau = \{X, \phi, \{b\}, \{a, c\}\}$ and $\sigma = \{Y, \phi, \{b\}\}$.

Here $\hat{\mu}o(Y, \sigma) = \{X, \phi, \{b\}, \{b, c\}, \{a, b\}\}$. Clearly the identity map $f : X \to Y$ is perfectly continuous and continuous but not perfectly $\hat{\mu}$-continuous. Since the set $\{a, b\}$ is $\hat{\mu}$-open in Y but $f^{-1}(\{a, b\}) = \{a, b\}$ is neither open nor closed in X.

Proposition 4.10 :

If $f : X \to Y$ is perfectly $\hat{\mu}$-continuous then it is strongly $\hat{\mu}$-continuous but not conversely.

Proof :

Similar to the proof of the Proposition 4.8.

Example 4.11 :

Let (X, τ) and (Y, σ) be defined as in Example 4.6, then f is strongly $\hat{\mu}$-continuous but not perfectly $\hat{\mu}$-continuous. Since the set $\{a, c\}$ is -open in Y and $f^{-1}(\{a, c\}) = \{a, c\}$ is open but not closed in X.

From the above discussions we have the following implications. $A \to B$ represents A implies B but not conversely.

$$\begin{tikzcd}
\text{Perfectly Continuous} & \text{Perfectly } \hat{\mu}\text{-Continuous} \arrow[l] \arrow[r] \arrow[d] & \text{Strongly } \hat{\mu} \text{ Continuous} \\
& \text{Continuous} & \text{Strongly Continuous} \arrow[l] \arrow[u]
\end{tikzcd}$$

Proposition 4.12 :

Let X be a discrete topological space, Y be any topological space and $f : X \to Y$ be a map. Then the following are equivalent.

(i) f is perfectly $\hat{\mu}$-continuous.

(ii) f is strongly $\hat{\mu}$-continuous.

Proof : (i) $\Rightarrow$ (ii) : Follows from Proposition 4.10

(ii) $\Rightarrow$ (i) : Let V be any $\hat{\mu}$-open set in Y. By hypothesis $f^{-1}(V)$ is open in X. Since X is a discrete space, $f^{-1}(V)$ is also closed in X and so f is perfectly $\hat{\mu}$-continuous.

Proposition 4.13 :

If $f : X \to Y$ and $g : Y \to Z$ are perfectly $\hat{\mu}$-continuous then their composition $gf : X \to Z$ is also perfectly $\hat{\mu}$-continuous.

Proof :

Let V be a $\hat{\mu}$-open set in Z. Since g is perfectly $\hat{\mu}$-continuous, $g^{-1}(V)$ is both open and closed in Y. Also f is perfectly $\hat{\mu}$-continuous $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is both open and closed in X. Thus $g \circ f$ is perfectly $\hat{\mu}$-continuous.

Proposition 4.14 :

Let $f : X \to Y$ and $g : Y \to Z$ be any two maps. Then their composition $g \circ f : X \to Z$ is

(i) $\hat{\mu}$-irresolute if g is perfectly $\hat{\mu}$-continuous and f is $\hat{\mu}$-continuous.
(ii) strongly $\hat{\mu}$-continuous if g is perfectly $\hat{\mu}$-continuous and f is continuous.
(iii) perfectly $\hat{\mu}$-continuous if g is strongly continuous and f is perfectly $\hat{\mu}$-continuous.

Proof :
Similar to the proof of the Proposition 4.13.

Definition 4.15:
A map $f : X \to Y$ is said to be totally $\hat{\mu}$-continuous if the inverse image of every open subset of Y is $\hat{\mu}$-clopen subset of X.

Example 4.16 :
Let $X = Y = \{a, b, c\}$, $\tau = \{X, \phi, \{a\}, \{b, c\}\}$ $\sigma = \{Y, \phi, \{a\}\}$.

Here $\hat{\mu}c(X, \tau) = P(X)$. Define a map $f : X \to Y$ by $f(a) = b$ $f(b) = c$ and $f(c) = a$. Clearly f is totally $\hat{\mu}$-continuous.

Proposition 4.17 :
Every perfectly $\hat{\mu}$-continuous map is totally $\hat{\mu}$-continuous.

Proof :
Let $f : X \to Y$ be perfectly -continuous map. Let V be an open set in Y. Then V is $\hat{\mu}$-open in Y. Since f is perfectly $\hat{\mu}$-continuous, $f^{-1}(V)$ is clopen in X implies that $f^{-1}(V)$ is $\hat{\mu}$-clopen in X.

The following example shows that the converse of the above Proposition 4.17 is not true.

Example 4.18 :
Let (X, τ) and (Y, σ) be defined as in Example 4.16, then $f : (X, \tau) \to (Y, \sigma)$ is totally $\hat{\mu}$-continuous but not perfectly $\hat{\mu}$-continuous. Since the set $\{a\}$ is $\hat{\mu}$-open in (Y, σ) but $f^{-1}(\{a\}) = \{c\}$ is not clopen in (X, τ).

The following two examples shows that totally $\hat{\mu}$-continuous and strongly $\hat{\mu}$-continuous are independent[5].

Example 4.19 :
Let (X, τ) and (Y, σ) be defined as in Example 4.16, then $f : (X, \tau) \to (Y, \sigma)$ is totally $\hat{\mu}$-continuous but not strongly -continuous, since the set $\{a\}$ is $\hat{\mu}$-open in (Y, σ) but $f^{-1}(\{a\}) = \{c\}$ is not open in (X, τ).

Example 4.21 :
Let $X = Y = \{a, b, c\}$, $\tau = \{X, \phi, \{a\}, \{a, b\}, \{a, c\}\}$ and $\sigma = \{Y, \phi, \{a\}\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be an identity map. Then f is strongly $\hat{\mu}$-continuous but not totally $\hat{\mu}$-continuous. Since the set $\{a\}$ is open in Y but $f^{-1}(\{a\}) = \{a\}$ which is $\hat{\mu}$-open but not $\hat{\mu}$-closed in X.

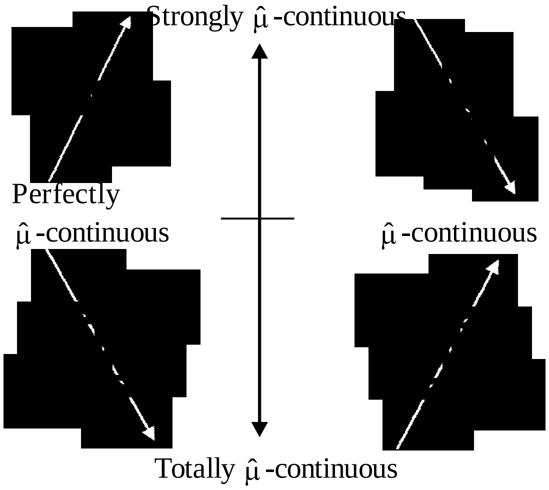
Proposition 4.22 :
Every totally $\hat{\mu}$-continuous map is $\hat{\mu}$-continuous.

Proof :
Clearly follows from definitions.
The following example supports that the converse of the Proposition 4.22 is not true.

Example 4.23 :
Let $X = Y = \{a, b, c\}$, $\tau = \{X, \phi, \{a\}, \{c\}, \{a, c\}\}$ and $\sigma = \{Y, \phi, \{b, c\}\}$. Define a

map $f : X \to Y$ by $f(a) = c$, $f(b) = a$ and $f(c) = b$. Here the map f is $\hat{\mu}$-continuous but not totally $\hat{\mu}$-continuous, since for the open set $\{b, c\}$ in Y, $f^{-1}(\{b,c\})=\{a, c\}$ which is $\hat{\mu}$-open but not $\hat{\mu}$-closed in X.

From the above Propositions and examples we have the following diagram. $A \to B$ (resp. A $\nleftrightarrow$ B) represents A implies B but not conversely (resp. A and B are independent of each others).



Remark 4.24 :

It is clear that the totally $\hat{\mu}$-continuous map is stronger than $\hat{\mu}$-continuous map and weaker than perfectly $\hat{\mu}$-continuous map.

5. *Contra $\hat{\mu}$-Continuous :*

Definition 5.1 :

A map $f : X \to Y$ is called contra $\hat{\mu}$-continuous if $f^{-1}(V)$ is $\hat{\mu}$-closed in X for each open set V of Y.

Proposition 5.2 :

If $f : X \to Y$ is contra continuous, then it is contra $\hat{\mu}$-continuous.

Proof :

The proof follows from the fact that every closed set is $\hat{\mu}$-closed.

The following example shows that the converse of the Proposition 5.2 is not true.

Example 5.3 :

Let $X = Y = \{a, b, c\}$, $\tau = \{X, \phi, \{b\}, \{a, c\}\}$ and $\sigma = \{X, \phi, \{a\}\}$. Here $\hat{\mu}c(X, \tau) = P(X)$. Let $f : X \to Y$ be an identity map. Here the map f is contra $\hat{\mu}$-continuous but not contra–continuous, since for the open set $\{a\}$ in Y, $f^{-1}(\{a\}) = \{a\}$ is not closed in X.

Proposition 5.4 :

If $f : X \to Y$ is contra α - continuous (resp. contra semi –continuous) then f is contra $\hat{\mu}$-continuous but not conversely.

Proof :

The proof follows from the fact that every α -closed (resp. semiclosed) set is $\hat{\mu}$-closed.

Example 5.5 :

Let (X, τ) and (Y, σ) be defined as in Example 5.3, then f is contra $\hat{\mu}$-continuous but it is neither contra α-continuous nor contra semi-continuous. Since for the open set $\{a\}$ in Y, $f^{-1}(\{a\}) = \{a\}$ is neither α-closed nor semiclosed in X.

Proposition 5.6 :

Let X be a $T\hat{\mu}$-space. If a map f : X

Y is contra $\hat{\mu}$-continuous, then f is contra continuous.

Proof :

The proof follows from definitions.

Remark 5.7 :

The following two examples shows that the contra $\hat{\mu}$-continuity and continuity are independent.

Example 5.8 :

Let (X, τ) and (Y, σ) be defined in Example 5.3 is contra $\hat{\mu}$-continuous, but not continuous, since for the open set {a} in Y, $f^{-1}(\{a\}) = \{a\}$ is not open in X.

Example 5.9 :

Let X = Y = {a, b, c}, $\tau = \{X, \phi, \{a\}, \{a, c\}\}$ and $\sigma = \{Y, \phi, \{c\}\}$. Here $\hat{\mu}c(X, \tau) = \{X, \phi, \{b\}, \{c\}, \{b, c\}\}$. Define a map f : X $\rightarrow$ Y by f(a) = c, f(b) = a and f(c) = b. Here the map f is continuous but not contra $\hat{\mu}$-continuous, since for the open set {c} in Y, $f^{-1}(\{c\}) = \{a\}$ is not $\hat{\mu}$-closed in X.

Proposition 5.10 :

If f : X $\rightarrow$ Y is contra -continuous map and g : Y Z is a continuous map then gf : X $\rightarrow$ Z is contra $\hat{\mu}$-continuous.

Proof :

Clearly follows from definitions.

Remark 5.11 :

The following example shows that the composition of two contra $\hat{\mu}$-continuous maps is not contra $\hat{\mu}$-continuous map.

Example 5.12 :

Let X = Y = Z = {a, b, c}, $\tau = \{X, \phi, \{a\}, \{a, c\}\}$, $\sigma = \{Y, \phi, \{a\}\}$ and $\eta = \{Z, \phi, \{b\}\}$. Define a map f : (X, τ) $\rightarrow$ (Y, σ) by f(a) = b, f(b) = c and f(c) = a and g : (Y, σ) $\rightarrow$ (Z, η) is the identity map. Here $\hat{\mu}c(X, \tau) = \{X, \phi, \{b\}, \{c\}, \{b, c\}\}$ and $\hat{\mu}c(Y, \sigma) = \{Y, \phi, \{b\}, \{c\}, \{b, c\}\}$. Clearly f and g are contra $\hat{\mu}$-continuous maps but $g \circ f$:(X, τ) $\rightarrow$ (Z, η) is not a contra $\hat{\mu}$-continuous map, since for the open set {b} in Z, $(g \circ f)^{-1}(b) = f^{-1}(g^{-1}(b)) = f^{-1}(b) = \{a\}$ is not -closed in X.

Lemma 5.13 :

The following properties hold for subsets A and B of a space X.

i) $x \in ker(A)$ if and only if $A \cap F \neq \phi$ for any closed set F of X containing x.

ii) $A \subset ker(A)$ and A = ker(A) if A is open in X.

iii) If $A \subset B$, then $ker(A) \subset ker(B)$.

Theorem 5.14 :

Assume that $\hat{\mu}o(X, \tau)$ is closed under arbitrary union. Then for a map f : X Y, the following are equivalent.

i) f is contra $\hat{\mu}$-continuous;

ii) for every closed subset F of Y, $f^{-1}(F) \in \hat{\mu}o(X, \tau)$;

iii) for each $x \in X$ and each closed set F of

Y containing f(x), there exists U $\hat{\mu}$o(X, τ) and x $\in$ U such that f(U) $\subset$ F;

iv) f($\hat{\mu}$cl(A)) $\subset$ kerf(A), for every subset A of X;

v) $\hat{\mu}$cl(f^{-1}(B))$\subset$ f^{-1}(ker(B)), for every subset B of Y.

Proof :

(i) $\Rightarrow$ (ii) : Obvious

(ii) $\Rightarrow$ (iii) : Let x be any point of X and F be any closed set of Y containing f(x). By (ii), f^{-1}(F) $\in$ $\hat{\mu}$o(X, τ) and x $\in$ f^{-1}(F) put U = f^{-1}(F). Then U $\in$ $\hat{\mu}$o(X, τ) and f(U) $\subset$ F.

(iii) $\Rightarrow$ (ii) : Let F be any closed set of Y and x $\in$ f^{-1}(F). Then f(x) $\in$ F and there exists U_x $\in$ $\hat{\mu}$o(X, τ) and x $\in$ U_x such that f(U_x) $\subset$ F. Therefore, by hypothesis f^{-1}(F) = $\cup$\{U_x/ x $\in$ f^{-1}(F)\} $\in$ $\hat{\mu}$o(X, τ).

(ii) $\Rightarrow$ (iv) : Let A be any subset of X. Suppose that y $\notin$ ker(f(A)). Then by Lemma 5.13, there exists a closed set F of Y containing y such that f(A) $\cap$ F =ϕ. Thus we have A $\cap$ f^{-1}(F) = ϕ and $\hat{\mu}$cl(A) $\cap$ f^{-1}(F) = ϕ. Therefore, we obtain f($\hat{\mu}$cl(A))$\cap$F=ϕ and y $\notin$ f($\hat{\mu}$cl(A)). This implies that f($\hat{\mu}$cl(A)) $\subset$ kerf (A)

(iv) $\Rightarrow$ (v) : Let B be any subset of Y. By (v) and Lemma 5.13, we have f($\hat{\mu}$cl f^{-1}(B)) $\subset$ kerf (f^{-1}(B)) $\subset$ ker(B) and $\hat{\mu}$cl(f^{-1}(B)) $\subset$ f^{-1} (ker(B)).

(v) $\Rightarrow$ (i) : Let V be any open subset of Y. Then by (v) and Lemma 5.13, we have $\hat{\mu}$cl(f^{-1}(V)) $\subset$ f^{-1}(ker(V)) = f^{-1}(V). Thus f^{-1}(V) = $\hat{\mu}$cl(f^{-1}(V)). This shows that f^{-1}(V) is $\hat{\mu}$-closed in X by Proposition 4.10 on -continuous functions in topological spaces[12].

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