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Efficient Complementary Perfect Triple Connected Domination Number of a Graph

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Abstract

In this paper we introduce new domination parameter called efficient complementary perfect triple connected domination number of a graph. A subset S of V of a nontrivial graph G is said to be an efficient complementary perfect triple connected dominating set, if S is a complementary perfect triple connected dominating set and every vertiex is dominated exactly once. The minimum cardinality taken over all efficient complementary perfect triple connected dominating sets is called the efficient complementary perfect triple connected domination number and is denoted by γ_{ecpt}. We investigate this number for some standard graphs. We also investigate its relationship with other graph theoretical parameters.

Key words: Complementary perfect triple connected domination number, Efficient complementary perfect triple connected domination number.

AMS (2010): *05C69*

Real Life Application of Efficient Complementary Perfect Triple Connected Domination Number

Application 1

People from developing countries

move to developed countries in search of employment. But now many Information Technology (IT) projects have been planted in developing countries which provide bread for many. These IT plants are in mofusil areas and even graduates from cities travel a lot for employment these days. Also, people in IT plants work overnight and good food had become their basic need. Hence it is essential to have triple connection between Information Technology, Automobile Industry and Catering to facilitate humans which inturn provide employment for a mass. We too connect places to this triple network so that these places share all the facilities available by means of landways. Also, we link only one IT plant to the pairs of areas needed so that the cost efficiency is also managed.

Example:
Consider the following figure 1(a).

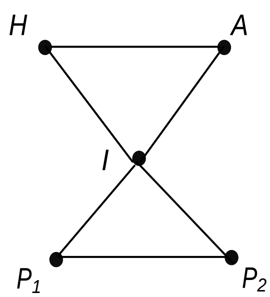


Fig. 1(a)

Let I, A and H represent Information Technology plant, Automobile industry and Hotel respectively. People from the places P_1 and P_2 move to the IT plant in search of employment and they make use of the transport and food facility which are in connection. Here we can connect as much places as we can to the triple connected network and facilitate people as per the requirement of the IT plant as shown below in figure 1(b).

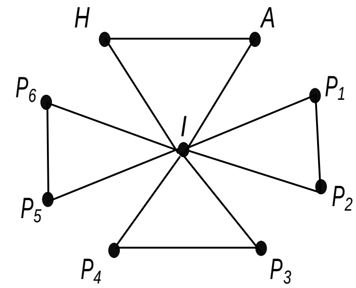


Fig. 1(b)

Since the network is connected to one IT plant, the cost factor is also reduced.

Application 2

Villages are backbone of India and a lot of industries and factories over the country run because of these villages. Harvesting is the major work carried on in the villages and the cultivated products are supplied to the factories. After manufacture, the extract from the factories are send to co- generation systems, most of which generate electric power and heat and soil meal too; wherein we can use this electric power and heat in the factory again and the soil meal in the field. Hence it will be of great use if we connect the harvesting field, the factory and the co-generation system. To this triple connection, we can connect distributors of the product and a network between the pairs distributors help to share the things on need based.

Example :

Consider the following figure 2(a).

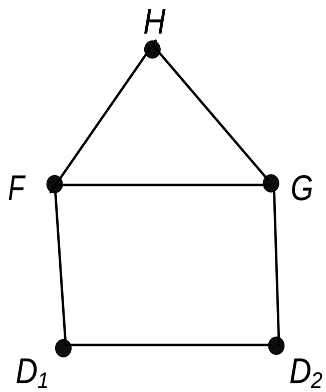


Fig. 2(a)

F - Factory
H - Harvesting Field
G - Co-gen
D_1 - Distributor 1
D_2 - Distributor 2

The sugar needed are brought from the cultivated area and the sugar production is carried on. For each 10 tonnes of sugarcane crushed, a sugar factory produces nearly 3 tonnes of wet baggase. Baggase is the fibrous matter that remains after sugarcane stalks are crushed to extract their juice. The high moisture content of bagasse, typically 40 to 50 %, is detrimental to its use as fuel and bagasse is often used as a primary fuel source of sugar mills. When burned in quantity, it produces sufficient heat energy to supply all the needs of a typical sugar mill, with energy to spare. Hence it is vital to link the cane production place, sugar factory and the Co-gen system which turns the bagasse into useful heat energy. The pairs of distributors of the produced goods are also connected in the network so that they distribute the goods through land or air transport as per the requirement.

Application 3 :

In the developing world, it is always necessary to have the forces in the borders to protect the country from foreign force, especially for country like India where we have problems in the border. It is vital to have the three forces namely army, navy and airforce to be connected. During critical situation to rescue the country we are in need of strong force and unite all these forces together. Instead of connecting a city to these three forces we can very well connect the pairs of cities and share the forces between the cities whenever necessary which will minimize the cost of man power and weapons too. This case is nothing but efficient complementary perfect triple connected domination number of a graph. This concept can be applied to many real life situations wherein we need of triple connection.

Example :

Consider the following problem. Let A, N, F in the following graph be the three forces namely army, navy and air force. Let S and V be the two cities in the border which are to be protected by the forces as shown in figure 3(a).

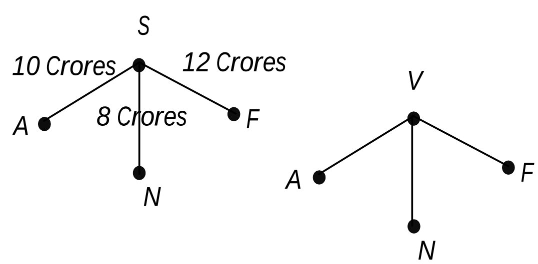


Fig. 3(a)

Suppose we need to spend 10 crores for army, 8 crores for navy and 12 crores for air force in effect to protect a city from external defence. Hence totally we are in need of 60 crores to have the three forces at two cities. Instead, we can connect the cities by a network and share the forces between the cities. By doing the above said, we get the following figure 3(b).

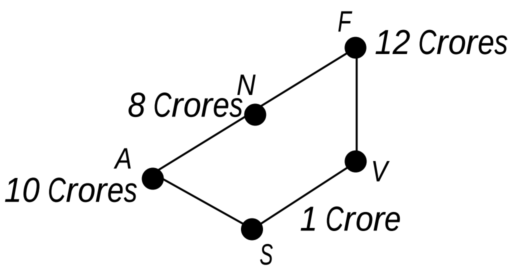


Fig. 3(b)

Here the three forces are in triple connection and the cities are also connected to share the force which will reduce 29 crores of rupees in the problem considered. This is nothing but efficient complementary perfect triple connected domination number of the graph in consideration.

1. Introduction

By a ***graph*** we mean a finite, simple, connected and undirected graph *G(V, E)*, where *V* denotes its vertex set and *E* its edge set. Unless otherwise stated, the graph *G* has *p* vertices and *q* edges. ***Degree*** of a vertex *v* is denoted by *d(v)*, the ***maximum degree*** of a graph *G* is denoted by $\Delta(G)$. A graph *G* is ***connected*** if any two vertices of *G* are connected by a path. A maximal connected subgraph of a graph *G* is called a ***component*** of *G*. The number of components of *G* is denoted by $\omega(G)$. The ***complement*** $\bar{G}$ of *G* is the graph with vertex set *V* in which two vertices are adjacent if and only if they are not adjacent in *G*. The **Cartesian graph product** $G = G_1 \square G_2$, sometimes simply called "the" graph product of graphs G_1 and G_2 with disjoint point sets V_1 and V_2 and edge sets X_1 and X_2 is the graph with point set $V_1 \times V_2$ and $u = (u_1, u_2)$ adjacent with $v = (v_1, v_2)$ whenever $[u_1 = v_1 \text{ and } u_2 \text{ adj } v_2]$ or $[u_2 = v_2 \text{ and } u_1 \text{ adj } v_1]$ We denote a ***cycle*** on *p* vertices by C_p, a ***path*** on *p* vertices by P_p, and a ***complete graph*** on *p* vertices by K_p. A ***wheel graph*** W_p of order *p*, sometimes simply called an p-wheel, is a graph that contains a cycle of order p-1, and for which every vertex in the cycle is connected to one other vertex. A ***tree*** is a connected acyclic graph. A ***bipartite graph*** (or ***bigraph***) is a graph whose vertex set can be divided into two disjoint sets V_1 and V_2 such that every edge has one end in V_1 and another end in V_2. A ***complete bipartite graph*** is a bipartite graph where every vertex of V_1 is adjacent to every vertex in V_2. The complete bipartite graph with partitions of order $|V_1|=m$ and $|V_2|=n$, is denoted by $K_{m,n}$. A ***star***, denoted by $K_{1,p-1}$ is a tree with one root vertex and p – 1 pendant vertices. The ***gear graph*** G_p, also sometimes known as a bipartite wheel graph, is a wheel graph with a graph vertex added between each pair of adjacent vertices of the outer cycle. The ***n–ladder graph*** can be defined as $P_2 \circ P_n$, where P_n is a path graph. An ***n,k-firecracker*** is a graph obtained by concatenation of n k-stars by linking one leaf from each. A ***bistar***, denoted by *B(m,n)* is the graph obtained by joining the root vertices of the stars $K_{1,m}$ and $K_{1,n}$. The

friendship graph, denoted by F_p can be constructed by identifying p copies of the cycle C_3 at a common vertex. A ***helm graph***, denoted by H_p is a graph obtained from the wheel W_p by joining a pendant vertex to each vertex in the outer cycle of W_p by means of an edge. ***Barbell graph*** is obtained by connecting two copies of a complete graphs K_p by an edge. The **m-book graph** B_m is defined as the graph Cartesian product $S_{m+1} \times P_2$, where S_m is a star graph and P_2 is the path graph on two nodes. The (m,n)-tadpole graph ($T_{m,n}$), also called a dragon graph, is the graph obtained by joining a cycle graph C_m to a path graph P_n with a bridge. The (m,n)--lollipop graph is the graph obtained by joining a complete graph K_m to a path graph P_n with a bridge[1-3].

A ***cut – vertex (cut edge)*** of a graph G is a vertex (edge) whose removal increases the number of components. A ***vertex cut***, or ***separating set*** of a connected graph G is a set of vertices whose rmoval results in a disconnected graph. The ***connectivity*** or ***vertex connectivity*** of a graph G, denoted by $\kappa(G)$ (where G is not complete) is the size of a smallest vertex cut. The ***chromatic number*** of a graph G, denoted by $\chi(G)$ is the smallest number of colours needed to colour all the vertices of a graph G in which adjacent vertices receive different colour. For any real number x, $\lfloor x \rfloor$ denotes the largest integer less than or equal to . A ***Nordhaus -Gaddum-type*** result is a (tight) lower or upper bound on the sum or product of a parameter of a graph and its complement. Terms not defined here are used in the sense of [14].

A subset S of V is called a ***dominating set*** of G if every vertex in $V - S$ is adjacent to at least one vertex in S. The ***domination number*** $\gamma(G)$ of G is the minimum cardinality taken over all dominating sets in G. A dominating set S of a connected graph G is said to be a ***connected dominating set*** of G if the induced sub graph $\langle S \rangle$ is connected. The minimum cardinality taken over all connected dominating sets is the ***connected domination number*** and is denoted by γ_c.

Many authors have introduced different types of domination parameters by imposing conditions on the dominating set[18,19].

In[18], the authors introduced the concept of complementary perfect domination number of a graph. A subset S of V of a nontrivial graph G is said to be a ***complementary perfect dominating set***, if S is a dominating set and the induced subgraph $\langle S \rangle$ has a perfect matching. The minimum cardinality taken over all complementary perfect dominating sets is called the ***complementary perfect domination number*** of G and is denoted by $\gamma_{cp}(G)$.

A subset S of V of a nontrivial graph G is said to be an ***efficient dominating set***, if every vertex is dominated exactly once. The minimum cardinality taken over all efficient dominating sets is called the ***efficient domination number*** of G and is denoted by $\gamma_e(G)$.

In[17], J. Paulraj Joseph, introduced the concept of triple connected graphs. A graph G is **triple connected** if any three vertices of the graph are lie on a path.

In[4], G. Mahadevan *et.al.*, introduced the concept of triple connected domination number of a graph. A subset S of V of a non trivial graph G is said to be a **triple connected dominating set**, if S is a dominating set and the induced subgraph $<S>$ is triple connected. The minimum cardinality taken over all triple connected dominating sets is called the ***triple connected domination number*** of G and is denoted by $\gamma_{tc}(G)$.

In[6], G. Mahadevan *et.al.*, introduced the concept of complementary perfect triple connected domination number of a graph. A subset S of V of a non trivial graph G is said to be a **complementary perfect triple connected dominating set**, if S is a triple connected dominating set and the induced subgraph $<V - S>$ has a perfect matching. The minimum cardinality taken over all complementary perfect triple connected dominating sets is called the ***complementary perfect triple connected domination number*** of G and is denoted by $\gamma_{cptc}(G)$.

In[5,7,8,9,10,11,12,13] G. Mahadevan *et. al.*, introduced **complementary triple connected domination number, paired triple connected domination number, triple connected two domination number, restrained triple connected domination number, dom strong triple connected domination number, strong triple connected domination number, weak triple connected domination number, triple connected complementary tree domination number of a graph** respectively and investigated new results on them.

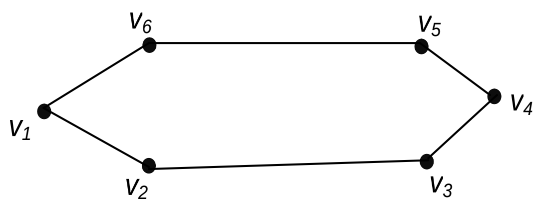
In this paper, we use this idea to develop the concept of **efficient complementary perfect triple connected dominating set** and **efficient complementary perfect triple connected domination number of a graph.**

Notation 1.1 [4] Let G be a connected graph with m vertices $v_1, v_2, \ldots, v_m$. The graph obtained from G by attaching n_1 times a pendant vertex of on the vertex v_1, n_2 times a pendant vertex of on the vertex v_2 and so on, is denoted by $G(n_1, n_2, n_3, \ldots, n_m)$ *where* n_i, $l_i \geq 0$ and $1 \leq i \leq m$.

2. Efficient complementary perfect triple connected domination number of a graph:

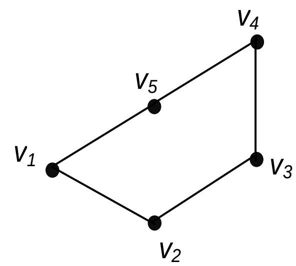
Definition 2.1 A subset S of V of a nontrivial graph G is said to be an *efficient complementary perfect triple connected dominating set,* if S is a complementary perfect triple connected dominating set and every vertex is dominated exactly once. The minimum cardinality taken over all efficient complementary perfect triple connected dominating sets is called the *efficient complementary perfect triple connected domination number* of G and is denoted by $\gamma_{ecpt}(G)$. Any efficient complementary perfect triple connected dominating set with γ_{ecpt} vertices is called a γ_{ecpt} -set of G.

Example 2.2 For the graph G_1 in figure 2.1, $S = \{v_1, v_2, v_3, v_4\}$ forms a γ_{ecpt} -set of G_1. Hence $\gamma_{ecpt}(G_1) = 4$.



G₁

Figure 2.1 : Graph with γ_{ecpt} = 4.



G₂

Figure 2.2. Graph in which V – S is not an ecptd set

Observation 2.3: Efficient complementary perfect triple connected dominating set (γ_{ecpt} -set or ecptd set) does not exists for all graphs.

Example 2.4 For K_6, there does not exist any efficient complementary perfect triple connected dominating set.

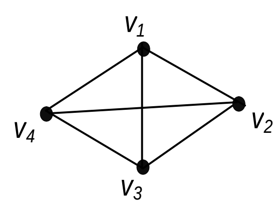
Remark 2.5: Throughout this paper we consider only connected graphs for which efficient complementary perfect triple connected dominating set exist.

Observation 2.6: The complement of an efficient complementary perfect triple connected dominating set need not be an efficient complementary perfect triple connected dominating set.

Example 2.7 For the graph G_2 in figure 2.2, = $\{v_1, v_2, v_3\}$ forms an efficient complementary perfect triple connected dominating set of G_2. But the complement $V - S = \{v_4, v_5\}$ is not an efficient complementary perfect triple connected dominating set.

Observation 2.8: Every efficient complementary perfect triple connected dominating set is a dominating set but not conversely.

Example 2.9: For the graph G_3 in figure 2.3, $S = \{v_1\}$ is a dominating set but not an efficient complementary perfect triple connected dominating set of G_3.



G₃

Figure 2.3

Observation 2.10: Every efficient complementary perfect triple connected dominating set is a complementary perfect triple connected dominating set but not conversely.

Example 2.11: For the graph G_4 in the figure 2.4, S = {v_1, v_2, v_3} is a complementary perfect triple connected dominating set but not an efficient complementary perfect triple connected dominating set.

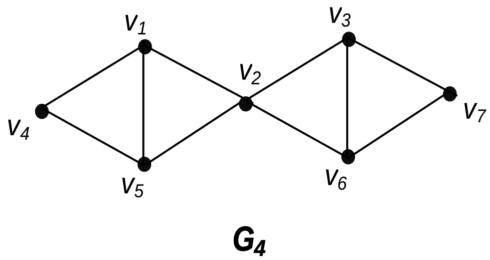


Figure 2.4

Observation 2.12: Every efficient complementary perfect triple connected dominating set is a complementary perfect dominating set but not conversely[15,16].

Example 2.13: For the graph G_5 in the figure 2.5, S = {v_1, v_2, v_3} is a complementary perfect but not an efficient complementary perfect triple connected dominating set.

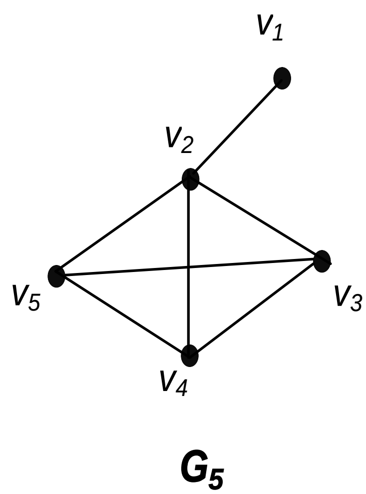


Figure 2.5

Exact value for some standard graphs:

1) For any cycle of order p ≥ 5, $\gamma_{ecpt}(C_p) = p - 2$.
2) For any book graph of order p ≥ 6, $\gamma_{ecpt}(B_p) = 4$.
3) For any ladder graph of order p ≥ 6, $\gamma_{ecpt}(G)$ $= \begin{cases} p-2 \text{ if } p \text{ is odd} \\ \frac{p}{2} \quad \text{if } p \text{ is even} \end{cases}$.
4) For any friendship graph of order p ≥ 5, $\gamma_{ecpt}(F_n) = 3$, *(where 2n +1 = p).*
5) For tadpole graphs of order p ≥ 5, $\gamma_{ecpt}(T_{m,n}) = p - 2$, *(where m + n = p).*

Remark 2.14: Efficient complementary perfect triple connected domination number does not exists for the following graphs.

1) Wheel graph
2) Fire cracker graph
3) Star and Bi-star graphs
4) Gear graphs
5) Barbell graph
6) Helm graph
7) Lollipop graph
8) Complete graph
9) Complete – bipartite graph
10) Path graph.

Theorem 2.15: For any connected graph *G* with $p \geq 5$, we have $3 \leq \gamma_{ecpt}(G) \leq p-2$ and the bounds are sharp.

Proof : The lower and upper bounds follows from *definition 2.1*. For C_5, the lower bound is attained and for C_9 the upper bound is attained.

Theorem 2.16: For any connected

graph *G* with *5* vertices, $\gamma_{ecpt}(G) = p - 2$ if and only if G is isomorphic to C_5, $C_4(P_2)$, $C_3(P_3)$, $C_3(2P_2)$ or F_2.

Proof : Suppose G is isomorphic to C_5, $C_4(P_2)$, $C_3(P_3)$, $C_3(2P_2)$ or F_2 then clearly, , $\gamma_{ecpt}(G) = p - 2$. Conversely, assume that G is a connected graph with 5 vertices and , $\gamma_{ecpt}(G) = 3$. Let S={x, y, z} be a $\gamma_{ecpt}(G)$-set. Then $\langle S\rangle = P_3$ or C_3. Let $V - S = V(G) - V(S) = \{u, v\}$, then $\langle V - S\rangle = K_2 = uv$.

Case 1 $\langle S\rangle = P_3 = xyz$.

Since G is connected, there exists a vertex x (or y, or z) in P_3 which is adjacent to u (or v) in K_2, then $\gamma_{ecpt}(G)$ –set does not exists. But when we increase the degrees of the vertices of S, let x be adjacent to u and z be adjacent to v. If d(x) = d(y) = d(z) = 2, then $G \cong C_5$. Now by increasing the degrees of the vertices, when d(x) = 2, d(y) = 3, d(z) = 1, then $G \cong C_4(P_2)$. If d(x) = 3, d(y) = 2, d(z) = 1, then $G \cong C_3(P_3)$. In all the other cases, $\gamma_{ecpt}(G)$ –set does not exists.

Since G is connected, there exists a vertex say y in P_3, which is adjacent to u (or v) in K_2, the $\gamma_{ecpt}(G)$ –set does not exists. But on increasing the degrees of vertices of S, let y be adjacent to u and v in such a way that d(x) = 1, d(y) = 4, d(z) = 1, then $G \cong C_3(2P_2)$. In all the other cases, no new graph exists.

Case 2: $\langle S\rangle = C = xyzx$

Since G is connected, there exists a vertex say x (or y or z) in C_3 is adjacent to u (or v) in K_2, then S = {x, u, v} forms a $\gamma_{ecpt}(G)$ –set. If d(x) = 3, d(y) = d(z) = 2, then $G \cong C_3(P_3)$. If d(x) = 4, d(y) = d(z) = 2, then $G \cong F_2$. In all other cases, no new graph exists.

Theorem 2.17: Any efficient complementary perfect triple connected dominating set must contain all the pendant vertices of G.

Proof: Let S be any efficient complementary perfect triple connected dominating set of G. Let u be a pendant vertex with the support v. Suppose u does not belong to S, then v must be in S, which is a contradiction for the concept of complementary perfect. Since u is a pendant vertex, then u belongs to S.

Theorem 2.18 $\gamma_{ecpt}(G) \geq \left\lceil \frac{p}{\Delta + 1} \right\rceil$.

Proof : Each vertex in V – S contributes one to degree sum of vertices of S. Then $|V\text{-}S| \leq \sum_{u \epsilon S} d(u)$ where S is an efficient complementary perfect triple connected dominating set. So $|V\text{-}S| \leq \gamma_{ecpt}\, \Delta$ which implies $(|V| - |S|) \leq \gamma_{ecpt}\, \Delta$. Therefore $p - \gamma_{ecpt} \leq \gamma_{ecpt}\, \Delta$ and hence $\gamma_{ecpt}(G) \geq \left\lceil \frac{p}{\Delta + 1} \right\rceil$.

The Nordhaus – Gaddum type result is given below:

Theorem 2.19: Let *G* be a graph such that *G* and have no isolates of order $p \geq 5$. Then

(i) $\gamma_{ecpt}(G) + \gamma_{ecpt}(\bar{G}) \leq 2p - 4$.

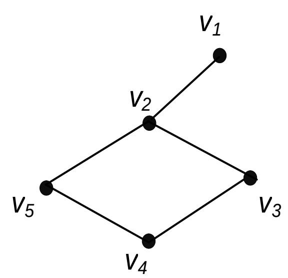
(ii) $\gamma_{ecpt}(G).\ \gamma_{ecpt}(\bar{G}) \leq (p-2)^2$ and the bound is sharp.

Proof : The bound directly follows from *theorem 2.15*. For **Cycle C_6,** both the bounds are attained.

3. Relation with Other dominational Parameters :

Observation 3.1: There exists a graph for which $\gamma_{cp}(G) = \gamma_{ecp}(G) = \gamma_{tc}(G) = \gamma_{cptc}(G) = \gamma_{ecpt}(G)$.

Example 3.2 : For the graph G_6 in figure 3.1, $S = \{v_1, v_2, v_3\}$ is a complementary perfect, efficient complementary perfect, triple connected, complementary perfect triple connected and efficient complementary perfect triple connected set. Hence $\gamma_{cp}(G) = \gamma_{ecp}(G) = \gamma_{tc}(G) = \gamma_{cptc}(G) = \gamma_{ecpt}(G)$.



G_6

Figure 3.1

Observation 3.3: For any connected graph G, $\gamma(G) \leq \gamma_{ecp}(G) \leq \gamma_{ecpt}(G)$ and the bounds are sharp.

Example 3.4 : For $G=C_8= v_1v_2v_3v_4v_5 v_6v_7v_8v_1$, $\gamma(G) = 3$, $\gamma_{ecp}(G)=4$ and $\gamma_{ecpt}(G)=6$.

Theorem 3.5 : For any connected graph G with $p\geq 5$ vertices, $\gamma(G)+\gamma_{ecpt}(G)\frac{3n-4}{2}$ and the bound is sharp.

Proof : Let G be a connected graph with $p \geq 5$ vertices. We know that $\gamma(G) \leq \frac{p}{2}$ and by *theorem 2.15*, $\gamma_{ecpt}(G) \leq p - 2$. Hence $\gamma(G) + \gamma_{ecpt}(G) \leq \frac{p}{2} + p - 2 = \frac{3n-4}{2}$. Hence $\gamma(G) + \gamma_{ecpt}(G) \leq \frac{3n-4}{2}$. For C_6, the bound is sharp.

Theorem 3.6 : For any connected graph G with $p \geq 5$ vertices, $\gamma_{ecp}(G) + \gamma_{ecpt}(G) \leq 2p - 4$ and the bound is sharp.

Proof : Let G be a connected graph with $p \geq 5$ vertices. We already know that $\gamma_{ecp}(G) \leq p - 2$ and $\gamma_{ecpt}(G) \leq p - 2$. So we have , $\gamma_{ecp}(G) + \gamma_{ecpt}(G) \leq (p - 2) + (p - 2) = 2p - 4$. Hence, $\gamma_{ecp}(G) + \gamma_{ecpt}(G) \leq 2p - 4$ and the bound is sharp for $C_3(P_3)$.

4. Relation with Other Graph Theoretical Parameters :

Theorem 4.1: For any connected graph G with $p \geq 5$ vertices, $\gamma_{ecpt}(G) + \kappa(G) < 2p - 3$.

Proof: Let G be a connected graph with $p \geq 5$ vertices. We know that $\kappa(G) \leq p-1$ and by *theorem 2.15*, $\gamma_{ecpt}(G) \leq p - 2$. Hence $\gamma_{ecpt}(G) + \kappa(G) \leq 2p - 3$. Let $\gamma_{ecpt}(G) + \kappa(G) = 2p - 3$. This is possible only if $\gamma_{ecpt}(G)=p-2$ and $\kappa(G) = p - 1$. But $\kappa(G) = p - 1$, and so $G \cong K_p$ for which $\gamma_{ecpt}(G)$ set does not exists.

Hence $\gamma_{ecpt}(G) + \kappa(G) < 2p - 3$.

Theorem 4.2: For any connected graph G with $p \geq 5$ vertices, $\gamma_{ecpt}(G) + \Delta(G) \leq 2p - 3$ and the bound is sharp.

Proof : Let G be a connected graph with $p \geq 5$ vertices. We know that $\Delta(G) \leq p - 1$ and by *theorem 2.15*, $\gamma_{ecpt}(G) \leq p - 2$. Hence $\gamma_{ecpt}(G) + \Delta(G) \leq 2p - 3$. For $C_3(2P_2)$, the bound is sharp.

Theorem 4.3: For any connected graph G with $p \geq 5$ vertices, $\gamma_{ecpt}(G) + \chi(G) < 2p - 2$.

Proof : Let G be a connected graph with $p \geq 5$ vertices. We know that $\chi(G) \leq p$ and by *theorem 2.15*, $\gamma_{ecpt}(G) \leq p - 2$. Hence $\gamma_{ecpt}(G) + \chi(G) \leq 2p - 2$. Let $\gamma_{ecpt}(G) + \chi(G) = 2p - 2$. This is possible only if $\gamma_{ecpt}(G) = p - 2$ and $\chi(G) = p$. Since $\chi(G) = p$, G is isomorphic to K_p for which $\gamma_{ecpt}(G)$ set does not exists. Hence $\gamma_{ecpt}(G) + \chi(G) < 2p - 2$.

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