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An inventory model for deteriorating items with weibull deterioration rate, linear demand rate, unit production cost and without shortages

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Abstract

The objective of this model is to investigate the inventory system for perishable items with linear demand pattern where Weibull deterioration is considered. The Economic order quantity is determined for minimizing the average total cost per unit time. The unit production cost is taken to be inversely related to the demand rate. The result is illustrated with numerical example.

Key words: Inventory system, Linear demand, Deterioration, Unit production cost, Shortage.

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1.0. Introduction

Perishable items widely exist in our daily life, such as fresh products, fruits, vegetables, seafood, etc, which decrease in quantity or utility during their delivery and storage stage periods. To improve products quality and reduce deteriorating loss in the fashion goods supply chain, the emphasis is on the whole process life cycle management which includes production, storage, transportation, retailing etc. Accompany by the increasing of the variety and quantity of the deteriorating and fashion goods, consumers' appetite for high quality perishable items are continually upgrading rapidly, so the topic of deteriorating inventory system management has became popular in the field of research and business.

Several researchers have studied stock deterioration over the years. Ghare and Schrader[3] were among the first authors to consider the role of deterioration in inventory modeling. Other authors, such as Covert and Philip[1], Kang and Kim[4] and Raafat *et.al.*[5], assumed either instantaneous re-supply or a finite production rate with different assumptions on how deterioration occurred. Wee[7] studied a replenishment policy with price-dependent demand and a varying rate of deterioration.

In the early 1970s, Silver and Meal[6] proposed an approximate solution procedure for the general case of a deterministic, time-varying demand pattern. Donaldson[2] then considered an inventory model with a linear trend in demand. After Donaldson, numerous research works have been carried out incorporating time varying demand into inventory models under a variety of circumstances[3].

In this paper an attempt has been made to develop an inventory model for perishable items with Weibull deterioration rate and the linear demand pattern is used over a finite planning horizon. The unit cost of production depends on the demand rate. Nature of the model is discussed for without shortage state. Optimal solution for the proposed model is derived and the applications are investigated with the help of some numerical examples.

2.0. Assumptions and notations :

Following assumptions are made for the proposal model:

i. Single inventory will be used.
ii. Lead time is zero.
iii. The model is studied when shortages are not allowed.
iv. Demand follows the linear demand pattern.
v. Weibull distribution is used for deterioration.
vi. Replenishment rate is infinite but size is finite.
vii. Unit production cost is assumed to depend on demand rate.
viii. Time horizon is finite.
ix. There is no repair of deteriorated items occurring during the cycle.

Following notations are made for the given model:

$I(t)$ = On hand inventory level at any time t, $t \geq 0$.

$R(t) = a + bt$ is the linear demand rate at any time $t \geq 0, a \geq 0, b > 0.$

$\theta(t)$ = Instantaneous rate of deterioration of the on-hand inventory given by $\alpha\beta \cdot t^{\beta-1}$ where

$0 < \alpha << 1,$ is known as the scale parameter and $\beta > 0$ is known as shape parameter.

S = Inventory at time $t = t_1$.

t_2 = Duration of a cycle.

c_d = The deterioration cost per unit item.

c_h = The holding cost per unit item.

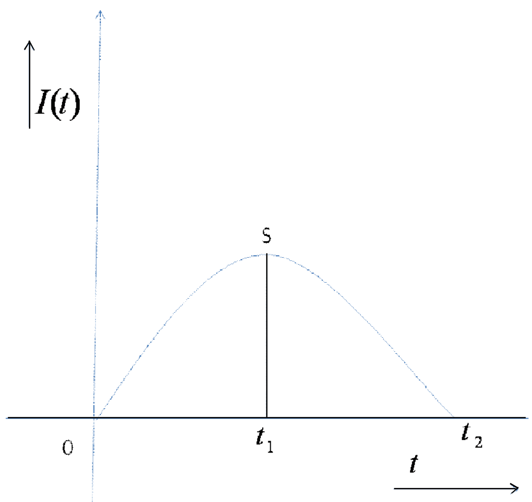
$K = \beta_1 R(t)$ is the production rate where $\beta_1 > 1$ is a constant.

$p = \alpha_1 R^{-\gamma}$ is the unit production cost which is inversely related to the demand rate where $\alpha_1 > 0, \gamma > 0$ *and* $\gamma \neq 2.$

U =The total average cost of the system.

3.0. Formulation :

The amount of stock is zero at time $t = 0$. Production starts at time $t = 0$ and stops at time t_1 when the stock attains a level S. During $[t_1,t_2]$, the inventory level gradually decreases mainly to meet demands and partly because of deterioration. By this process, the stock reaches zero at time t_2. The cycle then repeats itself after the scheduling period t_2.



If $I(t)$ be the on-hand inventory at time, $t \geq 0$, then at time $t + \Delta t$, the on-hand inventory in the interval $[0,t_1]$ will be

$$I(t+\Delta t) = I(t) - \theta(t)\, I(t)\, \Delta t - R(t)\, \Delta t + K\, \Delta t$$

Dividing by Δt and then taking as $\Delta t \to 0$ we get

$$(3.1)\quad \frac{dI(t)}{dt} + \alpha\, \beta\, t^{\beta-1}\, I(t) = (\beta_1 - 1)(a+bt)\,; \quad 0 \leq t \leq t_1$$

With conditions $I(0) = 0,\ I(t_1) = S.$

For the next interval $[t_1,t_2]$, we have

$$I(t+\Delta t) = I(t) - \theta(t)\, I(t)\, \Delta t - R(t)\, \Delta t$$

Dividing by Δt and then taking as $\Delta t \to 0$ we get

$$(3.2)\quad \frac{dI(t)}{dt} + \alpha\, \beta\, t^{\beta-1}\, I(t) = -(a+bt)\,; \quad t_1 \leq t \leq t_2$$

The boundary conditions are $I(t_1) = S$, $I(t_2) = 0.$

On solving equation (3.1) with boundary condition $I(0)=0$ we have

$$(3.3)\quad I(t) = \left(1 - \alpha\, t^{\beta}\right)\left(\beta_1 - 1\right)\left[at + \frac{b}{2}t^2 + \frac{a\alpha}{\beta+1}t^{\beta+1} + \frac{b\alpha}{\beta+2}t^{\beta+2}\right]; \quad 0 \leq t \leq t_1$$

On solving equation (3.2) with boundary condition $I(t_1) = S.$ we have

(3.4)

$$I(t) = \left(1 - \alpha\, t^{\beta}\right)\left[S(1+\alpha\, t^{\beta}) + a(t_1 - t) + \frac{b}{2}(t_1^2 - t^2) + \frac{a\alpha}{\beta+1}(t_1^{\beta+1} - t^{\beta+1}) + \frac{b\alpha}{\beta+2}(t_1^{\beta+2} - t^{\beta+2})\right]; \quad t_1 \leq t \leq t_2$$

Using $I(t_2) = 0$ in equation (3.4) we have

(3.5) $S = \left(1-\alpha\, t_1^{\beta}\right)\left[a(t_2 - t_1) + \frac{b}{2}(t_2^2 - t_1^{\,2}) + \frac{a\alpha}{\beta+1}(t_2^{\beta+1} - t_1^{\beta+1}) + \frac{b\alpha}{\beta+2}(t_2^{\beta+2} - t_1^{\beta+2})\right];$

The total amount of deteriorated units in $0 \le t \le t_2$ is given by

(3.6) Production in $[0, t_1]$ – Demand in $[0, t_2]$

$$= \beta_1 \int_0^{t_1} (a+bt)dt - \int_0^{t_2} (a+bt)dt$$

$$= a(\beta_1 t_1 - t_2) + \frac{b}{2}\left(\beta_1 t_1^2 - t_2^2\right)$$

The total inventory in the cycle is given by

(3.7) $\int_0^{t_1} I(t)dt - \int_{t_1}^{t_2} I(t)dt$

$$= (\beta_1 - 1)\left[\frac{a\,t_1^2}{2} + \frac{b\,t_1^3}{6} - \frac{a\alpha\beta}{(\beta+1)(\beta+2)} t_1^{\beta+2} - \frac{b\alpha\beta}{2(\beta+2)(\beta+3)} t_1^{\beta+3}\right]$$

$$+ \left\{t_2 - t_1 - \frac{\alpha}{\beta+1}(t_2^{\beta+1} - t_1^{\beta+1})\right\}\left[S(1+\alpha\, t_1^{\beta}) + a t_1 + \frac{b}{2} t_1^2 + \frac{a\alpha}{\beta+1} t_1^{\beta+1} + \frac{b\alpha}{\beta+2} t_1^{\beta+2}\right]$$

$$- \left[\frac{a}{2}(t_2^2 - t_1^{\,2}) + \frac{b}{6}(t_2^3 - t_1^{\,3}) - \frac{a\alpha\beta}{(\beta+1)(\beta+2)}(t_2^{\beta+2} - t_1^{\beta+2}) - \frac{b\alpha\beta}{2(\beta+2)(\beta+3)}(t_2^{\beta+3} - t_1^{\beta+3})\right]$$

The Production cost in $[0,t_1]$ is given by

(3.8) $\int_0^{t_1} \alpha_1\ \beta_1\ R(t)\ R^{-\gamma}\ dt = \alpha_1\ \beta_1\ t_1\ a^{1-\gamma}, \gamma \ne 2.$

Taking the relevant costs mentioned above, the total average cost per unit time of the system is given by

(3.9) $U(t_1, t_2) = \frac{1}{t_2}$ {Holding cost + Deterioration cost + Production cost}

$$= \frac{1}{t_2}\left[(\beta_1 - 1)\, C_h\left[\frac{a\,t_1^2}{2} + \frac{b\,t_1^3}{6} - \frac{a\alpha\beta}{(\beta+1)(\beta+2)} t_1^{\beta+2} - \frac{b\alpha\beta}{2(\beta+2)(\beta+3)} t_1^{\beta+3}\right]\right.$$

$$+C_h\left\{t_2-t_1-\frac{\alpha}{\beta+1}(t_2^{\beta+1}-t_1^{\beta+1})\right\}\left[S(1+\alpha\ t_1^{\ \beta})+at_1+\frac{b}{2}t_1^2+\frac{a\alpha}{\beta+1}t_1^{\beta+1}+\frac{b\alpha}{\beta+2}t_1^{\beta+2}\right]$$

$$-C_h\left[\frac{a}{2}(t_2^2-t_1^{\ 2})+\frac{b}{6}(t_2^3-t_1^{\ 3})-\frac{a\alpha\beta}{(\beta+1)(\beta+2)}(t_2^{\beta+2}-t_1^{\beta+2})-\frac{b\alpha\beta}{2(\beta+2)(\beta+3)}(t_2^{\beta+3}-t_1^{\beta+3})\right]$$

$$+C_d\left\{a(\beta_1 t_1-t_2)+\frac{b}{2}(\beta_1 t_1^2-t_2^2)\right\}+\alpha_1\ \beta_1\ t_1\ a^{1-\gamma}\ \Big]$$

Now equation (3.9) can be minimized but as it is difficult to solve the problem by deriving a closed equation of the solution of equation (3.9), Matlab Software has been used to determine optimal $t_1^{\ *},\ t_2^{\ *}$ and hence the optimal cost $U(t_1^{\ *},t_2^{*})$ can be evaluated.

4.0. EXAMPLES:

Example- 1:

The values of the parameters are considered as follows:

$$c_h=\$30/unit/year, c_d=\$50/unit,$$

$$\alpha=0.001, \beta=2\ ,\alpha_1=20\ ,\ \beta_1=10,$$

$$a=300, b=25, \gamma=3.8.$$

Now using equation (3.9) which can be minimized to determine optimal $t_1^{\ *}=2.47$ *Years*, $t_2^{\ *}=4.22$*Years*, and hence the average optimal cost $U(t_1^{\ *},t_2^{\ *})=\$\ 356.27/unit$.

Also the level of inventory at time $t=t_1$ *is*

$S^*=\ 647.96$ *units.*

5.0. Conclusion

Here an EOQ model is derived for perishable items with linear demand pattern. Weibull deterioration rate is used. The model is studied for minimization of total average cost. The Unit production cost is inversely related to the demand rate. Numerical examples are used to illustrate the result where we saw that when the value of increases, the optimal time decreases whereas initial inventory level increases.

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