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Application of vertex and group magic labeling

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Abstract

Graph labeling is a currently emerging area in the research of graph theory. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. If the labels of edges are distinct positive integers and for each vertex v the sum of the labels of all edges incident with v is the same for every vertex v in the given graph, then the labeling of the graph is called magic labeling. There are several types of magic labeling defined on graphs. In this paper we consider vertex magic labeling and group magic labeling of graphs as an application of magic labeling. We solve a problem of finding number of computers/workstations to be allocated to each department in a company under certain conditions.

Key words: vertex magic labeling, group magic labeling, Z-magic labeling.

AMS Subject Classification: 05C78

1. Introduction

Labeling of graphs is a special area in Graph Theory. A detailed study has been done in[6] by Joseph A.Gallion. Many types of magic labeling are defined in graphs. Originally Sedlacek has defined magic graph as a graph whose edges are labeled with distinct non-negative integers such that the sum of the labels of the edges incident to a particular vertex is the same for all vertices.

MacDougall, Miller, Slamin, and Wallis introduced the notion of a vertex-magic total labeling in 1999. For a graph G(V,E), an injective mapping f from $V \cup E$ to the set $\{1,2,...,|V|+|E|\}$ is a vertex-magic total labeling if there is a constant k, called the magic constant, such that for every vertex v, $f(v)+\Sigma f(vu) = k$ where the sum is over all vertices u adjacent to v. The vertex-magic total labeling is studied in detail in the monograph by Wallis[9] and vertex magic total labeling are found for family of graphs[1,2,8].

In a magic graph if the labels are non zero elements of a non trivial additive abelian group A, then the graph is called A -magic graph. In the literature recently A -magic graphs are studied and many results have been derived since J sedlacek introduced group magic graphs[3,4,5,6,7].

An A -magic graph G is said to be Z_k-magic graph if the group A is Z_k, the group of integers mod k. These Z_k-magic graphs are referred as k-magic graphs. E.Bala and K.Thirusangu have given the Z_3–magic labeling for the modified extended triplicate graph in [3]. In [5], Baskar Babujee, L.Shobana have shown that even cycles, bistar, ladder, biregular graphs and for a certain class of cayley digraphs, are Z_3–magic.

In this paper as an application of Z-magic labeling and vertex magic labeling we solve a problem to find the number of workstations/computers to be allocated to each department in a company under certain conditions.

2. Basic Definitions

The graphs considered here are finite, simple and undirected.

1. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions.
2. A connected graph is said to have semi-magic labeling if there is a labeling of the edges with integers such that for each vertex v the sum of the labels of all edges incident with v is the same for all v.
3. In semi-magic labeling if the labels are distinct positive integers then the labeling is called magic labeling.
4. A magic labeling is super magic if the set of edge labels consists of consecutive positive integers.
5. A vertex magic total labeling (VMT) of a graph G= (V,E) with |V| vertices and |E| edges is an assignment of the integers from 1,2,…,|V|+|E| to the vertices and edges of G with the property that the sum of the label on a vertex and the labels on its incident edges is a constant k, independent of the choice of the vertex and k is called as magic constant. A Super vertex magic labeling (SVM) is a VMT with set of edge labels {1, 2, …|E|}.
6. Let G = (V, E) be a connected simple graph. For any non-trivial additive abelian group A, let A*=A-{0}. A function f: E(G) $\rightarrow$ A* is called a labeling of G. Any such labeling induces a map f^+:V(G)$\rightarrow$A, defined by $f^+(v) = \sum_{(u,v)\,\in\, E(G)} f(u,v)$. If there exists a labeling f which induces a constant label c on V(G), we say that f is an A-magic labeling and that G is an A-magic graph with index c.
7. An A-magic graph G is said to be Z_k-magic graph if the group A is Z_k- the group of integers mod k. These Z_k- magic graphs are referred as k - magic graphs.
8. A A-magic graph G is said to be Z-magic graph if A=Z.
9. A graph G = (V; E) is called fully magic if it is A-magic for every abelian group A.
10. A k-magic graph G is said to be k-zero-sum (or just zero sum) if there is a magic labeling of G in Z_k that induces a vertex labeling with sum zero.

3. Main Results

We shall demonstrate an application of group magic labeling and vertex magic labeling in the following problems.

3.1 An application of Z-magic labeling :

A company wants to distribute an equal number of workstations to various departments to handle production/development work load in an effective manner. Departments can be categorized as $D_1, D_2, \ldots, D_n$. However in certain scenario based on production incidents/regulatory changes the workstations could be utilized by more than one department. Hence, company needs to create an inventory list to compute the number of workstations assigned to each department and also to determine the number of workstations to be shared between any two departments. The detail and serial numbers of workstation should be included in project plan during planning phase and it is very important for successful completion of project. This scenario is represented using a graph as shown below and solution is found using group magic labeling to that graph. The output generated by the graph and its Z-magic labeling provides, any company with number of workstations planned to be assigned to the departments and also the number of workstations shared by the pair of departments.

We draw the graph by considering for each $i=1,2,\ldots,n$ the department D_i as the vertex v_i and if departments D_i and D_j are sharing the workstations then we take an edge between v_i and v_j.

Problem 1 :

EVERTEC Solutions wants to provide an equal number of workstations to its departments Mortgage Application Development Team, Deposit Application Enhancement Team, Payment Gateway Support Team, Mi-Banco Testing team, Production Support and Operations Team in such a way that each workstation is utilized by two teams. The sharing of workstations by the teams is in the following manner. Deposit Application Enhancement Team shares with Mortgage Application development team, Payment Gateway Support Team and Operations Team. Payment Gateway Support Team shares with Mortgage Application Development Team, Deposit Application Enhancement Team and Mi-Banco Testing team. Mi-Banco Testing team shares with Mortgage Application Development Team, Payment Gateway Support Team and Production Support team. Production Support team shares with Mortgage Application Development Team, Mi-Banco Testing team and Operations Team. Operations Team shares with Mortgage Application Development Team, Production Support team and Deposit Application Enhancement Team. Find the number of workstations allotted to the departments and also the number of workstations shared by the pair of departments.

Solution:

The sharing of workstations between the teams described in the problem is shown in the following diagram.

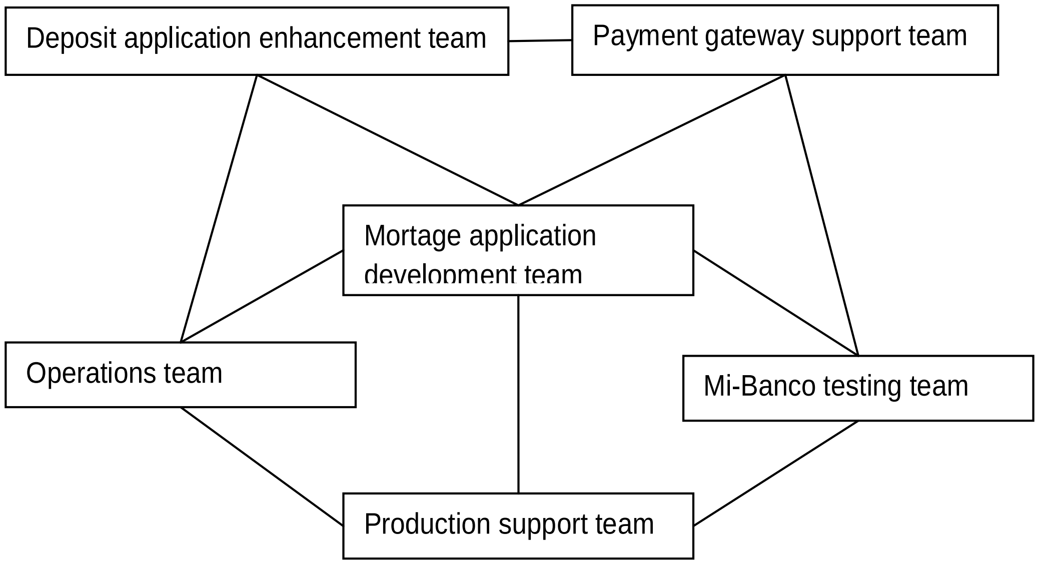

Fig. 1.

Table 1

Teams	Deposit application enhancement team	Payment gateway support team	Mi-Banco testing team	Production support team	Operations team	Mortgage application develop-ment team
Deposit application enhancement team		8			6	1
Payment gateway support team	8		3			4
Mi-Banco testing team		3		7		5
Production support team			7		6	2
Operations team	6			6		3
Mortgage application development team	1	4	5	2	3	

Empty cells in the above table shows the corresponding two teams are not sharing the workstations.

Let us denote Deposit Application Enhancement Team, Payment Gateway Support Team, Mi-Banco Testing team, Production Support and Operations Team as vertex v_1,v_2,v_3,v_4,v_5 respectively and the Mortage application development team as u. Representing this as a graph we obtain a wheel of size 5. By considering the Z-magic labeling of wheels we obtain the solution of this problem.

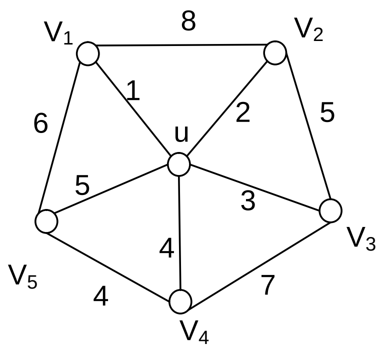

Fig. 2

The magic index is 15. Thus the company has to provide 15 number of workstations to each team. The number of workstations which are shared by two teams is taken from the edge labels of the above graph and it is shown in the following table.

Remark :

1. If the company needs computers in large numbers, then consider the multiple of the edge labels with any number.
2. If the company likes to provide exactly k number of workstations to the departments, then k-zero-sum labeling of the graph will provide the solution.
3. If the company likes to provide k or less number of workstations to the departments, then k-magic labeling of the graph will provide the solution.

3.2 Application of vertex magic total labeling:

A company wants to distribute equal number of computers to its departments such that each computer is utilized either by one department or by two departments. Here the problem is to find the number of computers allotted to each department and also to find number of computers utilized by one department and number of computers used by two departments. Representing this situation as a graph by considering the departments as vertices and if two departments shares computers then there is an edge between the corresponding vertices. Considering the vertex magic total labeling to this graph we are able to get the solution needed.

Problem 2 :

A company wants to provide equal number of computers to its 5 departments D_1, D_2, D_3, D_4, D_5. Departments are utilizing computers in such a manner that the departments D_1 and D_2, D_2 and D_3, D_3 and D_4, and D_4 and D_5 are sharing few computers. Find the number of computers allotted to each department. Find the number of computers utilized by one department and by two departments.

Solution:

For each i=1,2,3,4,5, we take the department D_i as the vertex v_i and if departments D_i & D_j are sharing computers then we take an edge between v_i & v_j. The graph corresponding to the given circumstance is given in fig 3.

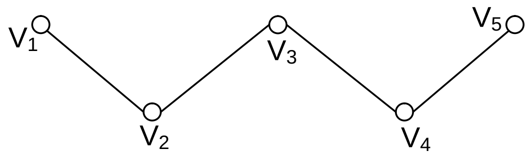

Fig. 3

Consider Vertex Magic Total labeling (VMT) of the given graph.

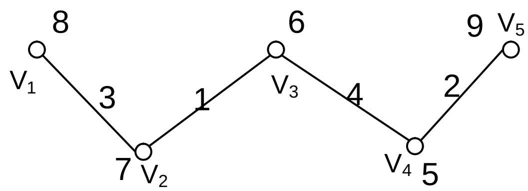

Fig. 4

Here at each vertex the sum of the label on the vertex and the labels on its incident edges is a constant k, this k gives the number of computers allotted to the departments which is same for each department. The vertex labels gives number of computers utilized by one department and edge labels gives number of computers utilized by two departments. Here k=11. So each department utilizes 11 computers. We catalog the vertex and edge labels as in the following table 2.

Table 2

Dept.	D_1	D_2	D_3	D_4	D_5
D_1	8	3	0	0	0
D_2	3	7	1	0	0
D_3	0	1	6	4	0
D_4	0	0	4	5	2
D_5	0	0	0	2	9

The entry in the diagonal of the table shows the number of computers utilized by one department. The other entries are the number of computers utilized by the respective pair of departments.

Problem 3:

A company wants to provide equal number of computers to its 5 departments Administrations, Human resource, Logistics, Finance and Accounts. In order to reduce the idle time of the computers the company wants few computers are utilized by two departments. The computers shared by the departments are Human resource and Logistics, Human resource and Administrations, Administrations and Accounts, Accounts and Finance and Finance and Logistics. Find the number of computers allotted to each department.Find the number of computers utilized by one department and by two departments.

Solution:

Let us denote the departments Administrations, Human resource, Logistics, Finance and Accounts as the vertices v_1, v_2, v_3, v_4, v_5 respectively. We take an edge between vertices if there is a sharing of computers between the corresponding departments. The graph and its Vertex magic total labeling are shown in fig 5.

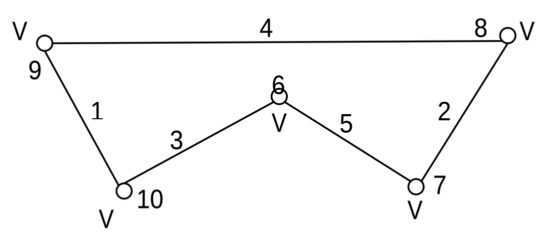

Fig. 5

This VMT labeling gives the magic constant k=14. Thus 14 computers are utilized by each department. The number of computers utilized by one department and which are utilized by two departments are given in the following table.

Table 3

Dept.	Ad	HR	LO	FI	ACC
AD	10	1	0	0	3
HR	1	9	4	0	0
LO	0	4	8	2	0
FI	0	0	2	7	5
ACC	3	0	0	5	6

The entry in the diagonal of the table shows the number of computers utilized by one department. The other entries are the number of computers utilized by the respective pair of departments.

Problem 4:

A company wants to provide equal number of computers to its 5 departments Administrations, Human resource, Logistics, Finance and Accounts. In order to reduce the idle time of the computers the company allows few computers can be utilized by two departments. Suppose every department utilizing few computers which are utilized by other departments. Find the number of computers allotted to each department. Find the number of computers utilized by one department and by two departments.

Solution:

We take the departments Administrations, Human resource, Logistics, Finance and Accounts as the vertices v_1, v_2, v_3, v_4, v_5 respectively. If two departments are sharing computers then we take an edge between the corresponding vertices. Here each pair of departments sharing computers. The graph corresponding to the given circumstance and its vertex magic labeling are shown in fig. 6.

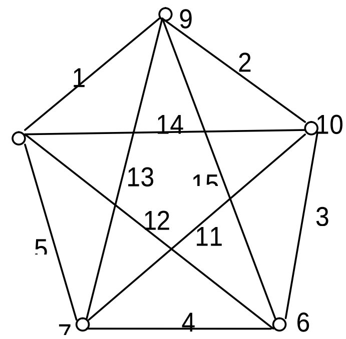

Fig 6

This VMT labeling gives the magic constant k=40. Thus each department provided with 40 computers. Number of computers utilized by a one department and which are utilized by two departments are given in the following table 4.

Table 4

Dept.	AD	HR	LO	FI	ACC
AD	9	2	15	13	1
HR	2	10	3	11	14
LO	15	3	6	4	12
FI	13	11	4	7	5
ACC	1	14	12	5	8

The entry in the diagonal of the table shows the number of computers utilized by one department. The other entries are the number of computers utilized by the respective pair of departments.

Remark :

1. The vertex magic labeling considered in the problems 2 and 3 are also a super vertex magic labeling (SVM). In this SVM labeling, the edge labels are smaller than the vertex labels. The computers shared by more than one department have to be installed with more software or with more equipment. So the company has to spent more money on the computers which are utilized by more than one department. If a SVM is considered for the graph (if it exists for the graph) which is drawn according to the situation described, then the number of computers which are utilized by more than one department is minimized. Hence the amount spent by the company on those computers is minimized.

2. Suppose the company needs computers in large numbers, then consider the multiple of the labeling with any number.

4. Conclusion

By applying Z-magic labeling or vertex magic total labeling to the graph that is drawn according to the situation described by a company, we obtained the number of workstations/ computers to be allotted to each department in the company which wants to provide equal number of workstations/computers to its departments in such a way that each workstation/ computer is utilized either by two or by one department.

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