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Graceful labelling of arbitrary Supersubdivision of disconnected graph

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Abstract

In this paper we prove that arbitrary Supersubdivision of disconnected graph is graceful.

Key words: Graceful labelling, Supersubdivision of graphs, disconnected graphs.

AMS Subject Classification (2010):05C78

Introduction

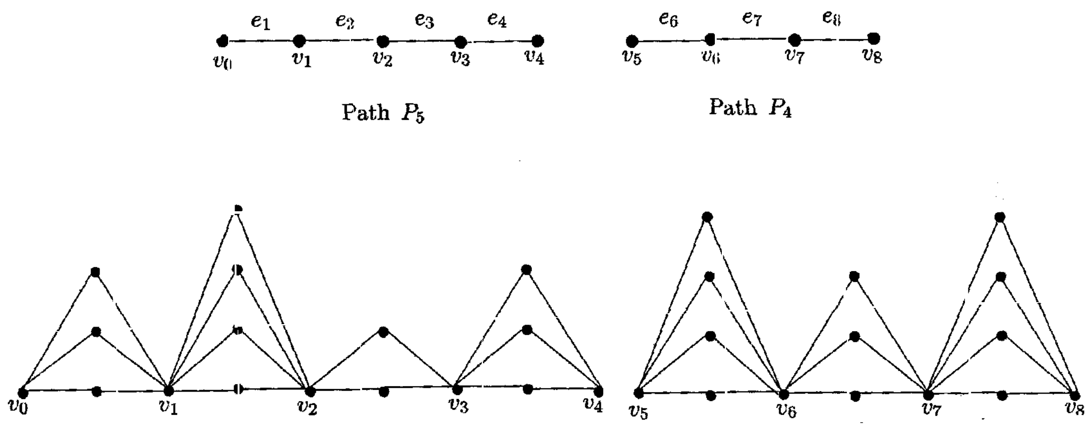
Let G be a graph with q edges. A graceful labelling of G is an injection from the set of its vertices to the set $\{0, 1, 2, ..., q\}$ such that the values of the edges are all integers from 1 to q, the value of an edge being the absolute value of the difference between the integers attributed to its end vertices. Recently G. Sethuraman and P. Selvaraju[6] have introduced a new method of construction called supersubdivision of a graph and showed that arbitrary supersubdivisions of paths are graceful. They conjectured that paths and stars are the only graphs for which every supersubdivision is graceful. Barrientos[1] disproved this conjecture by proving that every supersubdivision of a y-tree is graceful (recall that a y-tree is obtained from a path by appending an edge to a vertex of a path adjacent to an end point). Sethuraman and Selvaraju proved that every connected graph has some supersubdivision that is graceful. They pose that question as to whether some supersubdivision is valid for disconnected graphs[2].

In this paper we prove that arbitrary Supersubdivision of disconnected graph is graceful.

In the complete bipartite graph $K_{2,m}$ we call the part consisting of two vertices, the 2-vertices part of $K_{2,m}$ and the part consisting of m vertices the m-vertices part of $K_{2,m}$.

Let G be a graph with n vertices and t edges. A graph H is said to be supersubdivision of G if H is obtained by replacing every edge e_i of G by the complete bipartite graph $K_{2,m}$ for some positive integer m in such a way that the ends of e_i are merged with the two vertices part of $K_{2,m}$ after removing the edge e_i from G. A supersubdivision H of a graph G is said to be an arbitrary supersubdivision of the graph G if every edge of G is replaced by an arbitrary $K_{2,m}$ (m may vary for each edge arbitrarily).

A graph G is said to be connected if any two vertices of G are joined by a path. Otherwise it is called disconnected graph[3-6].



Arbitrary supersubdivision of $P_5 \cup P_4$

2. Main Results

Graceful labelling of arbitrary supersubdivision of disconneted paths.

Theorem 2.1. Arbitrary supersubdivision of $P_n \cup P_r$ is graceful provided the arbitrary supersubdivision is obtained by replacing each edge of G by $K_{2,m}$ with $m \geq 2$.

Proof. Let P_n be a path with successive vertices $v_0, v_1, v_2, ...v_{n-1}$ and let e_i $(1 \leq i \leq n-1)$ denote the edge $v_{i-1}\, v_i$ of P_n. Let P_r be a path with successive vertices $v_n, v_{n+1}, v_{n+2}, ...v_{n+r-1}$. Let H be an arbitrary supersubdivision of the disconnected graph $P_n \cup P_r$ where each edge e_i of $P_n \cup P_r$ is replaced by a complete bipartite graph K_{2,m_i} with $m_i \geq 2$ for $1 \leq i \leq n-1$ and $n+1 \leq i \leq n+r-1$. We observe that H has

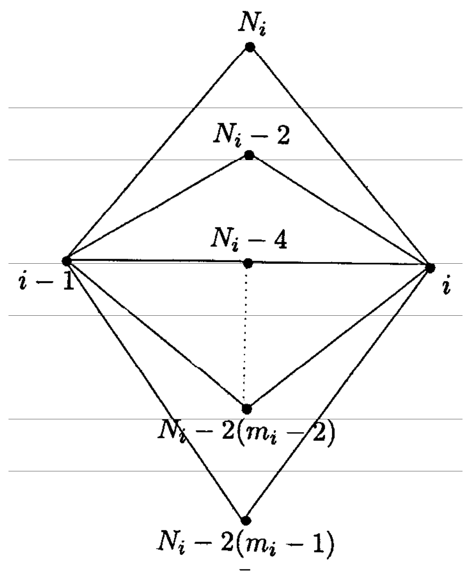
$M=2(m_1+m_2+....... + m_{n-1}+m_{n+1}+......+m_{n+r-1})$ edges.

Let $N_1 = M$

$N_i=M-2(m_1+m_2+.........+m_{i-1})+(i-1)$ for $2 \leq i \leq n-1$

$N_{n+1}=M-2(m_1+m_2+.........+m_{n-1})+(n-1)$

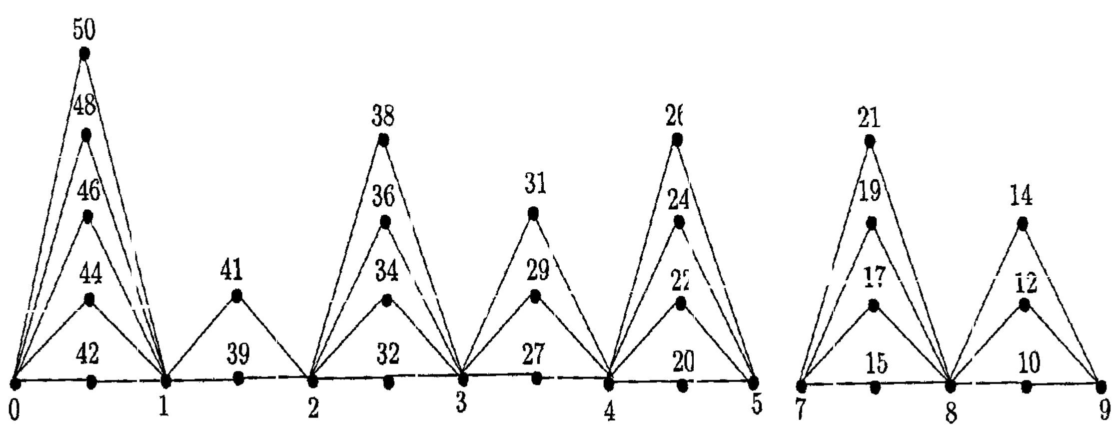
$N_i=N_{n+1}-2(m_{n+1}+m_{n+2}+.........+m_{i-1})+(i-n-1)$ for $n+2 \leq i \leq n+r-1$



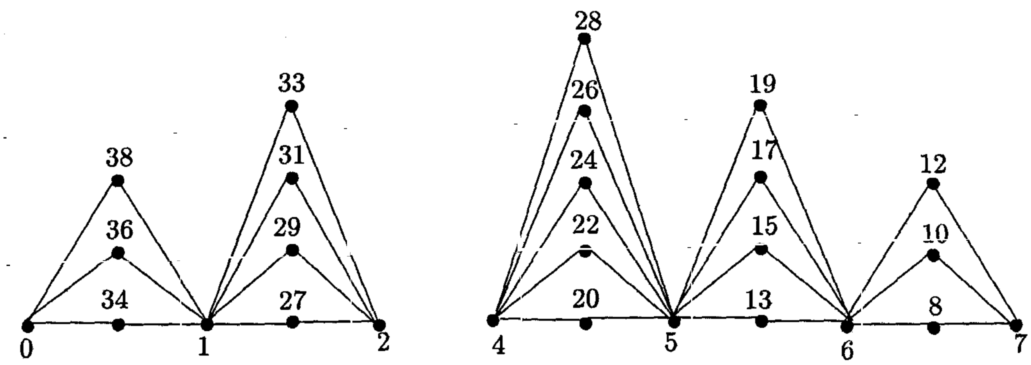
Graceful Labelling of K_{2,m_i} of H

Now for each i, $1 \le i \le n$-1, the two vertices of the 2-vertices part of K_{2,m_i} get the labels i-1 and i and the m_i vertices of the m_i-vertices part of K_{2,m_i} get the labels N_i, N_i-2, N_i-4,....,N_i-2 (m_i-1). For $n+1 \le i \le n+r-1$, the two vertices of the 2-vertices part of K_{2,m_i} get the labels i and i + 1 and the m_i vertices part of K_{2,m_i} get the labels N_i, N_i-2, N_i-4,....,N_i-2 (m_i-1). It is clear from the above labelling that the m_i+2 vertices of K_{2,m_i} have distinct labels and the 2m_i edges of K_{2,m_i} also have distinct labels for $1 \le i \le n$-1 and $n+1 \le i \le n+r-1$. Therefore the vertices of each K_{2,m_i}, $1 \le i \le n$-1 and $n+1 \le i \le n+r-1$ in the arbitrary supersubdivision H of $P_n \cup P_r$ have distinct labels and also the edges of each K_{2,m_i}, $1 \le i \le n$-1 and $n+1 \le i \le n+r-1$ in the arbitrary supersubdivision graph H of $P_n \cup P_r$ have distinct labels. Hence H is graceful[2].

Illustrative examples for the labelling of theorem 1.



Graceful labelling of the arbitrary supersubdivision of $P_6 \cup P_3$



Graceful labelling of the arbitrary supersubdivision of $P_6 \cup P_3$

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