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Almost Continuous Mappings in Intuitionistic Fuzzy Topological Spaces

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Abstract

The purpose of this paper is to introduce and study the concepts of intuitionistic fuzzy almost regular weakly generalized open mappings, intuitionistic fuzzy almost regular weakly generalized closed mappings and almost contra regular weakly generalized continuous mappings in intuitionistic fuzzy topological space and we investigate some of its properties. Also we provide the relations between intuitionistic fuzzy almost regular weakly generalized closed mappings and other intuitionistic fuzzy closed mappings.

Key words: Intuitionistic fuzzy topology, intuitionistic fuzzy regular weakly generalized closed set, intuitionistic fuzzy almost regular weakly generalized closed mappings, intuitionistic fuzzy almost regular weakly generalized open mappings, intuitionistic fuzzy almost contra regular weakly generalized continuous mappings intuitionistic fuzzy $_{rw}T_{1/2}$ ($IF_{rw}T_{1/2}$) space and intuitionistic fuzzy $rwgT_{1/2}$ (IF $rwgT_{1/2}$ in short) space.

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1. Introduction

After the introduction of Fuzzy set (FS) by Zadeh in 1965 and fuzzy topology by Chang[2] in 1967, several researches were conducted on the generalizations of the notions of fuzzy sets and fuzzy topology.The concept of intuitionistic fuzzy set (IFS) was introduced by Atanassov[1] in 1983 as a generalization of fuzzy sets. In 1997 Coker[3] introduced the concept of intuitionistic fuzzytopologica space. In tthis paper we introduce intuitionistic fuzzy almost regular weakly generalized closed mappings, intuitionistic fuzzy almost regular weakly generalized open mappings, intuitionistic fuzzy almost contra regular weakly generalized continuous.

2. *Preliminaries :*

Definition[1] *2.1:* Let X be a non empty fixed set. An *intuitionistic fuzzy set* (IFS in short) A in X is an object having the form $A=\{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$ where the functions $\mu_A(x): X \to [0, 1]$ and $\nu_A(x): X \to [0, 1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \le \mu_A(x) + \nu_A(x) \le 1$ for each $x \in X$. Denote by IFS(X), the set of all intuitionistic fuzzy sets in X.

Definition[1] *2.2* Let A and B be IFSs of the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x\in X\}$ and $B=\{\langle x, \mu_B(x), \nu_B(x)\rangle / x \in X\}$. Then

(a) $A \subseteq B$ if and only if $\mu_A(x) \le \mu_B(x)$ and $\nu_A(x) \ge \nu_B(x)$ for all $x \in X$

(b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$

(c) $A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle / x \in X\}$

(d) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle / x \in X\}$

(e) $A \cup B=\{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x)\wedge\nu_B(x)\rangle / x \in X\}$.

For the sake of simplicity, we shall use the notation $A = \langle x, \mu_A, \nu_A\rangle$ instead of $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$. Also for the sake of simplicity, we shall use the notation $A = \langle x, (\mu_A, \mu_B), (\nu_A, \nu_B)\rangle$ instead of $A = \langle x, (A/\mu_A, B/\mu_B), (A/\nu_A, B/\nu_B)\rangle$.

The intuitionistic fuzzy sets $0_\sim = \{\langle x, 0, 1\rangle / x \in X\}$ and $1_\sim = \{\langle x, 1, 0\rangle / x \in X\}$ are the empty set and the whole set of X, respectively.

Definition[3] *2.3* An *intuitionistic fuzzy topology* (IFT in short) on a non empty set X is a family τ of IFSs in X satisfying the following axioms:

(a) $0_\sim, 1_\sim \in \tau$

(b) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$

(c) $\cup G_i \in \tau$ for any arbitrary family $\{G_i / i\in J\} \subseteq \tau$.

In this case the pair (X, τ) is called an *intuitionistic fuzzy topological space* (IFTS in short) and any IFS in τ is known as an *intuitionistic fuzzy open set* (IFOS in short) in X.

The complement A^c of an IFOS A in an IFTS (X, τ) is called an *intuitionistic fuzzy closed set* (IFCS in short) in X.

Definition[3] *2.4:* Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A\rangle$ be an IFS in X. Then the *intuitionistic fuzzy interior* and an *intuitionistic fuzzy closure* are defined by

$\mathrm{int}(A) = \cup\{G / G$ is an IFOS in X and $G\subseteq A\}$

$\mathrm{cl}(A) = \cap\{K / K$ is an IFCS in X and $A \subseteq K\}$.

Note that for any IFS A in (X, τ), we have $\mathrm{cl}(A^c) = (\mathrm{int}(A))^c$ and $\mathrm{int}(A^c)=(\mathrm{cl}(A))^c$ [14].

Definition[4] *2.5* An IFS $A= \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$ in an IFTS (X, τ) is said to be

(a) *intuitionistic fuzzy semi closed set* (IFSCS in short) if int(cl (A)) $\subseteq$ A
(b) *intuitionistic fuzzy α-closed set* (IFαCS in short) if cl(int(cl(A))) $\subseteq$ A
(c)*intuitionistic fuzzy pre-closed set* (IFPCS in short) if cl(int(A)) $\subseteq$ A
(d) *intuitionistic fuzzy regular closed set* (IFRCS in short) if cl(int(A)) = A
(e) *intuitionistic fuzzy generalized closed set* (IFGCS in short) if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is an IFOS
(f) *intuitionistic fuzzy generalized semi closed set* (IFGSCS in short) if scl(A) $\subseteq$ U, whenever A $\subseteq$ U and U is an IFOS
(g) *intuitionistic fuzzy α generalized closed set* (IFαGCS in short) if αcl(A) $\subseteq$ U, whenever A $\subseteq$ U and U is an IFOS.

An IFS A is called intuitionistic fuzzy semi open set, intuitionistic fuzzy α-open set, intuitionistic fuzzy pre-open set, intuitionistic fuzzy regular open set, intuitionistic fuzzy generalized open set, intuitionistic fuzzy generalized semi open set and intuitionistic fuzzy α generalized open set (IFSOS, IFαOS, IFPOS, IFROS, IFGOS, IFGSOS and IαFGOS) if the complement A^c is an IFSCS, IFαCS, IFPCS, IFRCS, IFGCS, IFGSCS and IFαGCS respectively.

Definition[5] *2.6* An IFS A = $\{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$ in an IFTS (X, τ) is said to be an *intuitionistic fuzzy regular weakly generalized closed set* (IFRWGCS in short) if cl(int(A))$\subseteq$U whenever A $\subseteq$ U and U is an IFROS in X .

The family of all IFRWGCSs of an IFTS (X, τ) is denoted by IFRWGC(X).

Definition[5] *2.7* An IFS A is said to be an *intuitionistic fuzzy regular weakly generalized open set* (IFRWGOS in short) in (X, τ) if the complement A^c is an IFRWGCS in X.

The family of all IFRWGOSs of an IFTS (X, τ) is denoted by IFRWGO(X).

Result[5] *2.8* Every IFCS, IFαCS, IFGCS, IFRCS, IFPCS, IFαGCS is an IFRWGCS but the converses need not be true in general.

Definition[6] *2.9* Let (X, τ) be an IFTS and A = $\langle x, \mu_A, \nu_A\rangle$ be an IFS in X. Then the intuitionistic fuzzy regular weakly generalized interior and an intuitionistic fuzzy regular weakly generalized closure are defined by

rwgint(A) = $\cup$ { G / G is an IFRWGOS in X and G $\subseteq$ A }
rwgcl (A) = $\cap$ { K / K is an IFRWGCS in X and A $\subseteq$ K }.

Definition[3] *2.10* Let f be a mapping from an IFTS (X, τ) to an IFTS (Y, σ). If B $=\{\langle y,\mu_B(y), \nu_B(y)\rangle / y\in Y\}$ is an IFS in Y, then the pre-image of B under f denoted by f^{-1}(B), is the IFS in X defined by $f^{-1}(B) = \{\langle x, f^{-1}(\mu_B(x)), f^{-1}(\nu_B(x))\rangle / x \in X\}$.

If A = $\{\langle x, \lambda_A(x), \nu_A(x)\rangle / x \in X\}$ is an IFS in X, then the image of A under f denoted by f(A) is the IFS in Y defined by f (A) = $\{\langle y, f(\lambda_A(y)), f\_(\nu_A(y))\rangle / y \in Y\}$ where $f\_(\nu_A) = 1\text{-}f(1\text{-}\nu_A)$.

Definition[7] *2.11* A mapping f: (X, τ) $\rightarrow$ (Y, σ) from an IFTS (X, τ) into an IFTS (Y,

σ) is called an *intuitionistic fuzzy regular weakly generalized continuous* (IFRWG continuous in short) if $f^{-1}(B)$ is an IFRWGCS in (X, τ) for every IFCS B of (Y,σ).

Definition[6] *2.12* A mapping f: $(X, \tau) \to (Y, \sigma)$ from an IFTS (X, τ) into an IFTS (Y, σ) is called an *intuitionistic fuzzy regular weakly generalized irresolute* (IFRWG irresolute in short) if $f^{-1}(B)$ is an IFRWGCS in (X, τ) for every IFRWGCS B of (Y, σ).

Definition[4] *2.13* A mapping f: $(X, \tau) \to (Y, \sigma)$ from an IFTS (X, τ) into an IFTS (Y, σ) is said to be

(a) intuitionistic fuzzy semi closed mapping (IFSCM for short) if f(A) is an IFSCS in Y for every IFCS A in X.
(b) intuitionistic fuzzy pre-closed mapping (IFPCM for short) if f(A) is an IFPCS in Y for every IFCS A in X.
(c) intuitionistic fuzzy α-closed mapping (IFαCM for short) if f(A) is an IFαCS in Y for every IFCS A in X.

Definition[9] *2.14* A mapping f: $(X, \tau) \to (Y, \sigma)$ from an IFTS (X, τ) into an IFTS (Y, σ) is said to be intuitionistic fuzzy α-generalized closed mapping (IFαGCM for short) if f(A) is an IFαGCS in Y for every IFCS A in X.

Definition[11] *2.15* A mapping f: $(X, \tau) \to (Y, \sigma)$ from an IFTS (X, τ) into an IFTS (Y, σ) is said to be intuitionistic fuzzy closed mapping (IFCM for short) if f(A) is an IFCS in Y for every IFCS A in X.

Definition[5] *2.16* An IFTS (X, τ) is said to be an intuitionistic fuzzy $rwT_{1/2}$ (IF $rwT_{1/2}$ in short) space if every IFRWGCS in X is an IFCS in X.

Definition[8] *2.17* An IFTS (X, τ) is said to be an intuitionistic fuzzy $rwgT_{1/2}$ (IF $rwgT_{1/2}$ in short) space if every IFRWGCS in X is an IFPCS in X.

3. Intuitionistic fuzzy almost regular weakly generalized open mappings :

In this section we introduce intuitionistic fuzzy almost regular weakly generalized open mappings, intuitionistic fuzzy almost regular weakly generalized closed mappings and studied some of its properties.

Definition 3.1: A mapping f: $X \to Y$ is called an intuitionistic fuzzy almost regular weakly generalized open mappings (IFARWGOM for short) if f(A) is an IFRWGOS in Y for each IFROS A in X.

Example 3.2: Let X = { a, b }, Y = { u, v } and $G_1 = \langle x, (0.2, 0.3, 0.2), (0.7, 0.7, 0.8) \rangle$, $G_2 = \langle y, (0.2, 0.3, 0.2), (0.7, 0.7, 0.8) \rangle$. Then, $\tau = \{ 0_\sim, G_1, 1_\sim \}$ and $\sigma = \{ 0_\sim, G_2, 1_\sim \}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \to (Y, \sigma)$ by f(a) = u and f(b) = v. Then f is an IFARWGOM.

Definition[10] *3.3:* A mapping f: $(X, \tau) \to (Y, \sigma)$ is called an intuitionistic fuzzy almost regular weakly generalized closed mappings (IFARWGCM) if f (A) is an IFRWGCS in (Y, σ) for every IFRCS A of (X, τ).

Example 3.4: Let X={ a, b }, Y = { u, v } and $G_1 = \langle x, (0.7, 0.6, 0.5), (0.2, 0.3, 0.5) \rangle$,

$G_2 = \langle$ y, (0.7, 0.6, 0.5), (0.2, 0.3, 0.5) $\rangle$. Then, $\tau = \{ 0_\sim, G_1, 1_\sim \}$ and $\sigma = \{ 0_\sim, G_2, 1_\sim \}$ are IFTs on X and Y respectively. Define a mapping f : (X, τ) $\rightarrow$ (Y, σ) by f(a) = u and f(b) = v. Then f is an IFARWGCM.

Theorem 3.5: Every IFCM is an IFARWGCM but not conversely.

Proof : Assume that f : (X, τ) $\rightarrow$ (Y, σ) is an IFCM. Let A be an IFRCS in X. This implies A is an IFCS in X. Since f is an IFCM, f (A) is an IFCS in Y. Since IFCS is an IFRWGCS, f (A) is an IFRWGCS in Y. Hence f is an IFARWGCM.

Example : Let X = { a, b }, Y = {u, v} and $G_1 = \langle$ x, (0.4, 0.2, 0.1), (0.5, 0.4, 0.3) $\rangle$, $G_2 = \langle$ y, (0.3, 0.2, 0.1), (0.6, 0.7, 0.5) $\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: (X, τ) $\rightarrow$ (Y, σ) by f(a) = u and f(b) = v. Then, f is an IFARWGCM but f is not an IFCM since $G_1^c = \langle$ x, (0.5, 0.4, 0.3), (0.4, 0.2, 0.1) $\rangle$ is an IFCS in X.

Theorem 3.6: Every IFαGCM is an IFARWGCM but not conversely.

Proof : Let f: (X, τ) $\rightarrow$ (Y, σ) be an IFαGCM. Let A be an IFRCS in X. This implies A is an IFCS in X. Then by hypothesis f (A) is an IFαGCS in Y. Since every IFαGCS is an IFRWG, f(A) is an IFRWGS in Y. Hence f is an IFARWGCM

Example : Let X = {a, b}, Y = {u, v} and $G_1 = \langle$ x, (0.3, 0.4, 0.2), (0.4, 0.5, 0.3) $\rangle$, $G_2 = \langle$ y, (0.7, 0.6, 0.5), (0.3, 0.4, 0.5) $\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: (X, τ) $\rightarrow$ (Y, σ) by f(a) = u and f(b) = v. Then, f is an IFARWGCM but not an IFαGCM since $G_1^c = \langle$ x, (0.4, 0.5, 0.3), (0.3, 0.4, 0.2) $\rangle$ is an IFCS in X but f $(G_1^c) = \langle$ y, (0.4, 0.5, 0.3), (0.3, 0.4, 0.2) $\rangle$ is not an IFαGCS in Y.

Theorem 3.7: Every IFACM is an IFARWGCM but not conversely.

Proof: Let f: (X, τ) $\rightarrow$ (Y, σ) be an IFACM. Let A be an IFRCS in X. Since f is IFACM, f(A) is an IFCS in Y. Since every IFCS is an IFRWGCS, f(A) is an IFRWGCS in Y. Hence f is an IFARWGCM.

Example : Let X = { a, b }, Y = {u, v} and $G_1 = \langle$ x, (0.4, 0.2, 0.02), (0.5, 0.4, 0.4) $\rangle$, $G_2 = \langle$ y, (0.3, 0.2, 0.2), (0.6, 0.7, 0.7) $\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f : (X, τ) $\rightarrow$ (Y, σ) by f(a) = u and f(b) = v. Then, f is an IFARWGCM but f is not an IFACM since $G_1^c = \langle$ x, (0.5, 0.4, 0.4), (0.4, 0.2, 0.2) $\rangle$ is an IFRCS in X but f $(G_1^c) = \langle$ y, (0.5, 0.4, 0.4), (0.4, 0.2, 0.2) $\rangle$ is not an IFCS in Y.

Theorem 3.8: Every IFAαGCM is an IFARWGCM but not conversely.

Proof : Let f: (X, τ) $\rightarrow$ (Y,σ) be an IFAαGCM. Let A be an IFRCS in X. Since f is IFAαGCM, f(A) is an IFαGCS in Y. Since every IFαGCS is an IFRWGCS, f(A) is an IFRWGCS in Y. Hence f is an IFARWGCM.

Example : Let X = {a, b}, Y = {u, v} and $G_1 = \langle$ x, (0.3, 0.4, 0.3), (0.4, 0.5, 0.4) $\rangle$, $G_2 = \langle$ y, (0.7, 0.6, 0.7), (0.3, 0.4, 0.3) $\rangle$. Then

$\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: $(X, \tau) \to (Y, \sigma)$ by f(a) = u and f(b) = v. Then, f is an IFARWGCM but f is not an IFAαGCM since $G_1^c = \langle$ x, (0.4, 0.5, 0.4), (0.3, 0.4, 0.3) $\rangle$ is an IFRCS in Y but f $(G_1^c) = \langle$ y, (0.4, 0.5, 0.4), (0.3, 0.4, 0.3) $\rangle$ is not an IFGCS in Y.

Theorem 3.9: A bijective mapping f: $X \to Y$ is an IFARWG closed mapping if and only if the image of each IFROS in X is an IFRWGOS in Y.

Proof: Necessity: Let A be an IFROS in X. This implies A^c is IFRCS in X. Since f is an IFARWG closed mapping, $f(A^c)$ is an IFRWGCS in Y. Since $f(A^c) = (f(A))^c$, f(A) is an IFRWGOS in Y.

Sufficiency: Let A be an IFRCS in X. This implies A^c is an IFROS in X. By hypothesis, $f(A^c)$ is an IFRWGOS in Y. Since $f(A^c) = (f(A))^c$, f(A) is an IFRWGCS in Y. Hence f is an IFARWG closed mapping.

Theorem 3.10: Let f: $(X, \tau) \to (Y,\sigma)$ be an IFARWG closed mapping. Then f is an IFA closed mapping if Y is an $IF_{rw}T_{1/2}$ space.

Proof: Let A be an IFRCS in X. Then f(A) is an IFRWGCS in Y, by hypothesis. Since Y is an $IF_{rw}T_{1/2}$ space, f(A) is an IFCS in Y. Hence f is an IFA closed mapping.

Theorem 3.11: Let f: $X \to Y$ be a bijective mapping. Then the following are equivalent.
(i) f is an IFARWGOM
(ii) f is an IFARWGCM

Proof: Straightforward

Theorem 3.12: Let f : $(X, \tau) \to (Y, \sigma)$ be a mapping from an IFTS X into an IFTS Y. Then the following conditions are equivalent if Y is an $IF_{rw}T_{1/2}$ space.
(i) f is an IFARWGCM
(ii) f is an IFARWGOM
(iii) $f(int(A)) \subseteq int(cl(int(f(A))))$ for every IFROS A in X.

Proof: **(i) ⇒ (ii):** It is obviously true.
(ii) ⇒ (iii): Let A be any IFROS in X. This implies A is an IFOS in X. Then int(A) is an IFOS in X. Then f(int(A)) is an IFRWGOS in Y. Since Y is an $IF_{rw}T_{1/2}$ space, f(int(A)) is an IFOS in Y. Therefore $f(int(A)) = int(f(int(A)) \subseteq int(cl(int(f(A))))$.

(iii) ⇒ (i): Let A be an IFRCS in X. Then its complement A^c is an IFROS in X.By hypothesis $f(int(A^c)) \subseteq int(cl(int(f(A^c))))$. This implies $f(A^c) \subseteq int(cl(int(f(A^c))))$. Hence $f(A^c)$ is an IFαOS in Y. Since every IFαOS is an IFRWGOS, $f(A^c)$ is an IFRWGOS in Y. Therefore f(A) is an IFRWGCS in Y. Hence f is an IFARWGCM.

Theorem 3.13: Let f: $(X, \tau) \to (Y, \sigma)$ be a mapping from an IFTS X into an IFTS Y. Then the following conditions are equivalent if Y is an $IF_{rw}T_{1/2}$ space.
(i) f is an IFARWGCM
(ii) If B is an IFROS in X then f (B) is an IFRWGOS in Y
(iii) $f(B) \subseteq int(cl(f(B))$ for every IFROS B in X.

Proof: **(i) ⇒ (ii):** obviously.

(ii) $\Rightarrow$ (iii): Let B be any IFROS in X. Then by hypothesis f (B) is an IFRWGOS in Y. Since X is an $IF_{rw}T_{1/2}$ space, f(B) is an IFOS in Y (Result 2.23). Therefore f (B) = int(f(B)) $\subseteq$ int(cl(f(B))).

(iii) $\Rightarrow$ (i): Let B be an IFRCS in X. Then its complement B^c is an IFROS in X. By hypothesis $f(B^c) \subseteq int(cl(f(B^c)))$. Hence $f(B^c)$ is an IFRWGOS in Y. Therefore f (B) is an IFRWGCS in Y. Hence f is an IFARWGCM.

Theorem 3.14: Let f: $(X, \tau) \rightarrow (Y, \sigma)$ be an IFA closed mapping and g : $(Y, \sigma) \rightarrow (Z, \delta)$ is IFARWG closed mapping, then g o f : (X, τ) (Z, δ) is an IFA closed mapping. if Z is an $IF_{rw}T_{1/2}$ space.

Proof : Let A be an IFRCS in X. Then f(A) is an IFCS in Y. Since g is an IFRWG closed mapping, g(f(A)) is an IFRWGCS in Z. Therefore g(f(A)) is an IFCS in Z, by hypothesis.Hence g o f is an IFA closed mapping.

Theorem 3.15: Let f: $(X, \tau) \rightarrow (Y, \sigma)$ be an IFA closed mapping and g : $(Y, \sigma) \rightarrow (Z, \eta)$ be an IFRWG closed mapping. Then g o f : $(X, \tau) \rightarrow (Z, \eta)$ is an IFARWG closed mapping.

Proof: Let A be an IFRCS in X. Then f(A) is an IFCS in Y, by hypothesis. Since g is an IFRWG closed mapping, g(f(A)) is an IFRWGCS in Z. Hence g o f is an IFARWG closed mapping.

Theorem 3.16: If f : $(X, \tau) \rightarrow (Y, \sigma)$ is an IFARWG closed mapping and Y is an $IF_{rw}T_{1/2}$ space, then f(A) is an IFGCS in Y for every IFRCS A in X.

Proof : Let f : $(X, \tau) \rightarrow (Y, \sigma)$ be a mapping and let A be an IFRCS in X. Then by hypothesis f(A) is an IFRWGCS in Y. Since Y is an $IF_{rw}T_{1/2}$ space, f(A) is an IFGCS in Y.

Theorem 3.17: Let f: X $\rightarrow$ Y be a bijective mapping. Then the following are equivalent.

(i) f is an IFARWGOM

(ii) f is an IFARWGCM

(iii) f^{-1} is an IFARWG continuous mapping

Proof : (i) $\Leftrightarrow$ (ii) is obvious from the Theorem 3.11.

(ii) $\Rightarrow$ (iii) Let A $\subseteq$ X be an IFRCS. Then by hypothesis, f(A) is an IFRWGCS in Y. That is $(f^{-1})^{-1}(A)$ is an IFRWGCS in Y. This implies f^{-1} is an IFARWG continuous mapping. (iii) $\Rightarrow$ (ii) Let A $\subseteq$ X be an IFRCS. Then by hypothesis $(f^{-1})^{-1}(A)$ is an IFRWGCS in Y. That is f(A) is an IFRWGCS in Y. Hence f is an IFARWGCM.

4. Intuitionistic fuzzy almost contra regular weakly generalized continuous mappings:

In this section we introduce intuitionistic fuzzy almost contra regular weakly generalized continuous mappings, and studied some of its properties.

Definition[12] *4.1:* A mapping f :$(X, \tau) \rightarrow (Y, \sigma)$ is called an intuitionistic fuzzy almost contra regular weakly generalized continuous (IFACRWG continuous in short) mapping if $f^{-1}(B)$ is an IFRWGCS in (X, τ) for every IFROS B of (Y, σ).

Example 4.2: Let X={a, b}, Y={u,v} and $G_1 = \langle x, (0.2,0.3, 0.3), (0.8, 0.7, 0.7)\rangle$,
$G_2 = \langle y, (0.2, 0.2, 0.1), (0.8, 0.7, 0.9)\rangle$.
Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Consider a mapping $f : (X, \tau) \to (Y, \sigma)$ defined as f(a)= u and f(b) = v. This mapping f is an intuitionistic fuzzy almost contra regular weakly generalized continuous mapping.

Theorem 4.3: Every intuitionistic fuzzy almost contra continuous mapping is an intuitionistic fuzzy almost contra regular weakly generalized continuous mapping but not conversely.

Proof: Let $f : (X, \tau) \to (Y, \sigma)$ be an intuitionistic fuzzy contra continuous mapping. Let A be an IFROS in Y. By hypothesis, $f^{-1}(A)$ is an IFCS in X. Since every IFCS is an IFRWGCS, $f^{-1}(A)$ is an IFRWGCS in X. Hence f is an intuitionistic fuzzy almost contra weakly generalized continuous mapping.

Example: Let X = {a, b}, Y = {u, v} and
$G_1=\langle x, (0.4, 0.5, 0.5), (0.6, 0.5, 0.5)\rangle$,
$G_2=\langle y, (0.2, 0.3, 0.3), (0.8, 0.7, 0.7)\rangle$
Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Consider a mapping $f : (X, \tau) \to (Y, \sigma)$ defined as f(a) = u and f(b) = v. This f is an intuitionistic fuzzy almost contra weakly generalized continuous mapping but not an intuitionistic fuzzy contra continuous mapping,since the IFS G_2 is an IFOS in Y but $f^{-1}(G_2)= \langle x, (0.2, 0.3, 0.3), (0.8, 0.7, 0.7)\rangle$ is not an IFCS in X.

Theorem 4.4: Let $f : (X, \tau) \to (Y, \sigma)$ be an intuitionistic fuzzy almost contra regular weakly generalized continuous mapping and X an $IF_{rw}T_{1/2}$ space. Then f is an intuitionistic fuzzy contra continuous mapping.

Proof: Let B be an IFROS in Y. By hypothesis, $f^{-1}(B)$ is an IFRWGCS in X. Since X is an $IF_{rw}T_{1/2}$ space, $f^{-1}(B)$ is an IFCS in X. Hence f is an intuitionistic fuzzy contra continuous mapping.

Theorem 4.5: Let $f : (X, \tau) \to (Y, \sigma)$ be a mapping from an IFTS X into an IFTS Y and X an ${}_{rw}T_{1/2}$ space. Then the following statements are equivalent.

(a) f is an intuitionistic fuzzy almost contra regular weakly generalized continuous mapping,

(b) f is an intuitionistic fuzzy contra continuous mapping.

Proof: Obvious.

Theorem 4.6: Every intuitionistic fuzzy contra pre-continuous mapping is an intuitionistic fuzzy almost contra regular weakly generalized continuous mapping but not conversely.

Poof: Let $f : (X, \tau) \to (Y, \sigma)$ be an intuitionistic fuzzy contra pre continuous mapping. Let A be an IFROS in Y. By hypothesis, $f^{-1}(A)$ is an IFPCS in X. Since every IFPCS is an IFRWGCS, $f^{-1}(A)$ is an IFRWGCS in X. Hence f is an intuitionistic fuzzy almost contra regular weakly generalized continuous mapping.

Example: Let X = {a, b}, Y= {u, v} and
$G_1 = \langle x, (0.5, 0.3, 0.4), (0.5, 0.7, 0.6)\rangle$,
$G_2 = \langle x, (0.8, 0.8, 0.8), (0.2, 0.2, 0.8)\rangle$ and

$G_3 = \langle y, (0.9, 0.3, 0.3), (0.1, 0.6, 0.6) \rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Consider a mapping $f : (X,\tau) \rightarrow (Y,\sigma)$ defined as f(a) = u and f(b) = v. This f is an intuitionistic fuzzy almost contra regular weakly generalized continuous mapping but not an intuitionistic fuzzy contra pre-continuous mapping, since the IFS T_3 is an IFOS in Y but $f^{-1}(G_3) = \langle x, (0.9, 0.3, 0.3), (0.1, 0.6, 0.6) \rangle$ is not an IFPCS in X.

Theorem 4.7: Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be an intuitionistic fuzzy almost contra weakly generalized continuous mapping and X an IF_{wg} T_q space. Then f is an intuitionistic fuzzy contra pre-continuous mapping.

Proof: Let B be an IFOS in Y. By hypothesis, $f^{-1}(B)$ is an IFRWGCS in X. Since X is an IF_{wg} T_q space, $f^{-1}(B)$ is an IFPCS in X. Hence f is an intuitionistic fuzzy contra pre continuous mapping.

Theorem 4.8: Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a mapping from anIFTS X into an IFTS Y and X an IF $rwgT_{1/2}$ space. Then the following statements are equivalent.

(a) f is an intuitionistic fuzzy almost contra regular weakly generalized continuous mapping,

(b) f is an intuitionistic fuzzy contra pre-continuous mapping.

Proof: Obvious.

Theorem 3.9: Every intuitionistic fuzzy contra α continuous mapping is an intuitionistic fuzzy almost contra regular weakly generalized continuous mapping but not conversely.

Proof: Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be an intuitionistic fuzzy contra α continuous mapping. Let A be an IFROS in Y. By hypothesis, $f^{-1}(A)$ is an IFαCS in X. Since every IFαCS is an IFRWGCS, $f^{-1}(A)$ is an IFRWGCS in X. Hence f is an intuitionistic fuzzy almost contra regular weakly generalized continuous mapping.

Example: Let X = {a, b}, Y = {u, v} and $G_1 = \langle x, (0.5, 0.3, 0.2), (0.5, 0.7, 0.8) \rangle$, $G_2 = \langle y, (0.4, 0.2, 0.1), (0.6, 0.8, 0.9)$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Consider a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ defined as f(a) = u and f(b) = v. This f is an intuitionistic fuzzy contra weakly generalized continuous mapping but not an intuitionistic fuzzy contra α continuous mapping, Since the IFS T_2 is an IFOS in Y but $f^{-1}(G_2) = \langle x, (0.4, 0.2, 0.1), (0.6, 0.8, 0.9) \rangle$ is not an IFαCS in X.

Theorem 3.10: Every intuitionistic fuzzy contra αgeneralized continuous mapping is an intuitionistic fuzzy almost contra regular weakly generalized continuous mapping but not conversely.

Proof: Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be an intuitionistic fuzzy contra α generalized continuous mapping. Let A be an IFROS in Y. By hypothesis, $f^{-1}(A)$ is an IFαGCS in X. Since every IFαGCS is an IFRWGCS, $f^{-1}(A)$ is an IFRWGCS in X. Hence f is an intuitionistic fuzzy contra weakly generalized continuous mapping.

Example: Let X = {a, b}, Y = {u, v} and $G_1 = \langle x, (0.5, 0.6, 0.6), (0.5, 0.4, 0.4) \rangle$, $G_2 = \langle y, (0.3, 0.5, 0.5), (0.6, 0.5, 0.5)$. Then

$\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Consider a mapping $f : (X, \tau) \to (Y, \sigma)$ defined as f(a) = u and f(b) = v. This f is an intuitionistic fuzzy contra weakly generalized continuous mapping but not an intuitionistic fuzzy contra α generalized continuous mapping, since the IFS T_2 is an IFOS in Y but $f^{-1}(T_2) = \langle x, (0.3, 0.5, 0.5), (0.6, 0.5, 0.5)\rangle$ is not an IFαGCS in X.

Remark 3.11: An intuitionistic fuzzy contra semi continuous mapping and an intuitionistic fuzzy almost contra regular weakly generalized continuous mapping are independent to each other as seen from the following examples.

Example: Let X = {a, b}, Y = {u, v} and $G_1 = \langle x, (0.4, 0.3, 0.2), (0.6, 0.7, 0.8)\rangle$, $G_2 = \langle y, (0.4, 0.3, 0.2), (0.6, 0.7, 0.8)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Consider a mapping $f : (X, \tau) \to (Y, \sigma)$ defined as f(a)=u and f(b)=v. This f is an intuitionistic fuzzy contra semi continuous mapping but not an intuitionistic fuzzy contra weakly generalized continuous mapping, since the IFS G_2 is an IFOS in Y but $f^{-1}(G_2) = \langle x, (0.4, 0.3, 0.2), (0.6, 0.7, 0.8)\rangle$ is not an IFWGCS in X.

Example : Let X = {a, b}, Y = {u, v} and $G_1 = \langle x, (0.9, 0.7, 0.6), (0.1, 0.2, 0.3)\rangle$, $G_2 = \langle y, (0.7, 0.6, 0.5), (0.3, 0.4, 0.5)$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Consider a mapping $f : (X, \tau) \to (Y, \sigma)$ defined as f(a) = u and f(b) = v. This f is an intuitionistic fuzzy contra weakly generalized continuous mapping but not an intuitionistic fuzzy contra semi continuous mapping, Since the IFS T_2 is an IFOS in Y but $f^{-1}(G_2) = \langle x, (0.7, 0.6, 0.5), (0.3, 0.4, 0.5)\rangle$ is not an IFSCS in X.

Theorem 3.12: Let $f : (X, \tau) \to (Y, \sigma)$ be a mapping and let $f^{-1}(A)$ be an IFRCS in X for every IFROS A in Y. Then f is an intuitionistic fuzzy almost contra regular weakly generalized continuous mapping.

Proof: Let A be an IFROS in Y. By hypothesis, $f^{-1}(A)$ is an IFRCS in X. Since every IFRCS is an IFRWGCS, $f^{-1}(A)$ is an IFRWGCS in X. Hence f is an intuitionistic fuzzy almost contra regular weakly generalized continuous mapping.

5. Conclusion

In this paper we introduce and study the concepts of intuitionistic fuzzy almost regular weakly generalized open mappings, intuitionistic fuzzy almost regular weakly generalized closed mappings and almost contra regular weakly generalized continuous mappings in intuitionistic fuzzy topological space and we investigate some of its properties.

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