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Bianchi type-I inflationary cosmological model in general relativity

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Abstract

Bianchi type-I inflationary cosmological model in the presence of mass less scalar field with a flat potential is investigated. A determinate solution is obtained without taking any supplementary condition between the metric potentials. Various physical and geometrical features of the model are also discussed.

1. Introduction

Inflationary cosmological models play an important role in solving a number of outstanding problems in cosmology like the homogeneity, the isotropy and the flatness of the observed universe. The idea of an early inflationary phase was introduced by Alan Guth[1] in order to explain the observed status of the universe (its isotropy, homogeneity and flatness) in a natural way, that is without having to assume special conditions in the context of Grand Unified Theory. Different versions of the inflationary universe have been studied by many authors viz. Linde[2], Albrecht and Steinhardt[3], Linde[4], Abott and Wise[5], Matarrese and Lucchin[6], Rothman and Ellis[7]. In all these models the flatness problem is understood and solved, but situation concerning the isotropy and homogeneity is far less clear. We can have a solution of isotropy problem if we work with anisotropic metrics and show that they can be isotropized and inflated under very general circumstances. We assume a homogeneous space due to the obvious difficulties in dealing with non homogeneous metrics.

Various interesting results about the inflationary cosmological models have also been obtained by Wald[9], Moss and Sahni[10], Martinez-Ganzalez and Jones[11], Huang[12], Schmidt[13], Bali and Jain[14] and Bali[15].

By taking into consideration the scalar Higgs field ϕ with potential $V(\phi)$, inflation will take place if $V(\phi)$ has a flat region and ϕ field evolves slowly but the universe expands in an exponential way due to vacuum field energy[8]. It is assumed that the scalar field will take sufficient time to cross the flat region so that the universe expands sufficiently to become homogeneous and isotropic on the scale of the order of the horizon size.

In this paper, we have investigated Bianchi Type-I inflationary cosmological model in the presence of massless scalar field with a flat potential. To get a determinate solution no supplementary condition is used between the metric potentials and a condition for the consistency of the solution of the Einstein field equations is also established. Various physical and geometrical features of the model are also discussed.

2. *Metric and Field Equations :*

We consider Bianchi type – I metric in the form

$$ds^2 = -dt^2 + A^2dx^2 + B^2dy^2 + C^2dz^2 \tag{1}$$

where the metric potentials A, B and C are functions of t-alone. The Lagrangian will be that of gravity minimally coupled to a scalar field ϕ of potential function $V(\phi)$ (Stein-Schabes (1987)) is given by

$$L = \int \sqrt{-g}\left(R - \frac{1}{2}g^{ij}\partial_i\phi\partial_j\phi - V(\phi)\right)dx^4 \tag{2}$$

Here the notations have their usual meanings and the units are so chosen that $8\pi G = c = 1$. Now variation of lagrangian L with respect to the dynamical fields, provide us Einstein field equations

$$R_{ij} - \frac{1}{2}R\,g_{ij} = -T_{ij} \tag{3}$$

with the energy momentum tensor T_i^j for the inflation, given by

$$T_{ij} = \partial_i\phi\partial_j\phi - \left[\frac{1}{2}\partial_k\phi\partial^k\phi + V(\phi)\right]g_{ij} \tag{4}$$

and

$$\frac{1}{\sqrt{-g}}\partial_i[\sqrt{-g}\,\partial^i\phi] = -\frac{d\,V(\phi)}{d\phi} \tag{5}$$

The field equations (3), for the line-element (1), leads to

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4C_4}{BC} = -\frac{1}{2}\dot{\phi}^2 + V(\phi) \tag{6}$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4C_4}{AC} = -\frac{1}{2}\dot{\phi}^2 + V(\phi) \tag{7}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4B_4}{AB} = -\frac{1}{2}\dot{\phi}^2 + V(\phi) \tag{8}$$

$$\frac{A_4B_4}{AB} + \frac{B_4C_4}{BC} + \frac{A_4C_4}{AC} = \frac{1}{2}\dot{\phi}^2 + V(\phi) \tag{9}$$

and using equation (5), the equation for the scalar field ϕ leads to

$$\ddot{\phi} + \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right)\dot{\phi} + \frac{dV(\phi)}{d\phi} = 0 \tag{10}$$

The dot and suffix 4 after A,B and C denote ordinary differentiation with respect to cosmic time t.

3. *Solution of the Field Equations :*

We are interested in inflationary solution of the model, so flat region is considered where potential function $V(\phi)$ is constant (Stein-Schabes (1987)) *i.e.*

$$V(\phi) = V_0 (\text{ constant, say}) \quad (11)$$

Equation (10) leads to

$$\dot{\phi} = \frac{\alpha}{ABC} \quad (12)$$

where α is a constant of integration.

Equation (8) – (7) gives

$$\frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{B}}{AB} - \frac{\dot{A}\dot{C}}{AC} = 0 \quad (13)$$

On integration, equation (13) leads to

$$\frac{\dot{B}}{B} - \frac{\dot{C}}{C} = \frac{k_1}{ABC} \quad (14)$$

Similarly, equation (8) – (6) gives

$$\frac{\ddot{A}}{A} - \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{B}}{AB} - \frac{\dot{B}\dot{C}}{BC} = 0 \quad (15)$$

On integration, equation (15) leads to

$$\frac{\dot{A}}{A} - \frac{\dot{C}}{C} = \frac{k_2}{ABC} \quad (16)$$

Equation (6) +(7) + (8) +3(9) leads to

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{2\dot{A}\dot{B}}{AB} + \frac{2\dot{B}\dot{C}}{BC} + \frac{2\dot{A}\dot{C}}{AC} = 3V_0 \quad (17)$$

With the help of equation (12), equation (17) leads to

$$\frac{d}{dt}\left(\frac{\ddot{\phi}}{\dot{\phi}}\right) - \left(\frac{\ddot{\phi}}{\dot{\phi}}\right)^2 + 3V_0 = 0 \quad (18)$$

Solution of equation (18), is given by

$$\dot{\phi} = N \operatorname{cosec} h \sqrt{3V_0}\, t \quad (19)$$

On integration equation (19) gives

$$\phi = N \log \tan h \frac{\{\sqrt{3V_0}\, t\}}{2} + N_0 \quad (20)$$

where N and N_0 are constants of integration.

With the help of equation (12), equation (14) and (16) leads to

$$B = k_3 C e^{\left(\frac{k_1}{\alpha}\right)\phi} \quad (21)$$

and

$$A = k_4 C e^{\left(\frac{k_2}{\alpha}\right)\phi} \quad (22)$$

Now equations (21), (22) and (12) lead to

$$C = \left(\frac{\alpha}{k_3 k_4 N}\right)^{1/3} \sin h^{1/3}\{\sqrt{3V_0}\, t\}\, e^{-\left(\frac{k_1+k_2}{3\alpha}\right)\phi} \quad (23)$$

where ϕ is given by equation (20).
Hence the metric (1) reduces to the form

$$ds^2 = -dt^2 + \left(\frac{\alpha k_4^2}{k_3 N}\right)^{2/3} e^{\frac{2(2k_2-k_1)}{3\alpha}\phi} \sinh^{2/3}\{\sqrt{3V_0}\, t\} dx^2 + \left(\frac{\alpha k_3^2}{k_4 N}\right)^{2/3} e^{\frac{2(2k_1-k_2)}{3\alpha}\phi} \sinh^{2/3}\{\sqrt{3V_0}\, t\} dy^2 + \left(\frac{\alpha}{k_3 k_4 N}\right)^{2/3} e^{-\frac{2(k_1+k_2)}{3\alpha}\phi} \sinh^{2/3}\{\sqrt{3V_0}\, t\} dz^2 \quad (24)$$

4. Some Physical and Geometrical Features:

The scalar of expansion (θ) for the flow vector v^i for the model (24), is given by

$$\theta = \sqrt{3V_0} \cot h\{\sqrt{3V_0}\, t\} \quad (25)$$

The rotation ω, for the metric (24) is identically zero. The components of the shear tensor for the metric (24) are given by

$$\sigma_1^1 = \left(\frac{2k_2 - k_1}{3\alpha}\right) N \operatorname{cosec} h\{\sqrt{3V_0}\, t\}$$

$$\sigma_2^2 = \left(\frac{2k_1 - k_2}{3\alpha}\right) N \operatorname{cosec} h\{\sqrt{3V_0}\, t\}$$

$$\sigma_3^3 = \left(\frac{-k_1 - k_2}{3\alpha}\right) N \operatorname{cosec} h\{\sqrt{3V_0}\, t\}$$

$$\sigma_4^4 = 0$$

The shear σ is given by

$$\sigma = M \operatorname{cosec} h\{\sqrt{3V_0}\, t\}$$

where

$$M = \left(\frac{k_1^2 + K_2^2 - k_1 k_2}{3\alpha^2}\right) N^2$$

$$\frac{\sigma}{\theta} = \frac{M}{\sqrt{3V_0}} \sec h\{\sqrt{3V_0}\, t\}$$

Spatial volume R^3 is given by

$$R^3 = ABC$$

$$= \frac{\alpha}{N} \sin h\{\sqrt{3V_0}\, t\}$$

5. Discussion and Conclusion

The spatial volume increases exponentially with time indicating inflationary scenario of the model. The model in general represents shearing and non-rotating universe. Initially, the model is anisotropic but for large values of t, the model isotropizes which matches with the latest astronomical observation.

The deceleration parameter (q) is given by

$$q = -\frac{\ddot{R}/R}{\dot{R}^2/R^2}$$

$$= 2 + 3 \tan h^2 (\sqrt{3V_0}\, t)$$

$$> 0$$

Thus the model represents decelerating universe.

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References

1. Guth, A.H., *Phys. Rev. D, 23*, 347 (1981).
2. Linde, A.D., Phys. Lett.B, 108, 389 (1982)
3. Albrecht, A. and Steinhardt, P. J., Phys. Rev. Lett. 48, 1220 (1982).
4. Linde, A.D., Phys. Lett. B, 129, 177 (1983); Mod. Phys. Lett. A181 (1986).
5. Abott, L.F. and Wise, M.B., Nucl. Phys. B, 244, 541 (1984).
6. Matarrese, S. and Lucchin, F., Phys. Rev. D, 32, 1316 (1985).
7. Rothman,T. and Ellis, G.F.R., Phys. Lett. B, 180, 19 (1986).
8. Stein-Schabes, J.A., Phys. Rev. D, 35, 2345 (1987).
9. Wald, R.M., Phys. Rev. D, 28, 2118 (1983).
10. Moss, I. and Sahni, V., Phys. Lett. B, 178, 159 (1986).
11. Martinez-Gonzalez, E. and Jones, B.J.T., Phys. Lett. B, 167, 37 (1986).
12. Huang, B., Int. J. Theor. Phys.30, 8, 1121 (1991).
13. Schmidt, H.J., G.R.G. 25, 87 (1993).
14. Bali, R. and Jain, V.C., Pramana – J. Phys., 59, 1 (2002).
15. Bali, R., Int. J. Theor. Phys. 50, 3043 (2011).