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MHD Boundary Layer Stagnation Point Flow and Heat Generation/ Absorption of a Micropolar Fluid with Uniform Suction / Injection

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Abstract

A comprehensive study of the steady laminar flow with heat generation of an incompressible electrically conducting micropolar fluid impinging on a permeable flat plate is analyzed numerically. A uniform suction or blowing is applied normal to the plate, which is maintained at a constant temperature. Also, a uniform magnetic field is applied normal to the plate and the viscous dissipation effect is taken into account. The governing partial differential equations are transformed into ordinary differential equations by using similarity variables and then solved them numerically by standard technique. The effects of the uniform suction/ blowing parameter, magnetic parameter, material parameter on the flow and heat transfer are presented graphically and discussed.

Key words: Stagnation point flow, Micropolar fluid, MHD, Heat generation/absorption, Suction/injection.

Introduction

The concept of micropolar fluids introduced by Eringen[1] includes certain microscopic effects arising from the local structure and micromotions of the fluid elements. The theory of micropolar fluids and its extension to thermomicropolar fluids[2] provides a mathematical model for the non- newtonian fluid flow behaviour such as colloidal suspensions, polymeric fluids, liquid crystals, exotic lubricants and animal blood. The study of microcontinum fluid mechanics has received much more attention. A thorough review of the subject and applications of micropolar fluid mechanics has been given by Ariman *et al*.[3,4]. Studies of the flows of heat convection in micropolar fluids have focused mainly on flat

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plates (Ahmadi[5], Jena and Mathur[6,7], Gorla[8]) and regular surfaces (Mathur *et al.*[9], Lien *et al.*[10,11], Hsu and Chen[12]). Perdikis and Raptis[13] studied the heat transfer of a micropolar fluid in the presence of radiation. Raptis[14] studied the effect of radiation on the flow of a micropolar fluid past a continuously moving plate. Nazar *et al.*[15], Attia[16,17], Lok *et al.*[18] have studied the stagnation point flow of a micropolar fluid.

The problems of micropolar fluid with suction/injection have been taken by many researchers. Hassanien and Gorla[19] have investigated the heat transfer of a micropolar fluid from a non-isothermal stretching sheet with suction and blowing. Hady[20] studied the solution of heat transfer to micropolar fluid from a non-isothermal stretching sheet with injection. Arabay and Hassan[21] presented the effect of suction/injection on the flow of a micropolar fluid past a continuously moving plate in the presence of radiation. Salem and Odda[22] studied the influence of thermal conductivity and variable viscosity on the flow of a micropolar fluid past a continuously moving plate with suction or injection. Steady flow of micropolar fluid under uniform suction was studied by Chakraborty and Panja[23]. Patowary and Sut[24] presented the effect of variable viscosity and thermal conductivity of micropolar fluid past a continuously moving plate with suction or injection. Recently Kishan and Deepa[25] studied the viscous dissipation effects on stagnation point flow and heat transfer of a micropolar fluid with uniform suction or blowing.

The study of flow and heat transfer for an electrically conducting micropolar fluid under the influence of a magnetic field has attracted the interest of many investigators in view of its applications in many engineering problems such as nuclear reactors, oil exploration, plasma studies, generators and aerodynamics. Kasivishwanathan and Gandhi[26] presented a class of exact solutions for the magnetohydrodynamic flow of a micropolar fluid. Mohammadien and Gorla[27] studied the effects of transverse magnetic field on a mixed convection in a micropolar fluid on a horizontal plate with vectored mass transfer. Magnetohydrodynamic free convection boundary layer flow of a thermomicropolar fluid over a vertical plate was studied by Gorla *et al.*[28]. Sedeek[29] studied the flow of a magnetomicropolar fluid past a continuously moving plate. Ishaq *et al.*[30] studied the magnetohydrodynamic (MHD) flow of a micropolar fluid towards a stagnation point on a vertical surface. Murthy and Bahali[31] have studied the steady flow of a micropolar fluid under a transverse magnetic field with constant suction/injection. Khedr *et al.*[32] presented MHD flow of a micropolar fluid past a stretched permeable surface with heat generation or absorption. Islam *et al.*[33], Ashraf *et al.*[34] and recently Ashraf and Rashid[35] have taken the magnetohydrodynamic problems of micropolar fluid.

The purpose of the present paper is to study the effects of suction/injection and heat generation of an electrically conducting micropolar fluid impinging a flat plate in presence of transverse magnetic field.

Mathematical Formulation

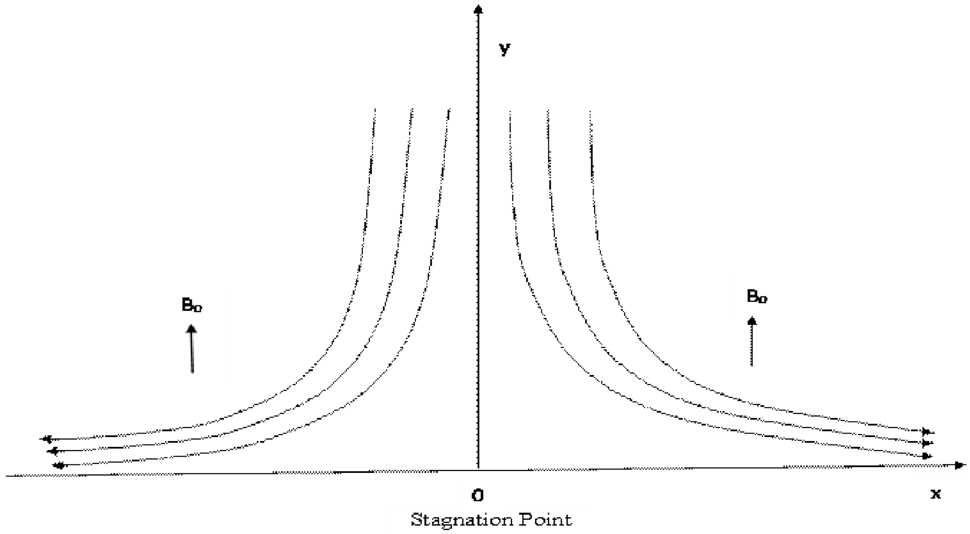

Figure 1. Physical model and coordinate system

Consider a steady two-dimensional flow of an incompressible electrically conducting micropolar fluid with heat generation/absorption impinging normally on a permeable horizontal flat plate placed at y=0 divides into two streams on the plate and leaves in both directions. The fluid is acted upon by a uniform transverse magnetic field of strength B_0. The magnetic Reynolds number is assumed to be small so that the induced magnetic field can be neglected as compared to the applied field. We assume that there is no polarization voltage, so the induced electric field is zero. The x-axis is chosen along the plate and y-axis is taken normal to it. Let u and v be the x- and y- components of velocity respectively and N be the component of the micro-rotation vector normal to the xy-plane. A uniform suction or injection is applied at the plate with a transpiration velocity at the boundary of the plate given by v_w (>0 for mass injection and <0 for mass suction). The potential flow velocity components in the vicinity of the stagnation point is given by U(x) = ax and V(y) = -ay, where the constant a (>0) is proportional to the free stream velocity far away from the plate. Under the usual boundary layer approximation including the viscous dissipation, the governing equations are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U\frac{dU}{dx} + \frac{(\mu + h)}{\rho}\frac{\partial^2 u}{\partial y^2} + \frac{h}{\rho}\frac{\partial N}{\partial y} - \frac{1}{\rho}\sigma B_0^2 u \tag{2}$$

$$\rho\left(u\frac{\partial N}{\partial x} + v\frac{\partial N}{\partial y}\right) = \frac{\gamma}{j}\frac{\partial^2 N}{\partial y^2} - \frac{h}{j}(2N + \frac{\partial u}{\partial y}) \tag{3}$$

$$\rho c_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \kappa\frac{\partial^2 T}{\partial y^2} + (\mu + h)(\frac{\partial u}{\partial y})^2 + Q(T - T_\infty) + \sigma B_0^2 u^2 \tag{4}$$

Subject to the boundary conditions:

$$y = 0 \;:\; u = 0,\; v = v_w,\; N = -m\frac{\partial u}{\partial y},\; T = T_w$$
$$y \to \infty \;:\; u = U(x) = ax,\; v \to 0,\; N \to 0,\; T = T_\infty \quad (5)$$

where T and T_W are the temperatures of the fluid and the plate, respectively whereas the temperature of the fluid far away from the plate is T_∞ Q is the volumetric rate of heat generation /absorption, μ is the viscosity, ρ is the density, σ is the electrical conductivity, κ is the thermal conductivity, c_p is the specific heat at constant pressure, j is the micro-inertia per unit mass, γ is the spin gradient viscosity, h is the vortex viscosity and m (0≤ m≤1) is the boundary parameter. Here we noted that when m=0, which indicates N=0 at the wall, represents concentrated particle flows in which the microelements close to the wall surface are unable to rotate. The case m=1 is used for the modelling of turbulent boundary layer flows. Here γ, j and h are assumed to be constants and γ is assumed to be given by Nazar *et al*.[15]

$$\gamma = \left(\mu + \frac{h}{2}\right) j \quad (6)$$

We take $j = \dfrac{\nu}{a}$ as a reference length, where ν is the kinematic viscosity.

Analysis :

The equation of continuity (1) is identically satisfied by stream function $\Psi(x, y)$ defined as

$$u = \frac{\partial \Psi}{\partial y} \;,\; v = -\frac{\partial \Psi}{\partial x} \quad (7)$$

For the solution of momentum, micro-rotation (spin) and the energy equation (2) to (4), the following similarity transformations in order to convert the partial differential equations into the ordinary differential equations are defined :

$$\Psi(x, y) = x\sqrt{a\nu}\, f(\eta),\;\; N(x, y) = ax\sqrt{\frac{a}{\nu}}\, g(\eta),$$
$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty},\;\; \eta = y\sqrt{\frac{a}{\nu}} \quad (8)$$

Using equation (8), the equations (2) to (4) reduce to (after some simplifications):

$$(1 + K)f''' + ff'' - f'^2 + Kg' + 1 - Mf' = 0 \quad (9)$$
$$\left(1 + \frac{K}{2}\right) g'' + fg' - f'g - K\left(2g + f''\right) = 0 \quad (10)$$
$$\theta'' + \Pr f\theta' + (1 + K)\, Pr\, Ec\, f''^2 + M\, Ec\, Prf'^2 + PrB\theta = 0 \quad (11)$$

Boundary conditions given in equation (5) in view of equation (8) can be written as

$$\eta = 0 \;:\;\; f = S\;,\; f' = 0,\; g = -mf''\;,\; \theta = 1$$
$$\eta \to \infty :\;\; f' \to 1,\; g \to 0\;,\; \theta \to 0 \quad (12)$$

where primes denote differentiation with respect to η, $K = \frac{h}{\mu}$ (+ve) is the material parameter, $M = \dfrac{\sigma B_0^2}{\rho a}$ is the magnetic parameter, $Pr = \dfrac{\mu c_p}{\kappa}$ is the prandtl number, $S = \dfrac{-v_w}{\sqrt{a\nu}}$ is the injection/ suction parameter (S<0 for injection and S>0 for suction), $Ec = \dfrac{U^2}{c_p(T_w - T_\infty)}$ is the Eckert number, $B = \dfrac{Q}{a\rho c_p}$ is the heat generation/ absorption parameter.

Equation (10) is that obtained by Attia[17] for non-magnetic case and obtain $g(\eta) = -\frac{1}{2}f''(\eta)$ whereas the equation (9) and (11) are non-linear differential equations with values prescribed at two boundaries which can be converted to initial value problem by shooting technique and then integrated numerically by Runge-Kutta method.

The physical quantities of interest are the local skin-friction coefficient C_f and the local Nusselt number Nu which are defined, respectively, as

$$C_f = \frac{\tau_w}{\rho U^2}\ , \quad Nu = \frac{x q_w}{\kappa(T_w - T_\infty)} \tag{13}$$

where U(x)=ax is a characteristic velocity and τ_w is the wall shear stress which is given by

$$\tau_w = \left\{(\mu + h)\frac{\partial u}{\partial y} + hN\right\}_{y=0}$$

and $q_w = -\kappa \left\{\frac{\partial T}{\partial y}\right\}_{y=0}$ is the heat transfer from the plate.

Thus, we get

$$C_f = \frac{\left\{1+\frac{K}{2}\right\} f''(0)}{\sqrt{Re}} \text{ and } Nu = -\sqrt{Re}\ \theta'(0) \tag{14}$$

where $Re = \frac{xU}{\nu}$ is the local Reynolds number.

Results and Discussion

The non linear ordinary differential equations (9) and (11) subject to the boundary conditions (12) were integrated numerically for various values of the parameters involved. The computations were done by a programme which uses a symbolic and computational computer language Matlab. To study the behaviour of the velocity and the temperature profiles, graphs are drawn for various values of the parameters that describe the flow. Figures 2 and 3 present the velocity profiles for various values of different parameters. It is observed from these graphs that the velocity increases with the increasing values of the parameter S while it decreases with the increase of the parameters K and M. Figures 4 to 9 show the profiles of temperature distribution for various parameters. It is evident from these graphs that the temperature distribution shows reverse phenomenon with the parameters S and Pr *i.e.* the temperature decreases with the increasing values of these parameters while it increases with the increasing values of parameters K, M, Ec and B. Tables 1 and 2 present the variation of skin friction coefficient which is proportional to and heat transfer rate which is proportional to at the surface, respectively for various values of the parameters S and M. It is observed from these tables that both the skin friction coefficient and heat transfer rate increases with the increasing values of the parameter S for all M and decreases with the increase of parameter M for all S.

Table 1. Numerical values for various values of parameters M and S when K=1

M	S= -1	S= -0.5	S=0	S=0.5	S=1
0	0.6766	0.8278	1.0063	1.2099	1.4352
0.1	0.6545	0.7952	0.9669	1.1606	1.3756
0.2	0.6339	0.7694	0.9299	1.1141	1.3191
0.5	0.5799	0.6925	0.8321	0.9902	1.1676
0.8	0.534	0.6347	0.744	0.8819	1.041

Table 2. Numerical values for various values of parameters M and S when K=1, Pr=0.7, Ec=0.2, B=0.1

M	S= -1	S= -0.5	S=0	S=0.5	S=1
0	-0.0058	0.1092	0.2549	0.4613	0.6959
0.1	-0.0147	0.0958	0.2400	0.4451	0.6796
0.2	-0.0228	0.0834	0.2262	0.4300	0.6644
0.5	-0.0430	0.0511	0.1899	0.3908	0.6251
0.8	-0.0589	0.0251	0.1769	0.3795	0.6116

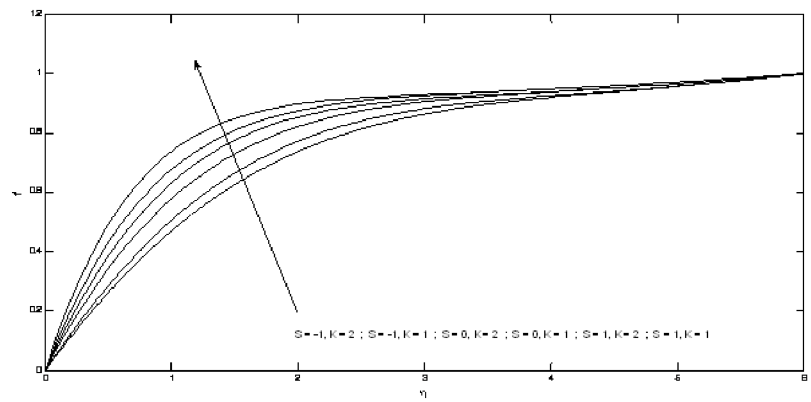

Figure 2. Velocity profile against η for various values of parameters S and K when M=0.2

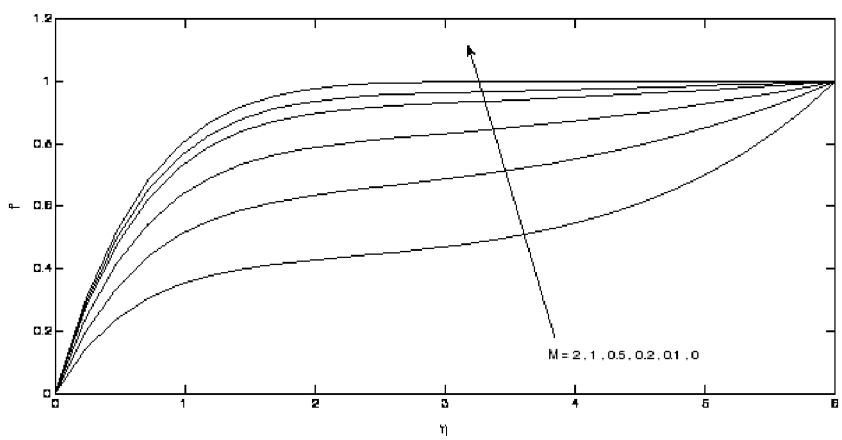

Figure 3. Velocity profile against η for various values of parameter M when K=1 and S=1

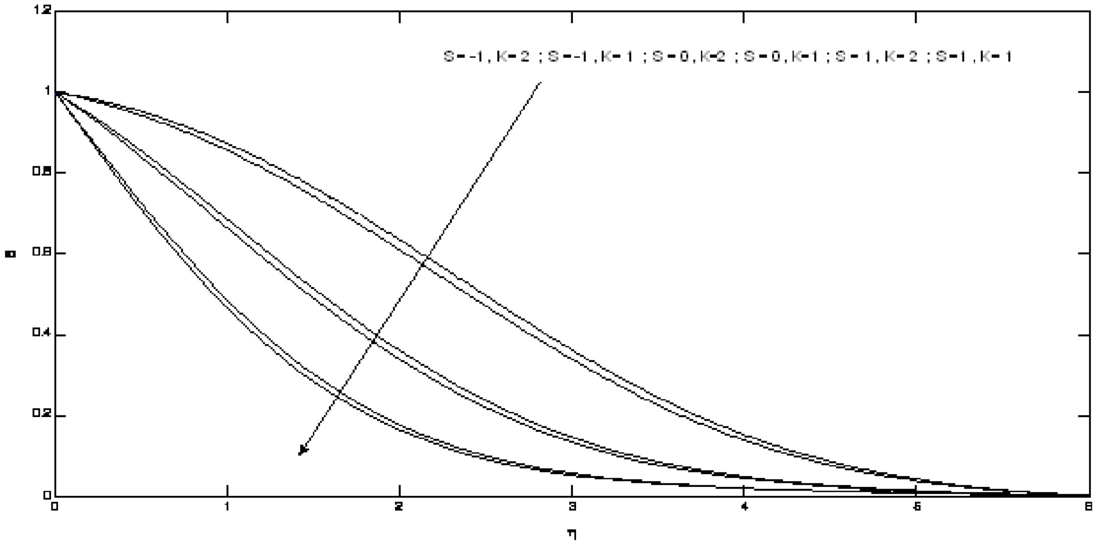

Figure 4. Temperature profile against η for various values of parameters S and K when Pr=0.5, Ec=0.2, M=0.2 and B=0

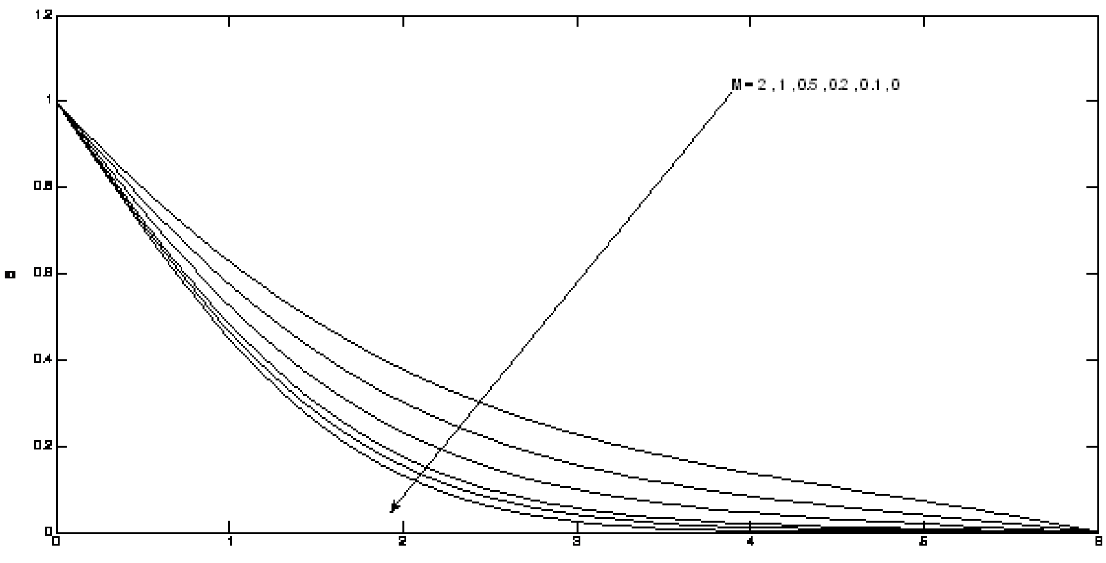

Figure 5. Temperature profile against η for various values of parameter M when Pr=0.5, Ec=0.2, K=1, B=0.1 and S=1

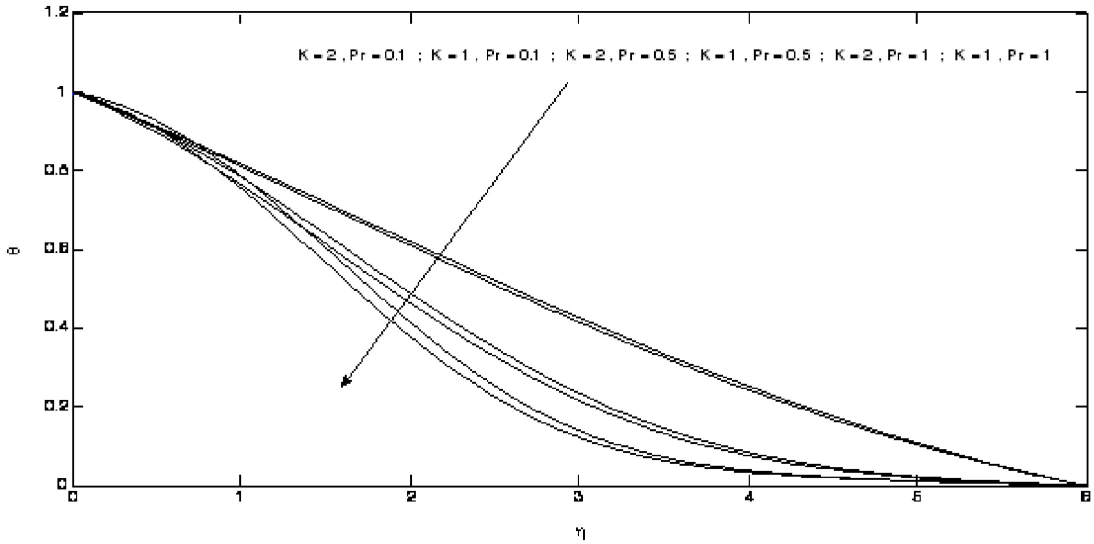

Figure 6. Temperature profile against η for various values of parameters K and Pr when Ec=0.2, M=0.2, B=0 and S= -0.5

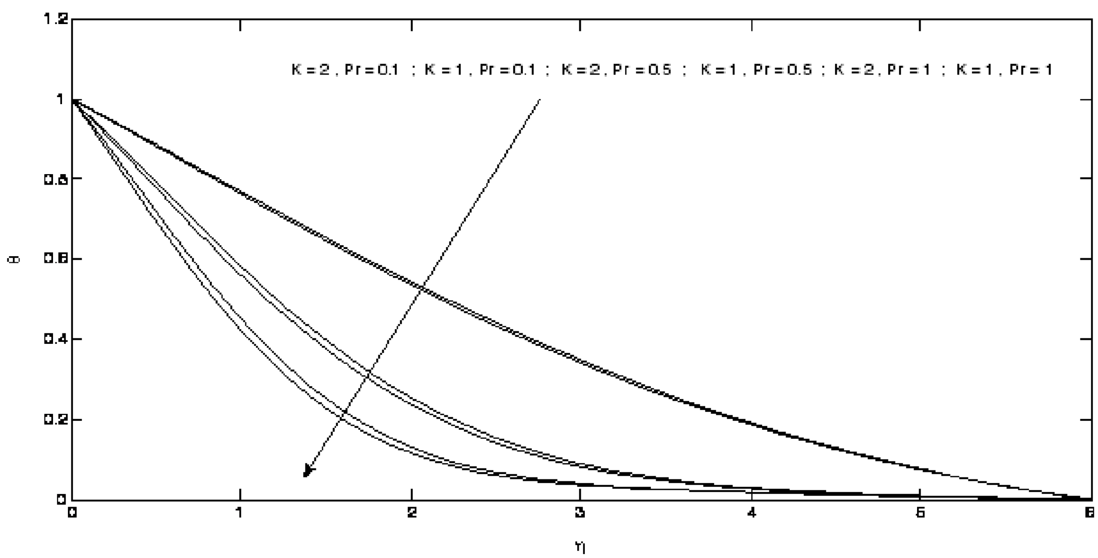

Figure 7. Temperature profile against η for various values of parameters K and Pr when Ec=0.2, S=0.5, M=0.2 and B=0

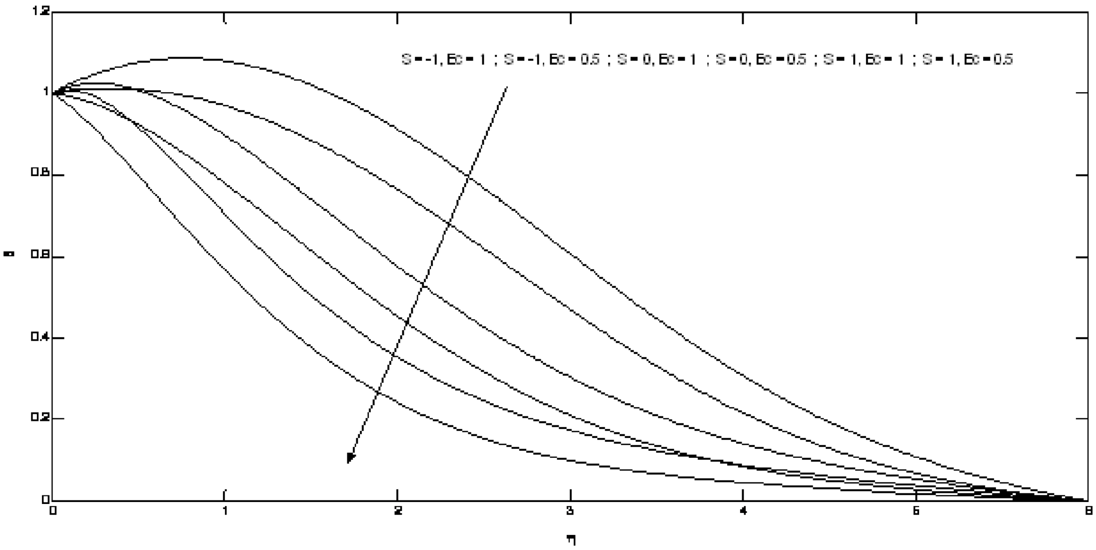

Figure 8. Temperature profile against η for various values of parameters S and Ec when Pr=0.5, B=0.1, M=0.2 and K=1

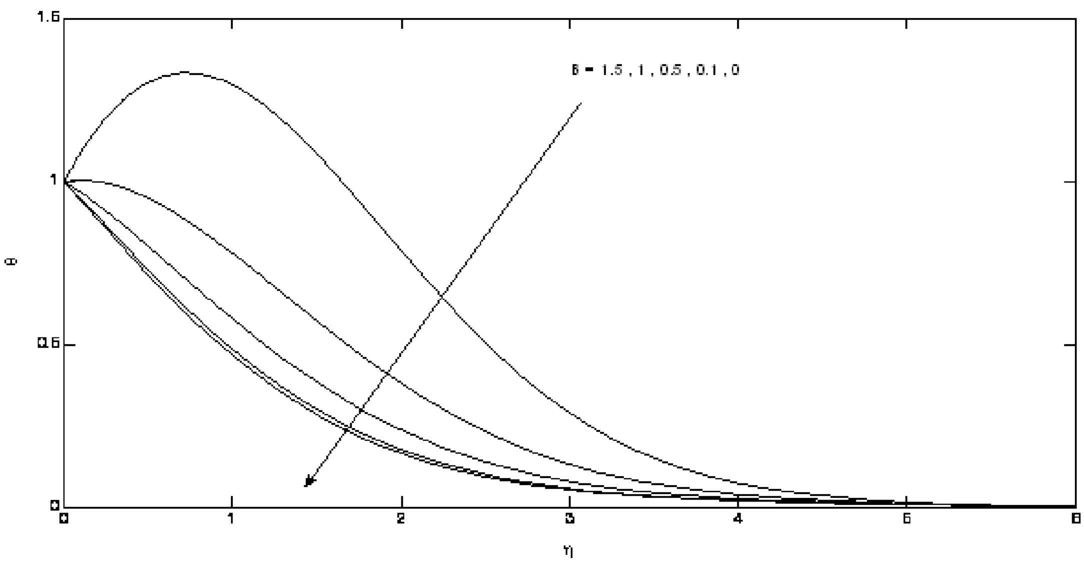

Figure 9. Temperature profile against η for various values of parameter B When Pr=0.5, Ec=0.2, S=1, K=1 and M=0.2

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