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Tilted Cosmological Model With Electromagnetic Field

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Abstract

Tilted Bianchi type I cosmological model in the presence of magnetic field and barotropic fluid is investigated. To determine complete solution, we have assumed that the condition $p = \gamma\rho$, where p being isotropic pressure, ρ the matter density and also assumed that the relation between metric potential as A=BC. Here, we have seen that Maxwell's equations $F_{[ij;k]} = 0$ is satisfied by F_{23} = constant. The physical and geometrical aspects of the model in the presence and absence of magnetic field together with singularity in the model are also discussed.

Key words: Tilted cosmological model; Magnetic field; Bianchi type-I universe.

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Introduction

Homogeneous and anisotropic cosmological models have been widely studied in classical general relativity in the search for a relativistic picture of the universe in its early stages because they can be explained a number of observed phenomena quite satisfactorily. So that in recent years, there has been a considerable interest in investigating spatially homogeneous and anisotropic cosmological models in which matter do not move orthogonally to the hypersurface of homogeneity. These types of models are called tilted cosmological models. The general dynamics of these cosmological models have been studied in details by King and Ellis[1] and they have shown that in such universe, the matter move with non-zero expansion, rotation and shear. Also, Ellis and King[2], Collins and Ellis[3], Ellis and Baldwin[4] have shown that we are likely to be living in a tilted universe and they have indicated that how we may detect it.

Bianchi type-I cosmological models create a special interest because these models contain isotropic special cases and therefore permits arbitrarily small anisotropy levels at any instant of cosmic time. The anisotropic magneto fluid models have significant contribution in the evolution of galaxies of stellar bodies. It is well known that the presence of strong magnetic field is exhibited by galaxies and interstellar space and gives rise to a kind of viscous effect in the fluid flow.Monaghan[5] has discussed the behaviour of magnetic field in stellar bodies. Tilted Bianchi type I models containing perfect fluid have been obtained by Dunn and Tupper[6-7]. Tilted solutions containing perfect fluid have been obtained by various authors like Farnsworth[8] and Reboucas[9]. Also, the barotropic fluid does not completely dominate the universe. From recent observational data we found that the universe is expanding with acceleration. Johri *et al.*[10] investigated the general Bianchi type I cosmological solution corresponding to spatially homogeneous and anisotropic models containing barotropic fluid. Bagora *et al.*[11-12] have also investigated Bianchi type I homogeneous tilted cosmological models in different context. In this paper, we have investigated magnetized tilted Bianchi type I cosmological model for barotropic fluid ($p = \gamma\rho$), where p being isotropic pressure, ρ the matter density with $0 \le \gamma \le 1$ .To determine the complete solution, we have assumed the relation between metric potential as A=BC. To get model in terms of cosmic time t, we have also assumed some special condition. The physical and geometrical aspects of both the model are discussed[13-16].

The metric and field equations :

We consider the Bianchi type-I metric in the form

$$ds^2 = -dt^2 + A\,dx^2 + B^2\,dy^2 + C^2\,dz^2 , \quad (1)$$

where A, B and C are functions of cosmic time ‘t’ alone.

The energy-momentum tensor for perfect fluid distribution with heat conduction given by Ellis[13] is given by

$$T_i^j = (\in + p)v_i v^j + p g_i^j + q_i v^j + v_i q^j + E_i^j , \quad (2)$$

together with

$$g_{ij} v^i v^j = -1 , \quad (3)$$

$$q_i q^i > 0 , \quad (4)$$

$$q_i v^i = 0 , \quad (5)$$

Here E_i^j is the electromagnetic field given by Lichnerowicz[14] as

$$E_i^j = \bar{\mu}\left[|h|^2 (v_i v^j + \frac{1}{2} g_i^j) - h_i h^j \right] , \quad (6)$$

where $\bar{\mu}$ is magnetic permeability and h_i is the magnetic flow vector defined by

$$h_i = \frac{\sqrt{-g}}{2\bar{\mu}} \in_{ijk\ell} F^{k\ell} v^j . \quad (7)$$

Here $F_{k\ell}$ is the electromagnetic field tensor and $\in_{ijk\ell}$ the Levi-Civita tensor density.

From (7) we find that $h_1 \neq 0$, $h_2 = 0$, $h_3 = 0$, $h_4 \neq 0$. This leads to $F_{12} = 0 = F_{13}$ by virtue of (7). We also find that $F_{14} = 0 = F_{24}$ due to the assumption of infinite conductivity (Maartens[15]) of the fluid. We take the incident magnetic field to be in the direction of x-axis so that the only non-vanishing component of F_{ij} is F_{23}.

The first set of Maxwell’s equation

$$F_{ij;k} + F_{jk;i} + F_{ki;j} = 0,$$

leads to

$$F_{23} = \text{constant } H \text{ (say)}.$$

Now $h_1 = \dfrac{H}{\bar{\mu}BC}\cosh\lambda$, $h_4 = \dfrac{H}{\bar{\mu}BC}\sinh\lambda$.

Since $|h|^2 = h_\ell h^\ell = h_1h^1 + h_4h^4 = g^{11}(h_1)^2 + g^{44}(h_4)^2$

$$= \frac{H^2\cosh^2\lambda}{\bar{\mu}^2B^2C^2} - \frac{H^2\sinh^2\lambda}{\bar{\mu}^2B^2C^2} = \frac{H^2}{\bar{\mu}^2B^2C^2}.$$

Equation (6) leads to

$$E_1^1 = \frac{-H^2}{2\bar{\mu}B^2C^2} = -E_2^2 = -E_3^3 = E_4^4. \quad (8)$$

In the above, p is the isotropic pressure, ρ the matter density, q^i the heat conduction vector orthogonal to v^i. The fluid flow vector v^i has the components $\left(\dfrac{\sinh\lambda}{A}, 0, 0, \cosh\lambda\right)$ satisfying (3), λ being the tilt angle.

The Einstein's field equation

$$R_i^j - \frac{1}{2}Rg_i^j = -8\pi T_i^j, \quad (c = G = 1)$$

The field equation for the line element (1) leads to

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4C_4}{BC} = -8\pi\left[(\rho+p)\sinh^2\lambda + p + 2q_1\frac{\sinh\lambda}{A} - \frac{H^2}{2\bar{\mu}(BC)^2}\right], \quad (9)$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4C_4}{AC} = -8\pi p\left[p + \frac{H^2}{2\bar{\mu}(BC)^2}\right], \quad (10)$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4B_4}{AB} = -8\pi p\left[p + \frac{H^2}{2\bar{\mu}(BC)^2}\right], \quad (11)$$

$$\frac{A_4B_4}{AB} + \frac{A_4C_4}{AC} + \frac{B_4C_4}{BC} = -8\pi\left[-(\rho+p)\cosh^2\lambda + p - 2q_1\frac{\sinh\lambda}{A} - \frac{H^2}{2\bar{\mu}(BC)^2}\right], \quad (12)$$

$$(\rho+p)A\sinh\lambda\cosh\lambda + q_1\cosh\lambda + q_1\frac{\sinh^2\lambda}{\cosh\lambda} = 0, \quad (13)$$

where the suffix '4' stands for ordinary differentiation with respect to the cosmic time 't' alone.

Solution of the field equation :

Equations from (9)-(13) are five equations in seven unknown A, B, C, ρ, p, q_1 and λ. For the complete determination of these quantities, we need two extra conditions.

Firstly, we assume that the model is filled with barotropic perfect fluid which leads to

$$p = \gamma\rho, \quad (14)$$

where $0 \le \gamma \le 1$.

Secondly, we assume that relation between metric potential as

$$A = BC. \quad (15)$$

Equations (9) and (11) lead to

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{2B_4C_4}{BC} + \frac{A_4C_4}{AC} + \frac{A_4B_4}{AB} = 8\pi(\rho - p) + \frac{8\pi H^2}{\bar{\mu}(BC)^2}. \quad (16)$$

Using condition of barotropic fluid from (14) in equation (16), we have

$$\frac{B_{44}}{B}+\frac{C_{44}}{C}+\frac{2B_4C_4}{BC}+\frac{A_4C_4}{AC}+\frac{A_4B_4}{AB}$$

$$=8\pi(\frac{p}{\gamma}-p)+\frac{8\pi H^2}{\bar{\mu}\,(BC)^2}\,. \qquad (17)$$

Again using eq. (14) in eq. (11), we have

$$\frac{B_{44}}{B}+\frac{C_{44}}{C}+\frac{2B_4C_4}{BC}+\frac{A_4C_4}{AC}+\frac{A_4B_4}{AB}$$

$$=\left[\frac{A_{44}}{A}+\frac{C_{44}}{C}+\frac{A_4C_4}{AC}+\frac{4\pi H^2}{\bar{\mu}\,(BC)^2}\right](1-\frac{1}{\gamma})+\frac{8\pi H^2}{\bar{\mu}\,(BC)^2}\,. \qquad (18)$$

Let us assume that $BC=\mu\,,\ \frac{B}{C}=\nu\ .$ (19)

Equations (10) and (11) with (19) lead to

$$\frac{\nu_4}{\nu}=\frac{a}{\mu^2}\ , \qquad (20)$$

where 'a' is constant of integration.

With the help of (19) and (20), equation (18) becomes

$$2\mu_{44}+\ell\,\frac{\mu_4{}^2}{\mu}=\frac{m}{\mu^3}+\frac{2Kn}{\mu}\,. \qquad (21)$$

Here $\ell=\frac{3\gamma+1}{3-\gamma}\,,\qquad m=\frac{(\gamma-1)a^2}{3-\gamma}\,,$

$$n=\frac{2(3\gamma-1)}{3-\gamma}\ \text{and}\ \ K=\frac{4\pi H^2}{\bar{\mu}}\,.$$

Let $\mu_4=f(\mu)\,,\ \mu_{44}=ff'\,.$ (22)

Using (19) and (20), equation (18) leads to

$$\mu_4{}^2=\frac{ma^2}{(\ell-2)\mu^2}+\frac{2Kn}{\ell}+\frac{b}{\mu^{\ell}}\,, \qquad (23)$$

where 'b' is constant of integration.
Equation (17) leads to

$$\log\nu=\int\frac{a\sqrt{\ell(\ell-2)}\,d\mu}{\mu^{1-\ell/2}\sqrt{m\ell\mu^{\ell}+2(\ell-2)Kn\mu^{\ell+2}+\ell(\ell-2)b\mu^{2}}}\,. \qquad (24)$$

Hence the metric (1) reduces to the form

$$ds^2=-\frac{d\mu^2}{f^2}+\mu^2dx^2+\mu\nu dy^2+\frac{\mu}{\nu}dz^2\,. \qquad (25)$$

The metric (25) reduces to the form

$$ds^2=\frac{-\ell(\ell-2)T^{\ell+2}dT^2}{\left[m\ell T^{\ell}+2(\ell-2)KnT^{\ell+2}+\ell(\ell-2)bT^2\right]}$$

$$+T^2dX^2+T\nu(dY^2+\frac{1}{\nu^2}dZ^2)\,, \qquad (26)$$

where $\mu=T$, $x=X$, $y=Y$, $z=Z$ and ν is determined by (24) when $\mu=T$.

Some physical and geometrical features :

The matter density ρ and isotropic pressure p for the model (26) are given by

$$8\,\pi\,p=\frac{5\gamma(3\gamma+1)b-(9\gamma-1)KT^{\ell}}{2(3\gamma+1)T^{\ell+2}}\ , \qquad (27)$$

$$8\,\pi\,\rho=\frac{5\gamma(3\gamma+1)b-(9\gamma-1)KT^{\ell}}{2\gamma(3\gamma+1)T^{\ell+2}}\,. \qquad (28)$$

The tilt angle λ is given by

$$\cosh\lambda=\frac{1}{2}\sqrt{\frac{5\gamma(5-\gamma)(3\gamma+1)b-(3-\gamma)(36\gamma^2-21\gamma+1)KT^{\ell}}{2\gamma(3\gamma+1)T^{\ell+2}}}\,, \qquad (29)$$

$$\sinh\lambda = \frac{1}{2}\sqrt{\gamma-1}. \tag{30}$$

The scalar of expansion θ calculated for the flow vector v^i for the model (26) is given by

$$\theta = \frac{1}{T^{13-\gamma/2(3-\gamma)}}\sqrt{\frac{(\gamma+3)[a^2T^{3\gamma+1/3-\gamma}+5bT^2]}{5}}, \tag{31}$$

The flow vectors v^i and heat conduction vectors q_i for the model (26) are given by

$$v^1 = \frac{1}{2T}\sqrt{\gamma-1}\ , \tag{32}$$

$$v^4 = \frac{1}{2}\sqrt{3+\gamma}\ , \tag{33}$$

$$q_1 = \frac{-(\gamma+3)\sqrt{\gamma-1}}{48(3-\gamma)\pi\gamma T^{2(2+\gamma)/3-\gamma}}, \tag{34}$$

$$q_4 = \frac{-(\gamma+3)\sqrt{\gamma+3}}{48(3-\gamma)\pi\gamma T^{7+\gamma/3-\gamma}}\ . \tag{35}$$

The non-vanishing components of shear tensor (σ_{ij}) and rotation tensor (ω_{ij}) are given by

$$\sigma_{11} = \frac{1}{6T^{3\gamma+1/2(3-\gamma)}}\sqrt{\frac{[a^2T^{3\gamma+1/3-\gamma}+5bT^2](\gamma+3)}{5}}, \tag{36}$$

$$\sigma_{22} = \frac{\gamma}{12T^{2(\gamma+2)/(3-\gamma)}}\left[3a\sqrt{5}T^{3\gamma+1/2(3-\gamma)}-\sqrt{a^2T^{3\gamma+1/3-\gamma}+5bT^2}\right]\sqrt{\frac{\gamma+3}{5}}, \tag{37}$$

$$\sigma_{33} = \frac{-1}{12\gamma T^{2(\gamma+2)/(3-\gamma)}}\left[3a\sqrt{5}T^{3\gamma+1/2(3-\gamma)}+\sqrt{a^2T^{3\gamma+1/3-\gamma}+5bT^2}\right]\sqrt{\frac{\gamma+3}{5}}, \tag{38}$$

$$\sigma_{44} = \frac{\gamma-1}{24T^{13-\gamma/2(3-\gamma)}}\sqrt{\frac{[a^2T^{3\gamma+1/3-\gamma}+5bT^2](\gamma+3)}{5}}\ , \tag{39}$$

$$\omega_{14} = \frac{1}{2T^{2(\gamma+2)/(3-\gamma)}}\sqrt{\frac{[a^2T^{3\gamma+1/3-\gamma}+5bT^2](\gamma-1)}{5}}. \tag{40}$$

The rates of expansion H_i in the direction of x, y and z axes are given by

$$H_1 = \frac{1}{T^2}\sqrt{\frac{a^2T^{\alpha}+5bT^2}{5T^{\alpha}}}, \tag{41}$$

$$H_2 = \frac{1}{2T^2}\left[\sqrt{\frac{a^2T^{\alpha}+5bT^2}{5T^{\alpha}}}+a\right], \tag{42}$$

$$H_3 = \frac{1}{2T^2}\left[\sqrt{\frac{a^2T^{\alpha}+5bT^2}{5T^{\alpha}}}-a\right]. \tag{43}$$

Special model :

To find the model in terms of cosmic time t we take a=0, then we have

$$\nu = \text{cons}\tan t\ , \mu = \sqrt{K_1t+K}\ .$$

By suitable transformations the metric (1) becomes

$$ds^2 = \frac{-dT^2}{K_1} + T^2 dX^2 + T(dY^2 + dZ^2). \quad (44)$$

The physical parameters of the model (44) are given by

$$p = \frac{1}{32T^2}, \cosh\lambda = .9, \sigma_{11} = \frac{27}{200}, \sigma_{22} = \frac{-3K_2}{40T^{1/2}},$$

$$\sigma_{33} = \frac{-3}{40K_2 T^{1/2}}, \sigma_{44} = \frac{3}{2000T}.$$

Conclusion

The model (26) represents a tilted model, $\rho \to \infty$ when $T \to 0$ and $\gamma < 1$. The model starts with a big-bang at T=0 and the expansion in the model decreases as time increases. The model has cigar type singularity at T=0 when $\gamma > \frac{1}{2}$ and it has point type singularity at T=0 when $\gamma < \frac{1}{2}$ (MacCallum[16]). Since $\underset{T\to\infty}{\mathrm{Lt}} \frac{\sigma}{\theta} \neq 0$, then model is anisotropic for large values of T. In general the model, represents expanding, shearing, rotating and tilted universe. Also at T=0, $H_1, H_2, H_3, \sigma_{ij}, \omega_{ij}, \rho$, p and θ are all tend to infinity whereas the spatial volume V=T ^{2} is zero and volume increases as time increases. This shows that model has finite physical singularity at T=0.

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