Ultra Scientist Vol. 25(1)A, 59-72 (2013).

Further characterization of induced paired domination number of a graph

G. MAHADEVAN[1], A. NAGARAJAN[2], A. RAJESWARI[3] and SELVAM AVADAYAPPAN[4]

(Acceptance Date 27th February, 2013)

Abstract

A set $S \subseteq V$ is a induced -paired dominating set if S is a dominating set of G and the induced subgraph <S> is a perfect matching. The induced - paired domination number $\gamma_{ip}(G)$ is the minimum cardinality taken over all paired dominating sets in G. The minimum number of colours required to colour all the vertices so that adjacent vertices do not receive the same colour and is denoted by $\chi(G)$. The authors[4] characterized the classes of graphs whose sum of induced paired domination number and chromatic number equals to $2n-6$, for any $n \geq 4$. In this paper we extend the above result and characterize the classes of all graphs whose sum of induced paired domination number and chromatic number equals to 2n - 7, for any $n \geq 4$.

Key words: Induced Paired domination number, Chromatic number

AMS(2010): 05C69

1. Introduction

Throughout this paper, by a graph we mean a finite, simple, connected and undirected graph G(V, E). For notations and terminology, we follow[2] the number of vertices in G is denoted by n. Degree of a vertex v is denoted by deg(v). We denote a cycle on n vertices by C_n, a path of n vertices by P_n, complete graph on n vertices by K_n. If S is a subset of V, then

<S> denotes the vertex induced subgraph of G induced by S. A subset S of V is called a dominating set of G if every vertex in V-S is adjacent to at least one vertex in S. The domination number $\gamma(G)$ of G is the minimum cardinality of all such dominating sets in G. A dominating set S is called a total dominating set if the induced subgraph <S> has no isolated vertices. The minimum cardinality taken over all total dominating sets in G is called the total domination number and is denoted by $\gamma_t(G)$. One can get a comprehensive survey of results on various types of domination number of a graph[12]. The chromatic number $\chi(G)$ is defined as the minimum number of colors required to color all the vertices such that adjacent vertices receive the same color.

Recently many authors have introduced different types of domination parameters by imposing conditions on the dominating set and/ or its complement. Teresa W. Haynes[11] introduced the concept of paired domination number of a graph. If we think of each vertex $s \in S$, as the location of a guard capable of protecting each vertex dominated by S, then for domination a guard protects itself, and for total domination each guard must be protected by another guard. For a paired domination the guards location must be selected as adjacent pairs of vertices so that each guard is assigned one other and they are designated as a backup for each other. Thus a paired dominating set S with matching M is a dominating set $S=\{v_1,v_2, v_3,\ldots v_{2t-1}, v_{2t}\}$ with independent edge set $M = \{e_1, e_2, e_3, \ldots. e_t\}$ where each edge e_i is incident to two vertices of S, that is M is a perfect Matching in <S>. A set $S \subseteq V$ is a paired dominating set if S is a dominating set of G and the induced subgraph <S> has a perfect matching. The paired domination number $\gamma_{pr}(G)$ is the minimum cardinality taken over all paired dominating sets in G. A set $S \subseteq V$ is a induced -paired dominating set if S is a dominating set of G and the induced subgraph <S> is a perfect matching. The induced - paired domination number $\gamma_{ip}(G)$ is the minimum cardinality taken over all paired dominating sets in G.

Several authors have studied the problem of obtaining an upper bound for the sum of a domination parameter and a graph theoretic parameter and characterized the corresponding extremal graphs. In[9], Paulraj Joseph J and Arumugam S proved that $\gamma + \kappa \leq p$, where denotes the vertex connectivity of the graph. They proved[8] that $\gamma_c + \chi \leq p + 1$ and characterized the corresponding extremal graphs. They also proved similar results for γ and γ_t. In[7], Mahadevan G Selvam A, Iravithul Basira A characterized the extremal of graphs for which the sum of the complementary connected domination number and chromatic number. In[3], Paulraj Joseph J and Mahadevan G proved that $\gamma_{pr} + \chi \leq 2n - 1$, and characterized the corresponding extremal graphs of order up to $2n - 6$. Motivated by the above results, in this paper, we characterize all graphs for which $\gamma_{ip}(G) + \chi(G) = 2n - 7$ for any $n \geq 4$.

We use the following preliminary results and notations for our consequent characterization:

Theorem[10] *1.1* If G is a connected graph of order $n \geq 3$, then $\gamma_{ip}(G) \leq n - 1$ and equality holds if and only if G is isomorphic to P_3, C_3, P_5 or G' where G' is the graph as in the following Figure 1.1.

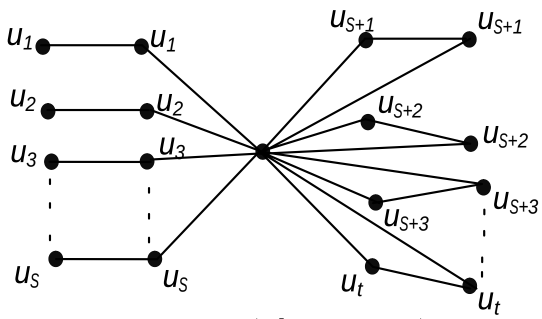


Figure 1.1 (where s, t $\geq$ 2)

Notation 1.2: $C_3(n_1P_{m1}, n_2P_{m2}, n_3P_{m3})$ is a graph obtained from C_3 by attaching n_1 times the pendent vertex of P_{m1}(Path on m_1 vertices) to a vertex u_i of C_3 and attaching n_2 times the pendent vertex of P_{m2}(Path on m_2 vertices) to a vertex u_j for $i \neq j$ of C_3 and attaching n_3 times the pendent vertex of P_{m3}(Path on m_3 vertices) to a vertex u_k for $i \neq j \neq k$ of C_3.

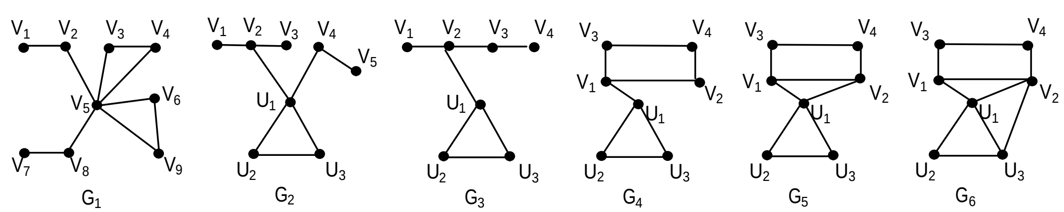
Notation 1.3: $C_3(u(P_{m1}, P_{m2}))$ is a graph obtained from C_3 by attaching the pendent vertex of P_{m1}(Path on m_1 vertices) and the pendent vertex of P_{m2}(Paths on m_2 vertices) to any vertex u of C_3.

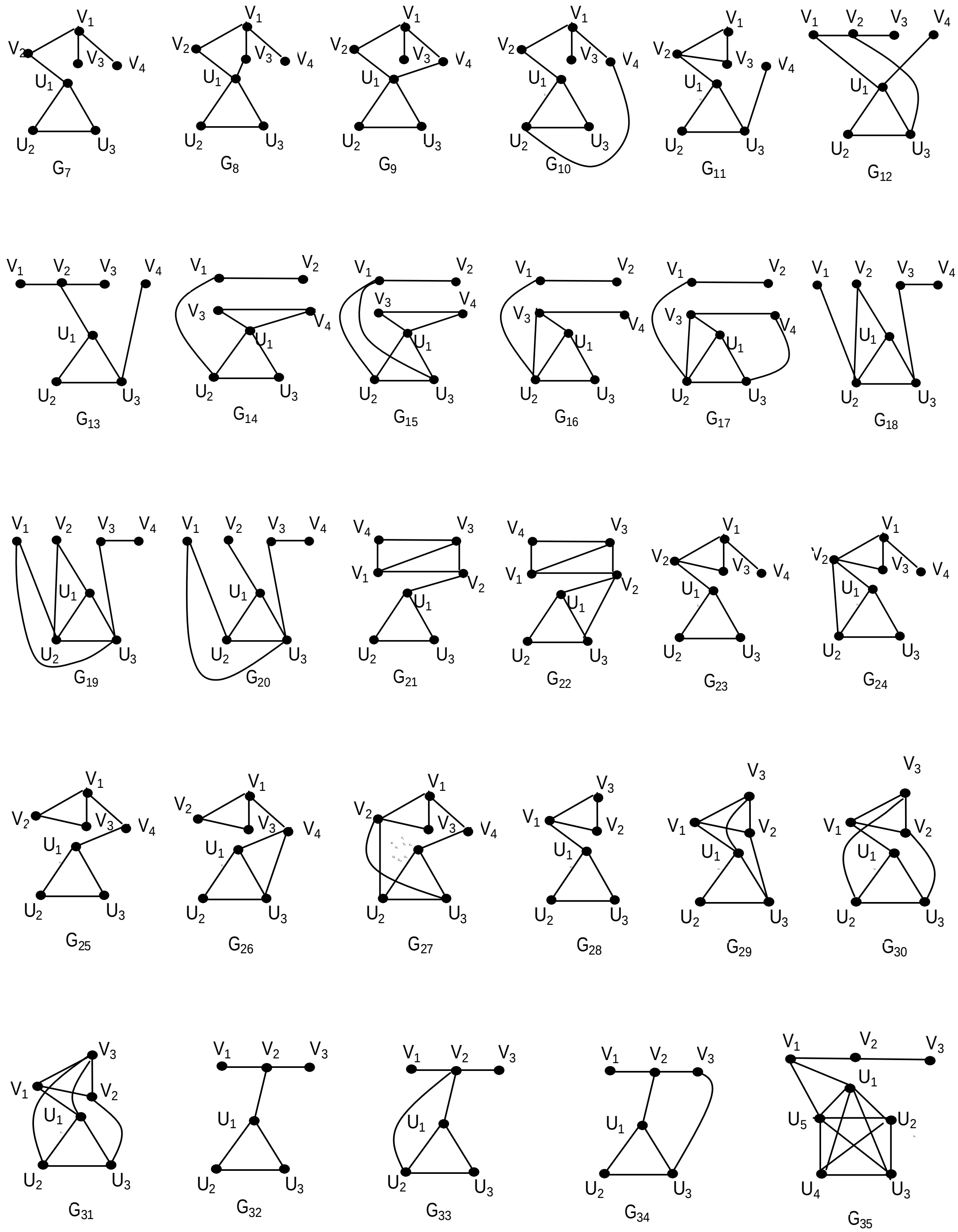
Notation 1.4: $K_5(n_1P_{m1}, n_2P_{m2}, n_3P_{m3}, n_4P_{m4}, n_5P_{m5})$ is a graph obtained from K_5 by attaching n_1 times the pendent vertex of P_{m1}(Paths on m_1 vertices) to a vertex u_i of K_5 and attaching n_2 times the pendent vertex of P_{m2}(Paths on m_2 vertices) to a vertex u_j for $i \neq j$ of K_5 and attaching n_3 times the pendent vertex of P_{m3}(Paths on m_3 vertices) to a vertex u_k for $i \neq j \neq k$ of K_5 and attaching n_4 times the pendent vertex of P_{m4}(Paths on m_4 vertices) to a vertex u_l for $i \neq j \neq k \neq l$ of K_5 and attaching n_5 times the pendent vertex of P_{m5}(Paths on m_5 vertices) to a vertex u_m for $i \neq j \neq k \neq l$ m of K_5.

Notation 1.5: $C_3(P_n)$ is the graph obtained from C_3 by attaching the pendant edge of P_n to any one vertices of C_3 and $K_n(P_m)$ is the graph obtained from K_n by attaching the pendant edge of Pm to any one vertices of K_n. $C_3(K_{1,n})$ is the graph obtained from C_3, by attaching the root vertex of $K_{1,n}$ to any one vertex of C_3. For $n \leq p$, $K_p(n)$ is the graph obtained from K_p by adding a new vertex and joint it with n vertices of K_p.

2. Main result

Theorem 2.1: For any connected graph G of order n ($n \geq 3$), $\gamma_{ip}(G) + \chi(G) = 2n-7$ if and only if $G \cong C_3(P_5)$, $C_3(3P_3)$, $C_3(P_6)$, $C_3(K_{1,3})$, $C_3(2P_2, P_2, 0)$, $C_3(P_3, P_3, 0)$, $C_3(P_3, P_2, P_2)$, $C_3(P_4, P_2, 0)$, $C_3(u(P_4, P_2))$, $C_3(u(P_3, P_2))$, $K_5(P_4)$, $K_5(P_3)$, $K_5(2P_3)$, $K_5(2P_2)$, $K_7(P_2)$, $K_7(2)$, $K_7(3)$, $K_7(4)$, $K_7(5)$, $K_7(6)$, $K_5(P_3, P_2, 0,0,0)$, $K_5(P_2, P_2, 0,0,0)$, K_9 or any one of the graphs shown in Figure 2.1




G_7 G_8 G_9 G_{10} G_{11} G_{12}
G_{13} G_{14} G_{15} G_{16} G_{17} G_{18}
G_{19} G_{20} G_{21} G_{22} G_{23} G_{24}
G_{25} G_{26} G_{27} G_{28} G_{29} G_{30}
G_{31} G_{32} G_{33} G_{34} G_{35}

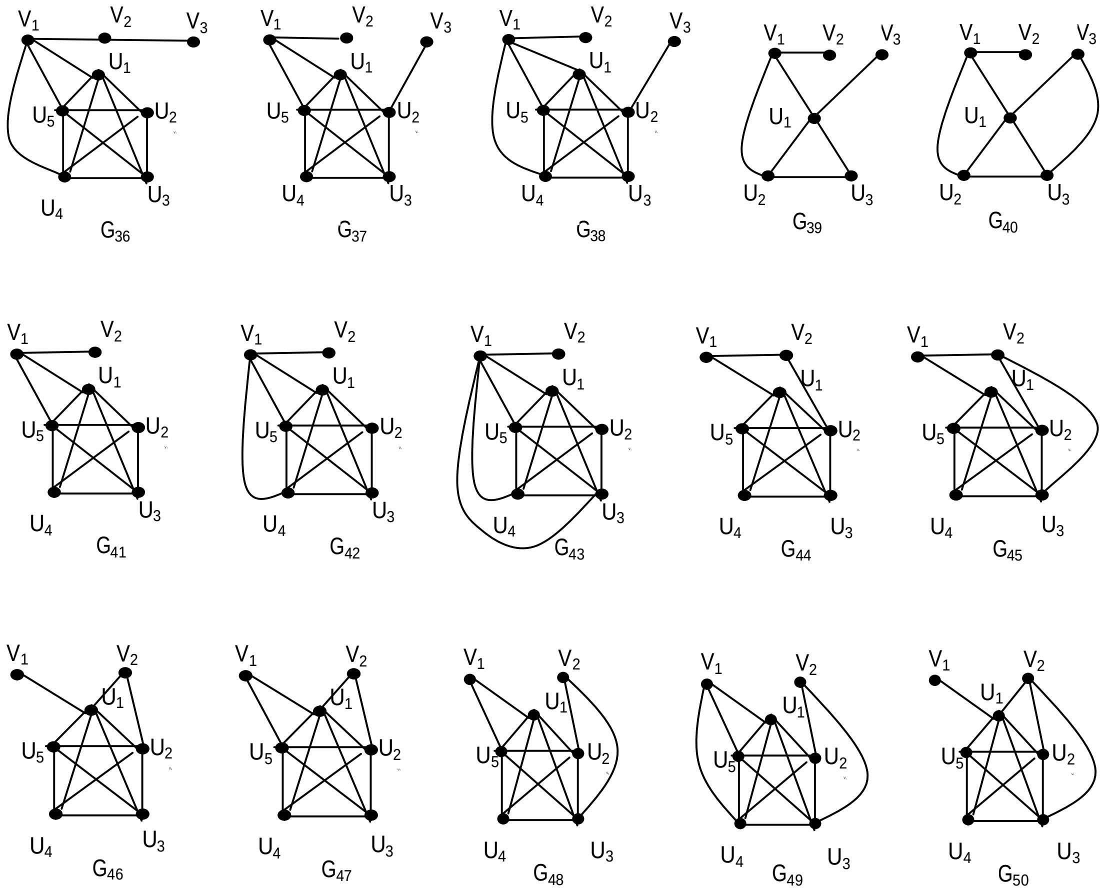


Figure 2.1

Proof: If G is any one of the graphs stated in the theorem, then it can be verified that $\gamma_{ip}(G) + \chi(G) = 9 = 2n - 7$. Conversely, let $\gamma_{ip}(G) + \chi(G) = 2n - 7$. Then the various possible cases are (i) $\gamma_{ip}(G) = n-1$ and $\chi(G) = n - 6$ (ii) $\gamma_{ip}(G) = n-2$ and $\chi(G) = n - 5$ (iii) $\gamma_{ip}(G) = n-3$ and $\chi(G) = n - 4$ (iv) $\gamma_{ip}(G) = n-4$ and $\chi(G) = n-3$ (v) $\gamma_{ip}(G) = n-5$ and $\chi(G) = n - 2$ (vi) $\gamma_{ip}(G) = n-6$ and $\chi(G) = n - 1$ (vii) $\gamma_{ip}(G) = n-7$ and $\chi(G) = n$.

Case (i): $\gamma_{ip} = n-1$ and $\chi = n - 6$.

Since $\gamma_{ip} = n-1$, by Theorem, 1. 1 $G \cong P_3$, C_3, P_5 or G' where G' is the graph as in Figure 1.1. Since $\chi = n - 6$, $G \cong G'$. But for G', $\chi = 3$, n = 9 so that $G \cong C_3(3P_3)$, G_1.

Case (ii): $\gamma_{ip} = n-2$ and $\chi = n - 5$.

Since $\chi(G) = n-5$, G contains a clique K on n-5 vertices (or) does not contains a clique K on n - 5 vertices. Let S = {v_1, v_2, v_3, v_4, v_5}. Then <S> has the following possible cases. <S> = K_5, $\overline{K}_5$, P_5, C_5, $P_3 \cup P_2$, $P_3 \cup \overline{K}_2$, $K_4 \cup K_1$, $P_4 \cup K_1$, $K_3 \cup K_2$, $K_3 \cup \overline{K}_2$, $C_4 \cup K_1$, and the remaining all possible spanning subgraphs on 5 vertices.

Subcase (i): If <S> = K_5.

Let v_1, v_2, v_3, v_4, v_5 be the vertices of K_5. Since G is connected, there exists a vertex u_i in K_{n-5} which is adjacent to any one of {v_1, v_2, v_3, v_4, v_5}. In this case {u_i, v_1} forms a γ_{ip} set of G so that γ_{ip} = 2 and n = 4, which is a contradiction. Hence no graph exists.

Subcase (ii): If <S> = $\overline{K}_5$.

Let v_1, v_2, v_3, v_4, v_5 be the vertices of $\overline{K}_5$. Since G is connected, all the vertices of $\overline{K}_5$ are adjacent to a vertex u_i in K_{n-5}. In this case {u_i, v_1} forms a γ_{ip} set of G so that γ_{ip} = 2 and n = 4, which is a contradiction. Hence no graph exists. Since G is connected, One vertiex of $\overline{K}_5$ is adjacent to a vertex u_i in K_{n-5} and one vertex is adjacent to u_j and one vertex is adjacent to u_k and one vertex is adjacent to u_l and one vertex is adjacent to u_m for $i \neq j \neq k \neq l \neq m$. In this case γ_{ip} set of G does not exists. Since G is connected, there exists a vertex u_i in $\overline{K}_5$ is adjacent to v_1, v_2, v_3, v_4 and u_j for $i \neq j$ is adjacent to v_5. In this case {u_i, u_j} for $i \neq j$ forms a γ_{ip} set of G so that γ_{ip} = 2 and n = 4, which is a contradiction. Hence no graph exists. Since G is connected, there exists a vertex u_i in $\overline{K}_5$ is adjacent to v_1, v_2, v_3 and u_j for $i \neq j$ is adjacent to v_4 and v_5. In this case {u_i, u_j} for $i \neq j$ forms a γ_{ip} set of G so that γ_{ip} = 2 and n = 4, which is a contradiction. Hence no graph exists. Since G is connected, there exists a vertex u_i in $\overline{K}_5$ is adjacent to v_1, v_2 and u_j for $i \neq j$ is adjacent to v_3 and v_4 and u_k for $i \neq j \neq k$ is adjacent to v_5. In this case γ_{ip} set of G does not exists.

Subcase (iii): If <S> = P_5.

Let v_1, v_2, v_3, v_4, v_5 be the vertices of P_5. Since G is connected, there exists a vertex u_i in K_{n-5} which is adjacent to any one of { v_1, v_5} or {v_2, v_4} or {v_3}. If u_i is adjacent to v_1, then {u_j, u_k, v_1, v_2, v_4, v_5} for $i \neq j$ forms a γ_{ip} set of G so that γ_{ip} = 6 and n = 8. Hence K = K_3 = $u_1u_2u_3$. If u_1 is adjacent to v_1, then G$\cong$ $C_3(P_6)$. Since G is connected, there exists a vertex u_i in K_{n-5} which is adjacent to v_2. In this case {u_i, v_2, v_4, v_5} forms a γ_{ip} set of G so that γ_{ip} = 4 and n = 6, which is a contradiction. Hence no graph exists.

Subcase (iv): If <S> = $P_3 \cup K_2$, (or) $P_3 \cup \overline{K}_2$, (or) $K_3 \cup K_2$, (or) C_5.

In all the possible cases, it can be verified that no graph exists.

Subcase (viii): If <S> = $P_3 \cup P_2$.

Let v_1, v_2, v_3 be the vertices of P_3 and

v_4, v_5 be the vertices of P_2. Since G is connected, there exists a vertex u_i in K_{n-5} which is adjacent to any one of { v_1, v_2, v_3} and any one of {v_4, v_5}. In this case {u_i, v_2, v_3, v_4} forms a γ_{ip} set of G so that γ_{ip} = 4 and n = 6, which is a contradiction. Hence no graph exists. Since G is connected, there exists a vertex u_i in K_{n-5} which is adjacent to any one of {v_1, v_2, v_3} and u_j for $i \neq j$ is adjacent to any one of {v_4, v_5}. In this case {u_j, v_2, v_3, v_4} forms a $_{ip}$ set of G so that $_{ip}$ = 4 and n = 6, which is a contradiction. Hence no graph exists. Since G is connected, there exists a vertex u_i in K_{n-5} which is adjacent to v_2 and v_4. In this case {u_j, u_k, v_2, v_3, v_4, v_5} for $i \neq j$ forms a γ_{ip} set of G so that γ_{ip} = 6 and n = 8. Hence $K = K_3 = u_1u_2u_3$. Let u_1 be adjacent to v_2 and v_4. If $deg(v_1) = 1 = deg(v_3)$, $deg(v_2) = 3$, $deg(v_4) = 2$, $deg(v_5) = 1$, then $G \cong G_2$.

Subcase (ix): If <S> = $K_4 \cup K_1$, (or) $P_4 \cup K_1$, (or) $K_3 \cup \overline{K}_2$, (or) $C_4 \cup K_1$ and all possible spanning subgraph S on 5 vertices[5-7].

In all the above possible cases it can be verified that no graph exits.

If G does not contain a clique K on n–5 vertices, then it can be verified that no new graph exists.

Case (iii): γ_{ip} = n-3 and χ=n–4.

Since χ=n–4, G contains a clique K on n – 4 vertices or does not contain a clique K on n – 4 vertices. Let G contain a clique K on n – 4 vertices. Let S = V(G) V(K) = {v_1, v_2, v_3, v_4}. Then the induced subgraph <S> has the following possible cases: <S> = K_4, $\overline{K}$, P_4, C_4, $K_{1,3}$, $P_3 \cup K_1$, $K_2 \cup K_2$, $K_3 \cup K_1$, $K_2 \cup \overline{K}_2$, $K_4 - \{e\}$, $C_3(1,0,0)$.

Subcase (i): If <S> = K_4, (or) $\overline{K}_4$.

In all the various possible cases, it can be verified that no graph exists.

Subcase (iii): If <S> = P_4,

Let {v_1, v_2, v_3, v_4} be the vertices of P_4. Since G is connected, there exists a vertex u_i in K_{n-4} which is adjacent to any one of {v_1, v_4} or any one of {v_2, v_3}. Let u_i be adjacent to v_1 for some i in K_{n-4}. Then {u_i, v_1, v_3, v_4} forms a γ_{ip} set of G so that γ_{ip} = 4 and n = 7 and hence $K = K_3 = u_1u_2u_3$. If u_1 is adjacent to v_1, then $G \cong C_3(P_5)$. Let u_i be adjacent to v_2 for some i in K_{n-4}. Then {u_j, u_k, v_2, v_3} for $i \neq j \neq k$ forms a γ_{ip} set of G so that γ_{ip} = 4 and n = 7 and hence $K = K_3 = u_1u_2u_3$. If u_1 is adjacent to v_2. If $deg(v_1) = 1 = deg(v_4)$, $deg(v_2) = 3$, $deg(v_3) = 2$, then $G \cong G_3$.

Subcase (iv): <S> = C_4.

Let {v_1, v_2, v_3, v_4} be the vertices of C_4. Since G is connected, there exists a vertex u_i in $K\text{-}_{n-4}$ which is adjacent to v_3. Let u_i for some i in K_{n-4}, be adjacent to v_3, then {u_i, u_j, v_2, v_4} for $i \neq j$ forms a γ_{ip}-set of G so that γ_{ip} = 4 and n = 7. Hence $K = K_3 = \langle u_1, u_2, u_3 \rangle$. If

u_1 is adjacent to v_1, then $\deg(v_1) = 3$, $\deg(v_2) = \deg(v_3) = \deg(v_2) = 2$,and so $G \cong G_4$. Let u_1 be adjacent to v_1 and v_2. If $\deg(v_1) = \deg(v_2) = 3$, $\deg(v_3) = \deg(v_4) = 2$,then $G \cong G_5$. Let u_1 be adjacent to v_1 and v_2 and u_3 be adjacent to v_2. If $\deg(v_1) = 3$, $\deg(v_2) = 4$, then $G \cong G_6$.

Subcase (v): $<S> = K_{1,3}$.

Let v_1 be the root vertex and v_2, v_3,v_4 are adjacent to v_1. Since G is connected, there exists a vertex u_i in K_{n-4} which is adjacent to v_1 (or) any one of $\{v_2, v_3, v_4\}$ and v_4. Let u_i for some i in K_{n-4} be the vertex adjacent to v_1, then $\{u_i, v_1\}$ is a γ_{ip} -set of G so that $\gamma_{ip} = 2$ and $n = 5$, which is a contradiction. Hence no such graph exists. Since G is connected, there exists a vertex u_i in K_{n-4} which is adjacent to any one of $\{v_2, v_3, v_4\}$. Then u_i for some i, is adjacent[1] to v_2. In this case, $\{u_i, u_j, v_1, v_4\}$ for $i \neq j$ is an γ_{ip} set of G so that $\gamma_{ip} = 4$ and $n = 7$, and hence $K = K_3 = <u_1, u_2, u_{3>}$. Let u_1 be adjacent to v_2. If $\deg(v_1) = 3$, $\deg(v_3) = \deg(v_4) = 1$, $\deg(v_2) =2$, then $G \cong G_7$. Let u_1 be adjacent to v_2 and v_3. If $\deg(v_1) = 3$, $\deg(v_2) = \deg(v_3) =2$, $\deg(v_4) = 1$, then G G_8. Let u_1 be adjacent to v_2 and v_4. If $\deg(v_1) = 3$, $\deg(v_2) = \deg(v_4) = 2$, $\deg(v_3) = 1$, then $G \cong G_9$. Let u_1 be adjacent to v_2 and u_2 be adjacent to v_4. If $\deg(v_1) = 3$, $\deg(v_2) = \deg(v_4) = 2$, $\deg(v_3) = 1$, then $G \cong G_{10}$.

Subcase (vi): $<S> = K_3 \cup K_1$.

Let v_1, v_2 and v_3 be the vertices of K_3 and v_4 be the vertex of K_1. Since G is connected, there exists a vertex u_i in K_{n-4} which is adjacent to any one of $\{v_1, v_2, v_3\}$ and $\{v_4\}$. In this case $\{u_i, v_1\}$ is a γ_{ip} set of G, so that $\gamma_{ip} = 2$ and $n = 5$, which is a contradiction. Hence no graph exists. Since G is connected, there exists a vertex u_i in K_{n-4} which is adjacent to v_2, and u_j for $i \neq j$, is adjacent to v_4. In this case, $\{u_i, u_j, v_1, v_3\}$ is a γ_{ip} -set of G so that $\gamma_{ip} = 4$ and $n = 7$. Hence $K = K_3 = <u_1, u_2, u_{3>}$. Let u_1 be adjacent to v_2 and u_3 be adjacent to v_4. If $\deg(v_1) = \deg(v_3) = 2$, and $\deg(v_2) =3$, $\deg(v_4) = 1$, then $G \cong G_{11}$.

Subcase (vii): $<S> = P_3 \cup K_1$.

Let v_1 ,v_2, v_3 be the vertices of P_3 and v_4 be the vertex of K_1. Since G is connected, there exists a vertex u_i in K_{n-4} which is adjacent to any one of $\{v_1, v_2, v_3\}$ and $\{v_4\}$. Let u_i be adjacent to v_1 and v_4. In this case $\{u_i, v_2, v_3, v_4\}$ is a γ_{ip} -set of G, so that $\gamma_{ip} = 4$ and $n = 7$. Hence $K = K_3 = <u_1, u_2, u_{3>}$. Let u_1 be adjacent to v_1 and v_4. If $\deg(v_1) = \deg(v_2) =2$, $\deg(v_3) = 1$, $\deg(v_4) = 1$, then $G \cong C_3(u(P_4, P_2))$. Let u_1 be adjacent to v_1 and v_4, and let u_3 be adjacent to v_2. If $\deg(v_1)=2$, $\deg(v_2) = 3$, $\deg(v_3) = \deg(v_4) = 1$, then $G \cong G_{12}$. Since G is connected, there exists a vertex u_i in K_{n-4} which is adjacent to v_1 and there exists u_j for $i \neq j$, is adjacent to v_4. In this case, $\{u_i, u_j, v_2, v_3\}$ is a γ_{ip} -set of G so that $\gamma_{ip} = 4$ and $n = 7$, and hence $K = K_3 = <u_1, u_2, u_{3>}$. Let u_1 be adjacent to v_1 and let u_3 be adjacent to v_4. If $\deg(v_1) = \deg(v_2) =2$, $\deg(v_3) = \deg(v_4) = 1$, then $G \cong C_3(P_4, P_2, 0)$. Since G is connected, there exists a vertex u_i in K_{n-4}

which is adjacent to v_2 and v_4. In this case, $\{u_i, v_2\}$ is a γ_{ip}-set of G so that $\gamma_{ip} = 2$ and $n = 5$, which is a contradiction. Hence no graph exists. Since G is connected, there exists a vertex u_i in K_{n-4} which is adjacent to v_2 and u_j for $i \neq j$ is adjacent to v_4. In this case, $\{u_j, u_k, v_2, v_3\}$ is a γ_{ip}-set of G so that $\gamma_{ip} = 4$ and $n = 7$, and hence $K=K_3=<u_1, u_2, u_3>$. Let u_1 be adjacent to v_2 and let u_3 be adjacent to v_4. If $\deg(v_1) = \deg(v_3) = \deg(v_4) = 1$, $\deg(v_2) = 3$, then $G \cong G_{13}$.

Subcase (viii): $<S> = K_2 \cup K_2$.

Let v_1 and v_2 be the vertices of K_2 and v_3, v_4 be the vertices of K_2. Since G is connected, there exists a vertex u_i in K_{n-4} which is adjacent to any one of $\{v_1, v_2\}$ and any one of $\{v_3, v_4\}$. Let u_i be adjacent to v_1 and v_3. In this case $\{u_j, u_k, v_1, v_2, v_3, v_4\}$ forms a γ_{ip}-set of G, so that $\gamma_{ip} = 6$ and $n = 9$. Hence $K = K_5 = <u_1, u_2, u_3, u_4, u_5>$. Let u_1 be adjacent to v_1 and v_3. If $\deg(v_1) = 2$, $\deg(v_3) = 2$, $\deg(v_2) = 1 = \deg(v_4)$, then $G \cong K_5(2P_3)$. Since G is connected, there exists a vertex u_i in K_{n-4} which is adjacent to v_1 and there exists u_j for $i \neq j$, is adjacent to v_3. In this case, $\{u_j, v_1, v_2, v_3\}$ for $i \neq j$ forms a γ_{ip}-set of G so that $\gamma_{ip} = 4$ and $n = 7$, and hence $K = K_3 = <u_1, u_2, u_3>$. Let u_1 be adjacent to v_3 and let u_2 be adjacent to v_1. If $\deg(v_1) = \deg(v_3) = 2$, $\deg(v_2) = \deg(v_4) = 1$, then $G \cong C_3(P_3, P_3, 0)$. Let u_1 be adjacent to v_3 and v_4, and let u_2 be adjacent to v_1. If $\deg(v_1) = \deg(v_3) = \deg(v_4) = 2$, $\deg(v_2) = 1$, then G G_{14}. Let u_1 be adjacent to v_3 and v_4, u_2 be adjacent to v_1, and let u_3 be adjacent to v_1-. If $\deg(v_1) = 3$, $\deg(v_3) = 2$ $\deg(v_4) = 2$, $\deg(v_2) = 1$, then $G \cong G_{15}$. Let u_1 be adjacent to v_3, and let u_2 be adjacent to v_1 and v_3. If $\deg(v_1) = 2$, $\deg(v_2) = \deg(v_4) = 1$, $\deg(v_3) = 3$, then $G \cong G_{16}$. Let u_1 be adjacent to v_3, u_2 be adjacent to v_1 and v_3, and let u_3 be adjacent to v_4. If $\deg(v_1)=2$, $\deg(v_2)=1$, $\deg(v_4)= 2$, $\deg(v_3) = 3$, then $G \cong G_{17}$.

Subcase (ix): $<S> = K_2 \cup \overline{K}_2$.

Let v_1 and v_2 be the vertices of $\overline{K}_2$ and v_3, v_4 be the vertices of K_2. Since G is connected, there exists a vertex u_i in K_{n-4} which is adjacent to v_1 and v_2 and any one of $\{v_3, v_4\}$. Let u_i be adjacent to v_1, v_2, v_3. In this case $\{u_i, v_3\}$ forms a γ_{ip} - set of G, so that $\gamma_{ip} = 2$ and $n = 5$, which is a contradiction. Hence no graph exists. Since G is connected, there exists a vertex u_i in K_{n-4} which is adjacent to v_1 and there exists u_j for $i \neq j$, is adjacent to v_2 and v_3. In this case, γ_{ip}- set of G does not exists. Since G is connected, there exists a vertex u_i in K_{n-4} which is adjacent to v_1 and there exists u_j for $i \neq j$, is adjacent to v_2 and u_k for $i \neq j \neq k$ is adjacent to v_3. In this case, $\{u_i, u_j, v_3, v_4\}$ for $i \neq j$ forms a γ_{ip}-set of G so that $\gamma_{ip} = 4$ and $n = 7$. Hence $K = K_3 = <u_1, u_2, u_3>$. Let u_1 be adjacent to v_2 and let u_2 be adjacent to v_1 and u_3 be adjacent to v_3. If $\deg(v_1) = \deg(v_2) = \deg(v_4) = 1$, $\deg(v_3) = 2$, then $G \cong C_3(P_3, P_2, P_2)$. Let u_1 be adjacent to v_2, u_2 be adjacent to v_1 and v_2 and let u_3 is adjacent to v_3. If $\deg(v_1) = \deg(v_4) = 1$, $\deg(v_2) = 2$, $\deg(v_3) = 2$, then $G \cong G_{18}$. Let u_1 be adjacent to v_2, u_2 be adjacent to v_1 and v_2 and let u_3 be adjacent to v_1 and v_3. If $deg(v_1) = 2$, $deg(v_4) = 1$, $deg(v_2) = 2$, $deg(v_3) = 2$, then $G \cong G_{19}$. Let u_1 be adjacent to v_2, u_2 be adjacent to v_1 and let u_3 be adjacent to v_3 and v_1. If $deg(v_1) = deg(v_3) = 2$, $deg(v_2) = deg(v_4) = 1$, then $G \cong G_{20}$.

Subcase (x): $\langle S\rangle = K_4 - \{e\}$.

Let v_1, v_2, v_3, v_4 be the vertices of K_4. Let e be any one of the edges inside the cycle C_4. Since G is connected, there exists a vertex u_i in K_{n-4} which is adjacent to v_1 or degree 3. In this case $\{u_i, v_1\}$ is a γ_{ip}-set of G, so that $_{ip} = 2$ and $n = 5$, which is a contradiction. Hence no graph exists. Since G is connected, there exists a vertex u_i in K_{n-4} which is adjacent to v_2 of degree 2. In this case, $\{u_i, u_j, v_1, v_3\}$ is a γ_{ip}-set of G so that $\gamma_{ip} = 4$ and $n = 7$. Hence $K = K_3 = \langle u_1, u_2, u_3\rangle$. Let u_1 be adjacent to v_2. If $deg(v_1) = deg(v_2) = deg(v_3) = 3$, $deg(v_4) = 2$, then $G \cong G_{21}$. Let u_1 be adjacent to v_2 and let u_3 adjacent to v_2. If $deg(v_1) = 3$, $deg(v_2) = 4$, $deg(v_3) = 3$, $deg(v_4) = 2$, then $G \cong G_{22}$.

Subcase (xi): $\langle S\rangle = C_3(1, 0, 0)$.

Let v_1, v_2, v_3 be the vertices of C_3 and let v_4 be adjacent to v_1. Since G is connected, there exists a vertex u_i in K_{n-4} which is adjacent to v_2. In this case $\{u_j, u_k, v_1, v_2\}$ for $i \neq j \neq k$ is a γ_{ip}-set of G, so that $\gamma_{ip} = 4$ and $n = 7$. Hence $K = K_3 = \langle u_1, u_2, u_3\rangle$. If u_1 is adjacent to v_2, $deg(v_1) = deg(v_2) = 3$, $deg(v_3) = 2$, $deg(v_4) = 1$, then $G \cong G_{23}$. Let u_1 be adjacent to v_2 and let u_2 be adjacent to v_2. If $deg(v_1) = 3$, $deg(v_2) = 4$, $deg(v_3) = 2$, $deg(v_4) = 1$, then $G \cong G_{24}$. Since G is connected, there exists a vertex u_i in K_{n-4} which is adjacent to v_1. In this case $\{u_i, v_1\}$ is a γ_{ip} - set of G, so that $\gamma_{ip} = 2$ and $n = 5$, which is a contradiction. Hence no graph exists. Since G is connected, there exists a vertex u_i in K_{n-4} which is adjacent to v_4. In this case $\{u_i, u_j, v_2, v_4\}$ for $i \neq j$ forms a γ_{ip}-set of G, so that $_{ip} = 4$ and $n = 7$. Hence $K = K_3 = \langle u_1, u_2, u_3\rangle$. If u_1 is adjacent to v_4, $deg(v_1) = 3$, $deg(v_2) = deg(v_3) = deg(v_4) = 2$, then $G \cong G_{25}$. Let u_1 be adjacent to v_4, and let u_3 be adjacent to v_4. If $deg(v_1) = 3$, $deg(v_2) = deg(v_3) = 2$, $deg(v_4) = 3$, then $G \cong G_{26}$. Let u_1 be adjacent to v_4, u_2 be adjacent to v_2 and let u_3 be adjacent to v_2. If $deg(v_1) = 3$, $deg(v_3) = 2$, $deg(v_4) = 2$, $deg(v_2) = 4$, then $G \cong G_{27}$. If G does not contain a clique K on $n - 4$ vertices, then it can be verified that no new graph exists.

Case (iv): $\gamma_{ip} = n-4$ and $\chi = n-3$.

Since $\chi = n-3$, G contains a clique K on $n-3$ vertices or does not contains a clique K on n-3 vertices. Let $S = V(G) - V(K) = \{v_1, v_2, v_3\}$. Then the induced sub graph $\langle S\rangle$ has the following possible cases. $\langle S\rangle = K_3$, $\overline{K}_3$, P_3, K_2 □ K_1.

Subcase (i): $\langle S\rangle = K_3$.

Let v_1, v_2, v_3 be the vertices of K_3. Since G is connected, there exists a vertex u_i in K_{n-3} which is adjacent to any one of $\{v_1, v_2, v_3\}$. Let u_i be adjacent to v_1, then $\{u_i, v_1\}$ is a

γ_{ip} -set of G, so that γ_{ip} = 2 and n = 6. Hence K = K_3 = <u_1, u_2, u_3>. Let u_1 be adjacent to v_1. If deg(v_1)=3, deg(v_3)= 2 =deg(v_2), then G ≅ G_{28}. Let u_1 be adjacent to v_1 and v_3 and let u_3 be adjacent to v_2. If deg(v_1) = deg(v_3) = 3, deg(v_2) = 3, then G ≅ G_{29}. Let u_1 be adjacent to v_1, u_2 be adjacent to v_3 and let u_3 be adjacent to v_2. If deg(v_1)=deg(v_2),= deg(v_3) = 3, then G ≅ G_{30}. Let u_1 be adjacent to both the vertices v_1 and v_3, u_2 be adjacent to v_3, and let u_3 be adjacent to v_2. If deg(v_1) = 3, deg(v_2) = 3, deg(v_3) = 4, then G ≅ G_{31}.

Subcase (ii): <S> = $\overline{K}_3$.

Let v_1, v_2, v_3 be the vertices of $\overline{K}_3$. Since G is connected, one of the vertices of K_{n-3} say u_I is adjacent to all the vertices of S (or) u_i be adjacent to v_1, v_2 and u_j be adjacent to v_3 for i≠j (or) u_i be adjacent to v_1 and u_j be adjacent to v_2 and u_k be adjacent to v_3 for i ≠j≠k. If u_i for some i, is adjacent to all the vertices of S, then {u_i, u_j} for i ≠ j, is a γ_{ip} -set of G, so that γ_{ip} = 2 and n = 6. Hence K = K_3 = <u_1, u_2, u_3>. If u_1 is adjacent to all the vertices v_1,v_2,v_3, then G ≅ $C_3(K_{1,3})$. Since G is connected, there exists a vertex u_i in K_{n-3} is adjacent to v_1 and u_j for i ≠ j is adjacent to v_2 and v_3, then {u_i, u_j} is a γ_{ip} -set of G, so that γ_{ip}= 2 and n = 6. Hence K = K_3 = <u_1, u_2, u_3>. Let u_1 be adjacent to v_1 and v_2 and let u_3 be adjacent to v_3. If deg(v_1) = deg(v_2) = deg(v_3) =1 then G $C_3(2P_2, P_2, 0)$. Since G is connected, there exists a vertex u_i in K_{n-3} is adjacent to v_1 and u_j for i≠ j is adjacent to v_2 and u_k for i ≠ j ≠ k in K_{n-3} is adjacent to v_3, then γ_{ip} -set of G does not exists.

Subcase (iii): <S> = P_3.

Let v_1, v_2, v_3 be the vertices of P_3. Since G is connected, there exists a vertex u_i in K_{n-3} which is adjacent to v_1 (or equivalently v_3) or v_2. If u_i is adjacent to v_2, then {u_i, v_2} is a γ_{ip} -set of G, so that γ_{ip} = 2 and n = 6. Hence K = K_3 = <u_1, u_2, u_3>. Let u_1 be adjacent to v_2. If deg(v_2) = 3, deg(v_1) = deg(v_3) =1, then G ≅ G_{32}. Let u_1 be adjacent to v_2 and let u_2 be adjacent to v_2. If deg(v_2) = 4, deg(v_1) = deg(v_3) =1, then G G_{33}. Let u_1 be adjacent to and let u_3 be adjacent to v_3. If deg(v_1) = 1, deg(v_2) = 3, deg(v_3)=2, then G≅G_{34}. Since G is connected, there exists a vertex u_i in K_{n-3} is adjacent to v_1 then {u_i, u_j, v_2, v_3} for i ≠ j is a γ_{ip} -set of G, so that γ_{ip} = 4 and n = 8. Hence K = K_5 = <u_1, u_2, u_3, u_4, u_5>. Let u_1 be adjacent to v_1. If deg(v_1) = deg(v_2) =2, deg(v_3)=1, then G ≅ $K_5(P_4)$. Let u_1 be adjacent to v_1 and let u_5 be adjacent to v_1. If deg(v_1)= 3, deg(v_2) = 2, deg(v_3) = 1, then G ≅ G_{35}. If u_1 is adjacent to v_1, u_4 be adjacent to v_1 and let u_5 be adjacent to v_1. If deg(v_1) = 4, deg(v_2) = 2, deg(v_3) = 1, then G ≅ G_{36}.

Subcase (iv): <S> = K_2 □ K_1.

Let v_1, v_2 be the vertices of K_2 and v_3 be the vertex of K_1. Since G is connected, there exists a vertex u_i in K_{n-3} which is adjacent to any one of {v_1, v_2} and {v_3} (or) u_i is adjacent to any one of $\{v_1, v_2\}$ and u_j for $i \neq j$ is adjacent to v_3. In this case, $\{u_j, v_1, v_2, v_3\}$ forms a γ_{ip} -set of G, so that $\gamma_{ip} = 4$ and n = 8. Hence $K = K_5 = <u_1, u_2, u_3, u_4, u_5>$. Let u_1 be adjacent to v_1 and let u_2 be adjacent to v_3. If $deg(v_1) = 2$, $deg(v_2) = deg(v_3) = 1$, then G $K_5(P_3, P_2, 0, 0, 0)$. Let u_1 be adjacent to v_1, u_2 be adjacent to v_3 and let u_5 be adjacent to v_1. If $deg(v_1) = 3$, $deg(v_2) = 1$, $deg(v_3) = 1$, then $G \cong G_{37}$. Let u_1 be adjacent to v_1, u_2 be adjacent to v_3 and let u_4 be adjacent to v_1 and u_5 be adjacent to v_1. If $deg(v_1) = 4$, $deg(v_2)=deg(v_3) = 1$, then $G \cong G_{38}$. Since G is connected, there exists a vertex u_i in K_{n-3} which is adjacent to v_1, v_3, so that $\{u_i, v_1\}$ is a γ_{ip} -set of G. Hence $\gamma_{ip} = 2$ and n = 6, so that $K = K_3 = <u_1, u_2, u_3>$. Let u_1 be adjacent to v_1 and v_3. If $deg(v_1) = 2$, $deg(v_2) = deg(v_3) = 1$, then $G \cong C_3(u(P_3, P_2))$. Let u_1 be adjacent to v_1 and v_3 and let u_2 be adjacent to v_1. If $deg(v_1) = 3$, $deg(v_2)=deg(v_3) = 1$, then $G \cong G_{39}$. Let u_1 be adjacent to v_1 and v_3, u_2 be adjacent to v_1 and let u_3 be adjacent to v_3. If $deg(v_1) = 3$, $deg(v_2) = 1$, $deg(v_3) = 1$, then $G \cong G_{40}$. If G does not contain a clique K on n 3 vertices, then it can be verified that no new graph exists.

Case (v): γ_{ip} = n–5 and χ=n–2.

Since χ=n–2 G contains a clique K on n–2 vertices or does not contains a clique K on n-2 vertices. Let G contains a clique K on n–2 vertices. Let S = V(G) V(K) = $\{v_1, v_2\}$. Then the induced subgraph <S> has the following possible cases. <S> = K_2, $\overline{K}_2$.

Subcase (i): <S> = K_2.

Let v_1, v_2 be the vertices of K_2. Since G is connected, there exists a vertex u_i in K_{n-2} which is adjacent to any one of $\{v_1, v_2\}$, then $\{u_i, v_1\}$ is a γ_{ip}-set of G, so that $\gamma_{ip} = 2$ and n = 7. Hence $K = K_5 = <u_1, u_2, u_3, u_4, u_5>$. Let u_1 be adjacent to v_1. If $deg(v_1) = 2$, $deg(v_2) = 1$, then $G \cong K_5(P_3)$. Let u_1 be adjacent to v_1 and let u_5 be adjacent to v_1. If $deg(v_1) = 3$, $deg(v_2) = 1$, then $G \cong G_{41}$. Let u_1 be adjacent to v_1, u_4 be adjacent to v_1 and let u_5 be adjacent to v_1. If $deg(v_1) = 4$, $deg(v_2) = 1$, then $G \cong G_{42}$. Let u_1 be adjacent to v_1, u_3 be adjacent to v_1 and let u_4 be adjacent to v_1, u_5 be adjacent to v_1. If $deg(v_1) = 5$, $deg(v_2) = 1$, then $G \cong G_{43}$. Let u_1 be adjacent to v_1 and let u_2 be adjacent to v_2. If $deg(v_1) = deg(v_2) = 2$, then $G \cong G_{44}$. Let u_1 be adjacent to v_1, u_2 be adjacent to v_2 and let u_3 be adjacent to v_2. If $deg(v_2) = 3$, $deg(v_1) = 2$, then $G \cong G_{45}$.

Subcase (ii): <S> = $\overline{K}_2$.

Let v_1, v_2 be the vertices of $\overline{K}_2$. Since G is connected, v_1 and v_2 are adjacent to a common vertex say u_i in K_{n-2} (or) v_1 is adjacent to u_i and v_2 is adjacent to u_j for some $i \neq j$ in K_{n-2}. In both cases, $\{u_i, u_j\}$ is a γ_{ip} -set of G, so that $\gamma_{ip} = 2$ and n = 7. Hence $K = K_5 = <u_1, u_2, u_3, u_4, u_5>$. Let u_1 be adjacent to v_1 and v_2. If $deg(v_1) = 1 = deg(v_2)$, then G $K_5(2P_2)$. Let u_1 be adjacent to v_1 and u_2 be adjacent to v_2. If $deg(v_1) = deg(v_2) = 1$, then G $K_5(P_2, P_2, 0, 0, 0)$.

Let u_1 be adjacent to v_1 and v_2 and let u_2 be adjacent to v_2. If $deg(v_1) = 1$, $deg(v_2) = 2$, then $G \cong G_{46}$. Let u_1 be adjacent to v_1 and v_2, u_2 be adjacent to v_2 and let u_5 be adjacent to v_1. If $deg(v_1) = deg(v_2) =2$, then $G \cong G_{47}$. Let u_1 be adjacent to v_1, u_2 be adjacent to $v_{2,}$ and let u_3 be adjacent to $v_{2,}$ u_5 be adjacent to v_1. If $deg(v_1) = deg(v_2) = 2$, then $G \cong G_{48}$. Let u_1 be adjacent to v_1, u_2 be adjacent to $v_{2,}$ and let u_3 be adjacent to v_2, u_4 be adjacent to $v_{1,}$ and let u_5 be adjacent to v_1. If $deg(v_1) = 3$, $deg(v_2) = 2$, then $G \cong G_{49}$. Let u_1 be adjacent to v_1 and v_2, u_2 be adjacent to v_2 and let u_3 be adjacent to v_2. If $deg(v_1) = 1$, $deg(v_2) = 3$, then $G \cong G_{50}$. If G does not contain a clique K on $n - 2$ vertices, then it can be verified that no new graph exists.

Case (vi): $\gamma_{ip} = n–6$ and $\chi=n–1$.

Since $\chi=n–1$ G contains a clique K on n-1 vertices. Let $\{v_1\}$ be a vertex not on K_{n-1}. Since G is connected, there exists a vertex v_1 is adjacent to one vertex u_i of K_{n-1}. In this case $\{u_i, v_1\}$ is a γ_{ip}-set of G so that $\gamma_{ip} = 2$ and $n = 8$. Hence $K = K_7 = <u_1, u_2 ,u_3, u_4, u_5, u_6, u_7>$. If u_1 is adjacent to v_1, $deg(v_1) = 1$, then $G \cong K_7(P_2)$. Let u_1 be adjacent to v_1 and let u_2 be adjacent to v_1. If $deg(v_1) = 2$, then $G \cong K_7(2)$. Let u_1 be adjacent to v_1, u_2 be adjacent to v_1 and let u_3 be adjacent to v_1. If $deg(v_1) = 3$, then $G \cong K_7(3)$. Let u_1 be adjacent to v_1, u_2 be adjacent to v_1 and let u_3 be adjacent to v_1, u_4 be adjacent to v_1. If $deg(v_1) = 4$, then G $K_7(4)$. Let u_1 be adjacent to v_1, u_2 be adjacent to v_1 and let u_3 be adjacent to v_1, u_4 be adjacent to v_1 and let u_5 be adjacent to v_1. If $deg(v_1) = 5$, then $G \cong K_7(5)$. Let u_1 be adjacent to v_1, u_2 be adjacent to v_1 and let u_3 be adjacent to v_1, u_4 be adjacent to v_1 and let u_5 be adjacent to v_1, u_6 be adjacent to v_1. If $deg(v_1) = 6$, then $G \cong K_7(6)$. If G does not contain a clique K on n 1 vertices, then it can be verified that no new graph exists.

Case (vii): $\gamma_{ip} = n–7$ and $\chi = n$

Since, $G \cong K_n$. But for K_n, $\gamma_{ip} = 2$ so that $n = 9$. Hence $G \cong K_9$.

References

1. Berge C., *Theory of graphs and its applications*, Methuen London, (1962).
2. Harary F., *Graph Theory,* Addison Wesley Reading Mass, (1972).
3. Mahadevan G., *On domination theory and related concepts in graphs*, Ph.D. thesis, Manonmaniam Sundaranar University, Tirunelveli, India, (2005).
4. Mahadevan G., Nagarajan A. and Rajeswari A., *Characterization of induced Paired domination number of a graph*, International Journal of Physical Sciences, (*Ultra Scientist of Physical Sciences*), Vol. *24(2A)*, 337-346, (2012).
5. Mahadevan G., Selvam Avadayappan, and Amra Parveen M., *Graphs whose sum of independent domination number and chromatic number equals to 2n-6 for any n > 3*, International *Journal of Physical Sciences,* (*Ultra Science*), Vol.

20(3)M, 757-762, (2008).

6. Mahadevan G., Selvam A., and Hajmeral, *On connected efficient domination number of a graph*, International Journal of Intelligent Information Processing, 2(2), July-December, pp.313-319, (2008).
7. Mahadevan G., Selvam Avadayappan, and Iravithual Basira A., *Sum of complementary connected domination number and chromatic number of a graph*, *International Journal of Computing and Mathematical Applications*, Vol. *2*, No. 1-2, pp.159-169, (2008).
8. Paulraj Joseph J., and Arumugam. S., *Domination in graphs*, *International Journal of Management Systems, 11*, 177-182, (1995).
9. Paulraj Joseph J., and Arumugam S., *Domination and connectivity in graphs*, *International Journal of Management and systems*, *8* No. *3*, 233-236 (1992).
10. Teresa W. Haynes, *Induced - paired domination in graphs*, *Ars. combin. 57*, 111–128, (2000).
11. Teresa W. Haynes, *Paired domination in graphs*, Congr. Numer 150, (2001).
12. Teresa W. Haynes, Stephen T. Hedetniemi, and Peter J. Slater, *Domination in graphs*, Advanced Topics, Marcel Dekker, New York, (1998).
13. Teresa W. Haynes, Stephen T. Hedetniemi, and Peter J. Slater, *Fundamentals of domination in graphs*, Marcel Dekker, New York, (1998).