Ultra Scientist Vol. 25(1)A, 73-80 (2013).

MHD flow of a conducting visco-elastic Rivlin-Ericksen (1955) fluid through porous medium in a long uniform rectangular duct due to an impulsive pressure gradient acting at the central part of a section

ANIL TRIPATHI*, A. K. SHARMA* and K.K. SINGH**

(Acceptance Date 25th February, 2013)

Abstract

The aim of present problem is to study the unsteady flow of a conducting visco-elastic Rivlin-Ericksen[10] type fluid through porous medium in a long uniform rectangular duct under the influence of an impulsive pressure gradient acting at the central part of a section and uniform magnetic field applied normally to the flow of fluid. The analytical expressions for velocity profile and flux have been determined by the application of transform technique. Some particular cases have been discussed in detail. The results for the flow of ordinary viscous fluid have also been deduced by taking parameter $\mu_{1\to 0}$.

Introduction

Ghosh[3] discussed the viscous fluid flow through a long rectangular channel under the influence of an impulsive pressure gradient acting at the central part of a section. The problem of viscous fluid flow subjected to uniform and periodic body force for a finite time has been studied by Dutta[2,3]. Roy, Sen and Lahiri (1990) investigated the problem of unsteady flow of Rivlin-Ericksen visco-elastic fluid through a rectangular duct due to an impulsive pressure gradient acting at the central part of a section. Many researchers have paid their attention towards the application of visco-elastic fluid flow through porous medium such as Ahmad and Taqui[1]; Thingarajan[15], Ramamurthi and Rahim (2002); Pundir and Pundir[9]; Sharma and Sharma[11]; Prakash and Aggarwal[8]; Kumar, Sharma and Singh[5]; Singh, Mishra and Sharma[14]; Singh, Kumar and Sharma[13]; Singh[12] and Kumar, Singh and Sharma[6]; Kumar, Gupta and Jain[4]; Panda and Mohapatra[7]; Kumar, Singh[10,11] and Sharma (2010,2011) have studied the flow problems of visco-elastic fluid through porous medium under the influence of magnetic field applied perpendicularly to the flow of fluid.

In the present paper, the MHD flow of conducting visco-elastic Rivlin-Ericksen type fluid through porous medium in a rectangular duct due to an impulsive pressure gradient acting at the central part of a section under the influence of uniform magnetic field applied perpendicularly to the flow of field has been studied. The analytical expressions for the velocity profiles and the flux have been determined by using Laplace integral transform. Some particular cases and deduction are also considered.

Basic Theory :

For slow motion, the rheological equations for Rivlin-Ericksen (1955) visco-elastic fluid are:

$$\tau_{ij} = -p\delta_{ij} + \tau'_{ij} \tag{1}$$

$$\tau'_{ij} = 2\mu\left(1 + \mu_1 \frac{\partial}{\partial t}\right) e_{ij} \tag{2}$$

$$e_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right) \tag{3}$$

where τ_{ij} is the stress tensor, τ'_{ij} the deviatoric stress tensor, e_{ij} the rate of strain tensor, μ the coefficient of visco-elasticity, p the fluid presser, u_i the velocity components of the fluid and δ_{ij} is the kronecker delta.

We have Navier-Stokes equation of motion for Rivlin-Ericksen fluid as

$$\frac{d\vec{q}}{dt} = -\frac{1}{\rho}\vec{\nabla}p + \nu\left(1 + \mu_1 \frac{\partial}{\partial t}\right)\nabla^2\vec{q} + \vec{F} \tag{4}$$

where $\vec{q}$ is the velocity, $\vec{F}$ the body force, $\nu\left(= \frac{\mu}{\rho}\right)$ the coefficent of viscosity and ρ is the density of fluid.

Formulation of the problem :

Consider the rectangular Cartesian coordinates system (*x,y,z*); *z-axis* is taken towards the direction of motion of the conducting fluid. The wall of the rectangular duct are taken to be the planes $x = 0$, $x = a$ and $y = 0$, $y = b$. Let u, v, w be the components of the velocity of the fluid along positive direction of *x-axis, y-axis* and *z- axis* respectively. Also consider rectangular duct is long and the fluid velocity w in the direction of *z-axis* only, therefore

$$u = 0,\ v = 0,\ w = W(x,y,t)$$

In view of the above assumptions the equation of motion of electrically conducting visco-elastic Rivlin-Ericksen fluid through porous medium is given by

$$\frac{\partial w}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\left(1 + \mu_1 \frac{\partial}{\partial t}\right)\left(\frac{\partial^2 W}{\partial x^2} + \frac{\partial^2 W}{\partial y^2}\right) - \frac{\nu}{K}W - \frac{\sigma B_0}{\rho}W \tag{5}$$

where W is the velocity of the fluid in *z*-direction, t the time, K the permeability of porous media, σ the electrical conductivity and B_0 is the magnetic inductivity.

Introducing the following non-dimensional quantities:

$$x^* = \frac{x}{a},\ y^* = \frac{y}{a},\ z^* = \frac{z}{a},\ t^* = \frac{\nu}{a^2}t,\ p^* = \frac{a^2}{\rho\nu^2}p,$$

$$W^* = \frac{a}{\nu}W,\ \mu_1^* = \frac{\nu}{a^2}\mu_1,\ K^* = \frac{1}{a^2}K$$

in equation (5), we get (after dropping stars)

$$\frac{\partial w}{\partial t} = -\frac{\partial p}{\partial z} + \left(1+\mu_1\frac{\partial}{\partial t}\right)\left(\frac{\partial^2 W}{\partial x^2}+\frac{\partial^2 W}{\partial y^2}\right) - \frac{1}{K}W - H^2 W \quad (6)$$

where $B_0 a\sqrt{\frac{\sigma}{\mu}} = H$ is the Hartmann number

$Let\ -\frac{\partial p}{\partial z} = f(x,y)\delta t,$ δt is Dirac delta function, be an impulsive pressure gradient acting at the central part of any section such that :

Boundary conditions are

$$\left.\begin{aligned} f(x,y) &= p \quad for \begin{cases} \frac{1}{2}-l_1 \le x \le \frac{1}{2}+l_1 \\ \frac{c}{2}-l_2 \le y \le \frac{c}{2}+l_2 \end{cases} \\ &= 0 \quad for \begin{cases} 0 \le x < \frac{1}{2}, \frac{1}{2}+l_1 < x \le 1 \\ 0 \le y < \frac{c}{2}, \frac{c}{2}+l_2 < y \le c \end{cases} \\ &= 0 \quad for\ \ t \le 0 \end{aligned}\right\} \quad (7)$$

where $c(= b/a)<1$, a non dimensional quantity.

To find the solution of equation (6), we define

$$f(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} B_{mn}\sin\frac{m\pi x}{1}\sin\frac{n\pi y}{c} \quad (8)$$

Multiplying both sides of equation (8) by $\sin\frac{m\pi x}{1}\,sin\frac{n\pi y}{c}$ and Integrating, we get

$$B_{mn}\int_0^c\int_0^1 \sin^2\frac{m\pi x}{1}\sin^2\frac{n\pi y}{c}\,dxdy = P\int_{\frac{c}{2}-l_2}^{\frac{c}{2}+l_2}\int_{\frac{1}{2}-l_1}^{\frac{1}{2}+l_1} \sin\frac{m\pi x}{1}\sin\frac{n\pi y}{c}\,dxdy$$

$$\Longrightarrow B_{mn} = \frac{16P}{mn\pi^2}\sin\frac{m\pi l_1}{2}\sin\frac{n\pi l_2}{2c} \quad (9)$$

Let us assume the velocity of fluid be

$$W = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn(t)}\sin\frac{m\pi x}{1}\sin\frac{n\pi y}{c} \quad (10)$$

Substituting (8) and (10) in equation (6), we have

$$\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{\partial}{\partial t}\{A_{mn}(t)\}\sin\frac{m\pi x}{1}\sin\frac{n\pi y}{c} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}B_{mn}\sin\frac{m\pi x}{1}\sin\frac{n\pi y}{c} +$$

$$\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}A_{mn}\,(\mathrm{t})\left\{\left(-\frac{\mathrm{m}^2\pi^2}{1}\right)+\left(-\frac{\mathrm{n}^2\pi^2}{\mathrm{c}^2}\right)\right\}\sin\frac{m\pi x}{1}\sin\frac{n\pi y}{c} +$$

$$\mu_1\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{\partial}{\partial t}\{A_{mn}(t)\}\left\{\left(-\frac{\mathrm{m}^2\pi^2}{1}\right)+\left(-\frac{\mathrm{n}^2\pi^2}{\mathrm{c}^2}\right)\right\}\sin\frac{m\pi x}{1}\sin\frac{n\pi y}{c} -$$

$$\frac{1}{K}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}A_{mn}(t)\sin\frac{m\pi x}{1}\sin\frac{n\pi y}{c} \tag{11}$$

Taking Laplace transform of equation (11), we have

$$s\bar{A}_{mn}(s) = \frac{B_{mn}}{s} - \bar{A}_{mn}(s)C_{mn} - \mu_1 s\bar{A}_{mn}(s)C_{mn} - \frac{1}{K}\bar{A}_{mn} - H^2\bar{A}_{mn}$$

$$or \quad \left\{(1+\mu_1 C_{mn})s + \left(\frac{1}{K} + H^2 + C_{mn}\right)\right\}\bar{A}_{mn} = \frac{B_{mn}}{s}$$

$$or \quad \bar{A}_{mn}(s) = \frac{B_{mn}}{s\left\{\left((1+\mu_1 C_{mn})s + \left(\frac{1}{K} + H^2 + C_{mn}\right)\right)\right\}}$$

$$or \quad \bar{A}_{mn}(s) = \frac{B_{mn}}{(1+\mu_1 C_{mn})s}\,\frac{1}{\left\{s + \left(\frac{\frac{1}{K} + H^2 + C_{mn}}{1+\mu_1 C_{mn}}\right)\right\}}$$

$$or\ \bar{A}_{mn}(s) = \frac{B_{mn}}{\left(\frac{1}{K} + H^2 + C_{mn}\right)} \left[\frac{1}{s} - \frac{1}{\left\{ s + \left(\frac{\frac{1}{K} + H^2 + C_{mn}}{1 + \mu_1 C_{mn}} \right) \right\}} \right] \tag{12}$$

where $\bar{A}_{mn}(s) = \int_0^\infty e^{-st} A_{mn}(t) dt$ (Laplace transform of function A_{mn} (t))

and $C_{mn} = \frac{m^2\pi^2}{1} + \frac{n^2\pi^2}{c^2} = \left(m^2 + \frac{n^2}{c^2}\right)\pi^2$

Now applying the inverse Laplace transform of (12), we get

$L^{-1}\{\bar{A}_{mn}(s)\} = A_{mn}(t)$

$$= \frac{B_{mn}}{\left(\frac{1}{K} + H^2 + C_{mn}\right)} \left[L^{-1}\left(\frac{1}{s}\right) - L^{-1} \frac{1}{\left\{ s + \left(\frac{\frac{1}{K} + H^2 + C_{mn}}{1 + \mu_1 C_{mn}} \right) \right\}} \right]$$

$$= \frac{B_{mn}}{\left(\frac{1}{K} + H^2 + C_{mn}\right)} \left[1 - e^{-\left(\frac{\frac{1}{K}+H^2+C_{mn}}{1+\mu_1 C_{mn}}\right)t} \right] \tag{13}$$

From (9), (10) and (13), we get velocity of visco-elastic Rivlin-Ericksen fluid

$$W = \frac{16P}{\pi^2} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\left[1 - e^{-\left(\frac{\frac{1}{K}+H^2+C_{mn}}{1+\mu_1 C_{mn}}\right)t} \right]}{mn\left(\frac{1}{K} + H^2 + C_{mn}\right)} \sin\frac{m\pi l_1}{2} \sin\frac{n\pi l_2}{2c} \sin\frac{m\pi x}{1} \sin\frac{n\pi y}{c} \tag{14}$$

Determination of flux :

The flux Q across any section at any instant is given by

$$Q = \int_0^1 \int_0^c w \, dx \, dy$$

$$= \frac{64cP}{\pi^4} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\left[1 - e^{-\left(\frac{\frac{1}{K}+H^2+C_{mn}}{1+\mu_1 C_{mn}}\right)t}\right]}{m^2 n^2 \left(\frac{1}{K} + H^2 + C_{mn}\right)} \sin\frac{m\pi l_1}{2} \sin\frac{n\pi l_2}{2c} \quad (15)$$

when m and n both are odd.

Particular cases :

Case I: When $\mu_{1\to 0}$ *i.e.* the coefficient of visco-elasticity becomes zero, the velocity of purely viscous fluid through porous medium and under the influence of uniform magnetic field is given from equation (14)

$$W = \frac{16P}{\pi^2} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\left[1 - e^{-\left(\frac{1}{K}+H^2+C_{mn}\right)t}\right]}{mn\left(\frac{1}{K} + H^2 + C_{mn}\right)} \sin\frac{m\pi l_1}{2} \sin\frac{n\pi l_2}{2c} \sin\frac{m\pi x}{1} \sin\frac{n\pi y}{c} \quad (16)$$

and from equation (15)

$$Q = \frac{64cP}{\pi^4} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\left[1 - e^{-\left(\frac{1}{K}+H^2+C_{mn}\right)t}\right]}{m^2 n^2 \left(\frac{1}{K} + H^2 + C_{mn}\right)} \sin\frac{m\pi l_1}{2} \sin\frac{n\pi l_2}{2c} \quad (17)$$

Case II: When K→∞ *i.e.* porous medium is withdrawn, the velocity of conducting visco-elastic Rivlin-Ericksen fluid under the influence of magnetic field is given from equation (14)

$$W = \frac{16P}{\pi^2} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\left[1 - e^{-\left(\frac{H^2+C_{mn}}{1+\mu_1 C_{mn}}\right)t}\right]}{mn(H^2 + C_{mn})} \sin\frac{m\pi l_1}{2} \sin\frac{n\pi l_2}{2c} \sin\frac{m\pi x}{1} \sin\frac{n\pi y}{c} \quad (18)$$

and flux from (15)

$$Q = \frac{64cP}{\pi^4} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\left[1 - e^{-\left(\frac{H^2 + C_{mn}}{1+\mu_1 C_{mn}}\right)t}\right]}{m^2 n^2 (H^2 + C_{mn})} \sin\frac{m\pi l_1}{2} \sin\frac{n\pi l_2}{2c} \tag{19}$$

which are the same result obtained by Roy, Sen and Lahiri (1990) with slight change of notations.

Case III: When M→0 *i.e.* magnetic field is withdrawn, the velocity of the visco-elastic Rivlin-Ericksen fluid through porous medium is given from equation (14)

$$W = \frac{16P}{\pi^2} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\left[1 - e^{-\left(\frac{1}{K} + C_{mn}\right)}\right]}{mn\left(\frac{1}{K} + C_{mn}\right)} \sin\frac{m\pi l_1}{2} \sin\frac{n\pi l_2}{2c} \sin\frac{m\pi x}{1} \sin\frac{n\pi y}{c} \tag{20}$$

and flux from equation (15)

$$Q = \frac{64cP}{\pi^4} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\left[1 - e^{-\left(\frac{1}{K} + C_{mn}\right)}\right]}{m^2 n^2 \left(\frac{1}{K} + C_{mn}\right)} \sin\frac{m\pi l_1}{2} \sin\frac{n\pi l_2}{2c} \tag{21}$$

which are the same result obtained by Singh (2009).

Case IV : When $\mu_1 \to 0$, $K \to \infty$ and M→0 then velocity of purely incompressible viscous fluid is given from equation (14)

$$W = \frac{16P}{\pi^2} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{q}{mn} \frac{[1 - e^{-C_{mn}t}]}{C_{mn}} \sin\frac{m\pi l_1}{2} \sin\frac{n\pi l_2}{2c} \sin\frac{m\pi x}{1} \sin\frac{n\pi y}{c} \tag{22}$$

and flux from (15)

$$Q = \frac{64cP}{\pi^4} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{[1 - e^{-C_{mn}t}]}{m^2 n^2 C_{mn}} \sin\frac{m\pi l_1}{2} \sin\frac{n\pi l_2}{2c} \tag{23}$$

which are the same result obtained by Gosh(1968) with slight change of notations.

References

1. Ahmad, N. and Taqvi, Z. A., *Jnanabha, 26*, p.-61 (1996).
2. Dutta, S., *Jour. of Tech. 3,* p.-58 (1998).
3. Ghosh, A.K., *Bull. Cal. Math. Soc. 60(4)* p.-163 (1968).
4. Kumar, N., Gupta, S. and Jain, T., *Ultra Scientist of Physical Sci.* Vol. *22,* No. *1(M),* p.-191 (2010).
5. Kumar, R., Sharma, A.K. and Singh, K.K., *Ultra Scientist of Physical Sci.* Vol. *20,* No. *2(M),* p.-545 (2008).
6. Kumar, R., Singh, K.K. and Sharma A.K., *Ultra Scientist of Physical Sci.* Vol. *21,* No. *2(M),* p.-571 (2009).
7. Panda, J. P. and Mohapatra, J. K., *Ultra Scientist of Physical Sci*. Vol. *22,* No. *1(M),* p.-155 (2010).
8. Prakash, K. and Aggarwal, A. K., Proc. *Nat. Acad. Sci. India, 77(A)* IV, p.-373 (2007).
9. Pundir, S. and Pundir, R., *Acta Ciencia Indica*, Vol. XXIX M No. 3, p.-593 (2003).
10. Rivlin, R. S. and Ericken, J. L., *Jour. Rational Mech. Annual. 4,* p.-323 (1955).
11. Sharma, P. R. and Sharma, S., Bull. Cal. Math. Soc., 97(6), p.-477 (2005).
12. Singh, N., Ph. D. Thesis, Agra Univ., p.-75 (2009).
13. Singh, K.K., Kumar, R. and Sharma, A.K., *Acta Ciencia Indica,* Vol. XXXIV M No. 1, p.-351 (2008).
14. Singh, K.K., Mishra G. and Sharma A.K., *Acta Ciencia Indica,* Vol. XXXIV M No. 1, p.-421 (2008).
15. Thingarajan, S. Ramamurthy, M. V. and Rahim, Mohd. A., *Bull. Pure Appl. Sci., 21E* No. *1*, p.-155 (2002).