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On $\overset{0}{g}$ - Homeomorphism in topological spaces

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Abstract

In this paper we introduce a new class of closed maps namely $\overset{0}{g}$ -closed maps which settled in between the class of g*-closed maps[12] and the class of gs-closed maps[12]. We also introduce and study new class of homeomorphisms called $\overset{0}{g}$ -homeomorphisms and $\overset{0}{g}$ *-homeomorphisms. Further we show that the set of all $\overset{0}{g}$ *-homeomorphisms form a group under the operation composition of maps.

Key Words : $\overset{0}{g}$ -closed maps; $\overset{0}{g}$ *-closed maps; $\overset{0}{g}$ *-homeomorphisms.

1. Introduction

The notion homeomorphism plays an important role in topology. A homeomorphism between two topological spaces X and Y is a bijective map $f : X \rightarrow Y$ when both f and f^{-1} are continuous. Malghan[16] and Devi *et al.*[8] introduced the concept of generalized closed maps and semi generalized closed maps respectively in topological spaces. Veera kumar introduced ψ-closed maps in topological spaces. In this paper we introduce a new class of closed maps namely $\overset{0}{g}$ -closed maps and then introduce and study $\overset{0}{g}$ -homeomorphisms and $\overset{0}{g}$ *-homeomorphisms in a topological space. We also prove that the set of all $\overset{0}{g}$ *-homeomorphisms forms a group under the operation composition of functions.

2. Preliminaries :

Throughout this paper (X, τ), (Y, σ) and (Z, η) represent topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of space

(X, τ) the cl(A), int(A) and A^c denote the closure of A, the interior of A and the complement of A in X respectively.

We recall the following definitions:

Definition 2.01 : A subset A of a topological space (X, τ) is called semi-open[1] (resp. semi-closed, semi pre-open[3]) if $A \subseteq cl(int(A))$ (resp. $int(cl(A)) \subseteq A$, $A \subseteq cl(int(cl(A)))$.

Definition 2.02 : A subset A of a topological space (X, τ) is called g-closed[2] (resp. sg-closed[4], gs-closed[5], ψ-closed[11], g*-closed[12]) set if $cl(A) \subseteq U$ (resp. $scl(A) \subseteq U$, $scl(A) \subseteq U$, $scl(A) \subseteq U$, $cl(A) \subseteq U$) whenever $A \subseteq U$ and U is open (resp. semi open, open, sg-open, g-open) set in (X, τ).

Definition 2.03 : A map $f : (X, \tau) \to (Y, \sigma)$ is called g-closed map[16] (resp. sg-closed map[8], gs-closed map[8], ψ-closed map[15], g*-closed map[12]) if the image of each closed set in (X, τ) is closed set (resp. sg-closed set, gs-closed set, ψ-set, g*-closed set) in (Y, σ).

Definition 2.04: A function $f : (X, \tau) \to (Y, \sigma)$ is called g-continuous[6] (resp. sg-continuous[7], gs-continuous[10], ψ-continuous[11], g-irresolute[6], sg-irresolute[7], gs-irresolute[10], ψ-irresolute[11]) if the inverse image of every σ-closed (resp. closed, closed, closed, g-closed, sg-closed, gs-closed, ψ-closed) set in Y is g-closed (resp. sg-closed, gs-closed, ψ-closed, g-closed, sg-closed, gs-closed, ψ-closed) set in X.

Definition 2.05 : A bijection $f : (X, \tau) \to (Y, \sigma)$ is called (i) g-homeomorphism[14] if f is both g-continuous and g-open (ii) sg-homeomorphism[10] if f is both sg-continuous and sg-open (iii) gs-homeomorphism[10] if f is both gs-continuous and gs-open (iV) ψ-homeomorphism[15] if f is both ψ-continuous and ψ-open.

Definition 2.06: Let (X, τ) be a topological space and $A \subseteq X$. Then $\overset{0}{g}$-closure of A (briefly $\overset{0}{g}$-cl(A)[12]) is defined as the intersection of all $\overset{0}{g}$-closed sets containing A.

Definition 2.07: Let (X, τ) be a topological space and $A \subseteq X$. Then $\overset{0}{g}$-interior of A (briefly $\overset{0}{g}$-int(A)[13]) is defined as the union of all $\overset{0}{g}$-open sets contained in A.

3. $\overset{0}{g}$-*Closed maps :*

In this section we introduce the notions of $\overset{0}{g}$-closed maps, $\overset{0}{g}$-open maps, $\overset{0}{g}$*-closed maps and $\overset{0}{g}$*-open maps in topological spaces and obtained certain characterizations of these maps.

Definition 3.01: A map $f : (X, \tau) \to (Y, \sigma)$ is called $\overset{0}{g}$-closed (resp. $\overset{0}{g}$-open) map if f(A) is $\overset{0}{g}$-closed (resp. $\overset{0}{g}$-open) set in (Y, σ) for every closed (open) set A of (Y, σ).

Theorem 3.02: (i) Every closed map

and g*-closed map is $\overset{0}{g}$-closed map.

(ii) Every closed map and g*-closed map is $\overset{0}{g}$-closed map.

Next examples show that the converse of the above theorem is not true in general.

Example 3.03: Let $X = Y = \{a, b, c\}$, $\tau = \{\phi, \{b\}, X\}$ and $\sigma = \{\phi, \{a\}, Y\}$. Define $f : (X, \tau) \to (Y, \sigma)$ by identity mapping then f is $\overset{0}{g}$-closed map. However f is not closed map and g*-closed map.

Example 3.04: Let $X = Y = \{a, b, c\}$, $\tau = \{\phi, \{a, b\}, X\}$ and $\sigma = \{\phi, \{a\}, \{c\}, \{a, c\}, Y\}$. Define $f : (X, \tau) \to (Y, \sigma)$ by identity mapping then f is gs-closed map. However f is not $\overset{0}{g}$-closed map.

Theorem 3.05: Every $\overset{0}{g}$-closed map is g-closed map[9].

Theorem 3.06: A map $f : (X, \tau) \to (Y, \sigma)$ is $\overset{0}{g}$-closed map iff $\overset{0}{g}\text{-cl}(f(A)) \subseteq f(\text{cl}(A))$ for every subset A of (X, τ).

Proof: Suppose that f is $\overset{0}{g}$-closed map and $A \subseteq X$. Then $f(\text{cl}(A))$ is $\overset{0}{g}$-closed in (Y, σ). We have $f(A) \subseteq f(\text{cl}(A))$ and by theorem (4.07) and (4.08) of [13], $\overset{0}{g}\text{-cl}(f(A)) \subseteq \overset{0}{g}\text{-cl}(f(\text{cl}(A))) = f(\text{cl}(A))$.

Conversely let A be any closed set in (X, τ). Then $A = \text{cl}(A)$ and so $f(A) = f(\text{cl}(A)) \supseteq \overset{0}{g}\text{-cl}(f(A))$, by hypothesis. We have $f(A) \subseteq \overset{0}{g}\text{-cl}(f(A))$ by theorem (4.07) of [13]. Therefore $f(A) = \overset{0}{g}\text{-cl}(f(A))$ *i.e.* f(A) is $\overset{0}{g}$-closed by theorem (4.07) of [13] and hence f is $\overset{0}{g}$-closed map.

Theorem 3.07 : A map $f : (X, \tau) \to (Y, \sigma)$ is $\overset{0}{g}$-closed iff for each subset A of (Y, σ) and for each open set B containing $f^{-1}(A)$ there exists a $\overset{0}{g}$-open set C of (Y, σ) such that $A \subseteq C$ and $f^{-1}(C) \subseteq B$.

Theorem 3.08: If $f : (X, \tau) \to (Y, \sigma)$ be a closed map and $g : (Y, \sigma) \to (Z, \eta)$ be a $\overset{0}{g}$-closed map then their composition $gof : (X, \tau) \to (Z, \eta)$ is $\overset{0}{g}$-closed map.

Remark 3.09: The following example show that the composition of two $\overset{0}{g}$-closed maps need not be $\overset{0}{g}$-closed map.

Example 3.10: Let $X = Y = Z = \{a, b, c\}$, $\tau = \{\phi, \{b\}, \{c\}, \{b, c\}, \{a, c\}, X\}$, $\sigma = \{\phi, \{c\}, Y\}$ and $\eta = \{\phi, \{a\}, \{c\}, \{a, c\}, Z\}$. Define $f : (X, \tau) \to (Y, \sigma)$ by identity mapping and $g : (Y, \sigma) \to (Z, \eta)$ by identity mapping then f and g both are $\overset{0}{g}$-closed maps but their composition $gof : (X, \tau) \to (Z, \eta)$ is not a $\overset{0}{g}$-closed map.

Theorem 3.11: If $f : (X, \tau) \to (Y, \sigma)$ and $g : (Y, \sigma) \to (Z, \eta)$ be two mappings such that their composition $gof : (X, \tau) \to (Z, \eta)$ be a $\overset{0}{g}$-closed map then the following are true

(i) If f is continuous and serjective, then g is $\overset{0}{g}$-closed map.

(ii) If g is $\overset{0}{g}$-irresolute and injective, then f is $\overset{0}{g}$-closed map.

Theorem 3.12: If $f : (X, \tau) \to (Y, \sigma)$ be a $\overset{0}{g}$-closed map and $g : (Y, \sigma) \to (Z, \eta)$ be a closed map, then their composition $gof : (X, \tau) \to (Z, \eta)$ need not be a $\overset{0}{g}$-closed map.

The following example supports the above theorem.

Example 3.13: Let X = Y = Z = {a, b, c}, $\tau = \{\phi, \{b\}, \{c\}, \{b, c\}, \{a, c\}, X\}$, $\sigma = \{\phi, \{a\}, \{b, c\}, Y\}$ and $\eta = \{\phi, \{b\}, \{c\}, \{b, c\}, Z\}$. Define $f : (X, \tau) \to (Y, \sigma)$ by identity mapping and $g : (Y, \sigma) \to (Z, \eta)$ by g(a) = a = g(b), g(c) = c. Then f is $\overset{0}{g}$-closed map and g is closed map but their composition $gof : (X, \tau) \to (Z, \eta)$ is not a $\overset{0}{g}$-closed map.

Theorem 3.14: Let f_A be the restriction of a map $f : (X, \tau) \to (Y, \sigma)$ to a subset A of (X, τ) then

(i) If $f : (X, \tau) \to (Y, \sigma)$ is $\overset{0}{g}$-closed and A is a closed subset of (X, τ), then $f_A: (A, \tau_A) \to (Y, \sigma)$ is $\overset{0}{g}$-closed.

(ii) If $f : (X, \tau) \to (Y, \sigma)$ is $\overset{0}{g}$-closed (resp. closed) and $A=f^{-1}(B)$ for some closed (resp. $\overset{0}{g}$-closed) set B of (Y, σ) then $f_A : (A, \tau_A) \to (Y, \sigma)$ is $\overset{0}{g}$-closed.

Proof: (i) Let B be a closed set of A. Then $B = A \cap C$ for some closed set C of (X, τ) and so B is closed in (X, τ). By hypothesis, f(B) is -closed in (Y, σ). But $f(B) = f_A(B)$ and so f_A is a $\overset{0}{g}$-closed map.

(ii) Let C be a closed set of A. Then $C = A \cup D$ for some closed set D of (X, τ). Now $f_A(C) = f(C) = f(A \cap D) = f(f^{-1}(B) \cap D) = B \cap f(D)$. Since f is $\overset{0}{g}$-closed, f(D) is $\overset{0}{g}$-closed and so $B \cap f(D)$ is $\overset{0}{g}$-closed in (Y, σ) by theorem (4.09) of [13]. Thus f_A is $\overset{0}{g}$-closed map.

Theorem 3.15: For any bijective $f : (X, \tau) \to (Y, \sigma)$ the following statements are equivalent.

(i) $f^{-1} : (Y, \sigma) \to (X, \tau)$ is $\overset{0}{g}$-continuous.

(ii) f is $\overset{0}{g}$-open map and

(iii) f is $\overset{0}{g}$-closed map.

Definition 3.16: Let x be a point of (X, τ) and A be a subset of X. Then A is called a $\overset{0}{g}$-neighborhood (briefly $\overset{0}{g}$-nbd) of x in (X, τ) if there exists a $\overset{0}{g}$-open set B of (X, τ) such that $x \in B \subseteq A$.

Theorem 3.17: Let $f : (X, \tau)\to (Y, \sigma)$ be a map then the following statements are equivalent.

(i) f is $\overset{0}{g}$-open map.

(ii) For a subset A of (X, τ), $f(int(A)) \subseteq \overset{0}{g}\text{-}int(f(A)$.

(iii) For each $x \in X$ and for each nbd A of x in (X, τ), there exists a $\overset{0}{g}$-nbd of f(x) in (Y, σ) such that B $\subseteq$ f(A). σ

Proof: (i) $\Rightarrow$ (ii) Suppose f is $\overset{0}{g}$-open. Let A $\subseteq$ X. Since int(A) is open in (X, τ), f(int(A)) is $\overset{0}{g}$-open in (Y, σ). Hence (int(A)) $\subseteq$ f(A) and we have, f(int(A)) $\subseteq$ $\overset{0}{g}$-intf(A).

(ii) $\Rightarrow$ (iii) Suppose (ii) holds. Let x $\in$ X and A be an arbitrary nbd. of x in (X, τ). Then there exists an open set G such that x $\in$ G $\subseteq$ A. By assumption, f(G) $\subseteq$ f(int(G)) $\subseteq$ -intf(G). This implies f(G) = $\overset{0}{g}$-intf(G). Therefore, f(G) is $\overset{0}{g}$-open in (Y, σ). Further, f(x) $\in$ f(G) $\subseteq$ f(A) and so (iii) holds, by taking B = f(G).

(iii) $\Rightarrow$ (i) Let A be an open set in (X, τ), x $\in$ A and f(x) = y. Then for each x $\in$ A, y $\in$ f(A), by assumption there exists a -nbd U_y of y in (Y, σ) such that $U_y \subseteq$ f(A). Since U_y is a $\overset{0}{g}$-nbd of y, there exists a $\overset{0}{g}$-open set V_y in (Y, σ) such that y $\in V_y \subseteq U_y$. Therefore f(A) = $\cup\{V_y : y \in f(A)\}$. Since any union of $\overset{0}{g}$-open sets is $\overset{0}{g}$-open set[13], f(A) is a -open set of (Y, σ).Thus, f is $\overset{0}{g}$-open mapping.

Theorem 3.18 : A function f : (X, τ) $\to$ (Y, σ) is $\overset{0}{g}$-open if and only if for any subset B of (Y, σ) and for any closed set C containing f^{-1}(B), there exists a $\overset{0}{g}$-closed set A of (Y, σ) containing B such that f^{-1}(A) $\subseteq$ C.

Corollary 3.19: A function f : (X, τ) $\to$(Y, σ) is $\overset{0}{g}$-open mapping if and only if f^{-1} ($\overset{0}{g}$-cl(B)) $\subseteq$ cl(f^{-1}(B) for every subset B of (Y, σ).

Definition 3.20: A map f : (X, τ) $\to$ (Y, σ) is said to be a $\overset{0}{g}$*-closed (resp. $\overset{0}{g}$*-open) map if the image f(A) is $\overset{0}{g}$-closed (resp. $\overset{0}{g}$-open) set in (Y, σ) for every -closed (resp. $\overset{0}{g}$-open) set A in (X, τ).

Theorem 3.21: Every $\overset{0}{g}$*-closed map is $\overset{0}{g}$-closed map.

The converse is not true in general as seen from the following example.

Example 3.22: Let X = Y = {a, b, c}, τ = {ϕ, {a}, {b, c}, X} and σ = {ϕ, {c}, Y}. Define f : (X, τ) $\to$ (Y, σ) by identity mapping then f is $\overset{0}{g}$-closed map but not $\overset{0}{g}$*-closed map.

Theorem 3.23: A map f : (X, τ) $\to$ (Y, σ) is $\overset{0}{g}$*-closed iff $\overset{0}{g}$-cl(f(A)) $\subseteq$ f($\overset{0}{g}$-cl(A)) for every subset A of (X, τ).

Proof: Similar to theorem (3.06).

Theorem 3.24: For any bijection f : (X, τ) $\to$ (Y, σ) the following are equivalent

(i) f^{-1} : (Y, σ) $\to$ (X, τ) is $\overset{0}{g}$-irresolute.

(ii) f is a $\overset{0}{g}$*-open map.

(iii) f is a $\overset{0}{g}$*-closed map.

Proof: Similar to theorem (3.15).

4. $\overset{0}{g}$*-Homeomorphisms :*

In this section we introduce the following definitions.

Definition 4.01: A bijection f : (X, τ) → (Y, σ) is said to be $\overset{0}{g}$-homeomorphisms if f is $\overset{0}{g}$-continuous and $\overset{0}{g}$-open maps *i.e*. both f and f^{-1} are $\overset{0}{g}$-continuous maps.

Theorem 4.02: Every homeomorphism is $\overset{0}{g}$-homeomorphism.

The converse of the above theorem is not true in general as it can be seen from the following example.

Example 4.03: Let X = Y = {a, b, c}, τ = {ϕ, X} and σ = {ϕ, {a}, Y}. Define f : (X, τ) → (Y, σ) by identity mapping then f is $\overset{0}{g}$-homeomorphism but not homeomorphism.

Theorem 4.04: Every $\overset{0}{g}$-homeomorphism is gs-homeomorphism.

The converse of the above theorem is not true in general as it can be seen from the following example.

Example 4.05: Let X = Y = {a, b, c}, τ = {ϕ, {b}, {a, c}, X} and σ = {ϕ, {a}, {a, b}, Y}.

Define f : (X, τ) → (Y, σ) by identity mapping then f is gs-homeomorphism but not $\overset{0}{g}$-homeomorphism.

Theorem 4.06: Every $\overset{0}{g}$-homeomorphism is g-homeomorphism.

Definition 4.07: A bijection f : (X, τ) → (Y, σ) is said to be $\overset{0}{g}$*****-homeomorphism if both f and f^{-1} are $\overset{0}{g}$-irresolute.

We denote the family of all $\overset{0}{g}$*-homeomorphism of a topological space (X, τ) onto itself by $\overset{0}{g}$*-h(X, τ).

Theorem 4.08: Every $\overset{0}{g}$*-homeomorphism is $\overset{0}{g}$-homeomorphism.

The converse of the above theorem is not true in general as it can be seen from the following example.

Example 4.09: Let X = Y = {a, b, c}, τ = {ϕ, X} and σ = {ϕ, {a, b}, Y}. Define f : (X, τ) → (Y, σ) by identity mapping then f is $\overset{0}{g}$-homeomorphism but not $\overset{0}{g}$*-homeomorphism.

Therefore the class of $\overset{0}{g}$-homeomorphism properly contain the class of homeomorphism and properly contained in the class of g-homeomorphism, the class of gs-homeomorphism

Remark 4.10: The following examples show that $\overset{0}{g}$-homeomorphism is independent from sg-homeomorphism and Ψ-homeomorphism.

Example 4.11: Let X = Y = {a, b, c}, τ = {ϕ, {a}, X} and σ = {ϕ, {c}, Y}. Define f:

$(X, \tau) \to (Y, \sigma)$ by identity mapping then f is $\overset{0}{g}$-homeomorphism but not sg-homeomorphism and Ψ-homeomorphism

Example 4.12: Let $X = Y = \{a, b, c\}$, $\tau = \{\phi, \{a\}, \{a, b\}, X\}$ and $\sigma = \{\phi, \{b\}, \{b, c\}, Y\}$. Define $f : (X, \tau) \to (Y, \sigma)$ by identity mapping then f is sg-homeomorphism and Ψ-homeomorpshism but not $\overset{0}{g}$-homeomorphism

All the above discussion can be summarized by the following diagram: ----

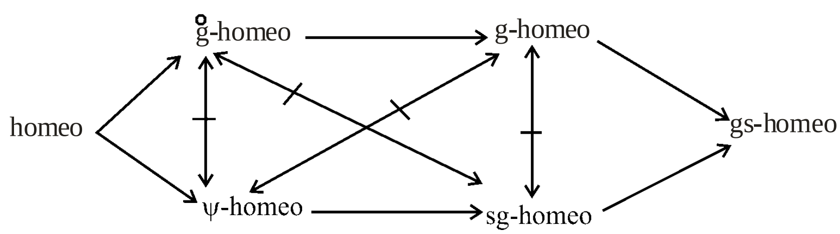

Diagram (4.13)

Theorem 4.14: If $f : (X, \tau) \to (Y, \sigma)$ and $g : (Y, \sigma) \to (Z, \eta)$ are $\overset{0}{g}$*-homeomorphism, then their composition $g \circ f : (X, \tau) \to (Z, \eta)$ is also $\overset{0}{g}$*-homeomorphism.

Theorem 4.15: If $f : (X, \tau) \to (Y, \sigma)$ be a bijective and $\overset{0}{g}$-continuous map then following statements are equivalent.

(i) f is $\overset{0}{g}$-open map.

(ii) f is $\overset{0}{g}$-homeomorphism.

(iii) f is $\overset{0}{g}$-closed map.

Theorem 4.16: The set $\overset{0}{g}$*-h(X, τ) is a group under the composition of maps.

Proof: Define a binary operation Θ : $\overset{0}{g}$*-h$(X, \tau) \times \overset{0}{g}$*-h$(X, \tau) \to \overset{0}{g}$*-h$(X, \tau)$ by $f \Theta g = g \circ f$ for all f and $g \in \overset{0}{g}$*-h(X, τ) and o is the usual operation of composition of maps. Then by theorem (4.14) $g\Theta f \in \overset{0}{g}$*-h$(X, \tau)$. Again composition of maps is associative and the identity map $I : (X, \tau) \to (X, \tau)$ belonging to $\overset{0}{g}$*-h(X, τ) serves as the identity element of $\overset{0}{g}$*-h(X, τ). If $f \in \overset{0}{g}$*-h(X, τ), then $f^{-1} \in \overset{0}{g}$*-h$(X, \tau)$ such that $f_\Theta f^{-1} = f^{-1}{}_\Theta f = I$ and so inverse exists for each element of *-h(X, τ). Thus ($\overset{0}{g}$*-h(X, τ), $_\Theta$) is a group under the operation of composition of maps.

Theorem 4.17: If $f : (X, \tau) \to (Y, \sigma)$ be a $\overset{0}{g}$*-homeomorphism, then f induces an isomorphism from the group $\overset{0}{g}$*-h(X, τ) onto the group $\overset{0}{g}$*-h(Y, σ).

Proof: Define $\psi_f : \overset{0}{g}$*-h$(X, \tau) \to \overset{0}{g}$*-h$(Y, \sigma)$ by $\psi_f(h) = fohof^{-1}$ for every $h \in \overset{0}{g}$*-h(X, τ). Then ψ_f is a bijection. Further, for all $h_1, h_2 \in \overset{0}{g}$*-h$(X, \tau)$, $\psi_f(h_1oh_2) = fo(h_1oh_2)of^{-1} = (foh_1of^{-1})\ o\ (foh_2of^{-1}) = \psi_f(h_1)\ o\ \psi_f(h_2)$ so ψ_f

is a homeomorphism and so it is an isomorphism induced by f.

Theorem 4.18: $\overset{0}{g}$*-homeomorphism is an equivalence relation in the collection of all topological spaces.

Proof: Follows from theorem (4.14).

Theorem 4.19: If $f : (X, \tau) \rightarrow (Y, \sigma)$ is a $\overset{0}{g}$*-homeomorphism, then $\overset{0}{g}$-cl(f^{-1}(A)) $\subseteq$ f^{-1} ($\overset{0}{g}$-cl((A)) for all A $\subseteq$ Y.

Theorem 4.20: If $f : (X, \tau) \rightarrow (Y, \sigma)$ is a $\overset{0}{g}$*-homeomorphism, then $\overset{0}{g}$-cl(f(A)) = f($\overset{0}{g}$-cl((A)) for all A $\subseteq$ X.

Proof: Follows from theorem (4.19).

Theorem 4.21: If $f : (X, \tau) \rightarrow (Y, \sigma)$ is $\overset{0}{g}$*-homeomorphism, then f($\overset{0}{g}$-int(A)) = $\overset{0}{g}$-int(f(A)) for all A $\subseteq$ X.

Proof: Follows from theorem (4.20).

Theorem 4.22: If $f : (X, \tau) \rightarrow (Y, \sigma)$ is a $\overset{0}{g}$*-homeomorphism, then f^{-1}(-int(A)) = $\overset{0}{g}$-int(f^{-1}(A)) for all A $\subseteq$ Y.

Proof: Follows from theorem (4.21).

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