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Viscous Fluid Cosmological Model with Two Degrees of Freedom in General Relativity

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Abstract

Viscous fluid cosmological model with two degrees of freedom is investigated. We assume the coordinates to be comoving. In presence of viscosity our model is non-degenerate Petrov Type- I. There is a real physical singularity at T=0. Moreover in the absence of viscosity, we get the stiff fluid case. Some significant physical and geometrical aspects of the model are also discussed.

Key words: Viscous Fluid cosmology, Cylindrically symmetric metric with two degrees of freedom.

1 Introduction

Most of the recent astrophysical observations indicate that the viscosity has played a crucial role in the evolution of the universe[1,2,3] and references therein. Large entropy per baryon and the remarkable degree of anisotropy of the cosmic microwave radiation suggest that we should analyze dissipative effects in cosmology. Further there are several processes such as the decoupling of neutrinos during the radiation era and the decoupling of radiation and matter during the recombination era, which are expected to give rise to the viscous effect[10]. Earlier the role of viscosity on the cosmological models has been investigated by Weinberg[6]. Misner[7,8] has also studied the effect of viscosity on the evolution of cosmological models. Heller *et al.*[9] have obtained viscous universes without initial singularity. Das *et al.*[4,5] have examined the role of neutrino viscosity on the late time cosmic acceleration and shown that the neutrino bulk viscous stresses can give rise to the late time acceleration of the universe. They also found that a number of spatially flat

FRW models with a negative deceleration parameter can be constructed using neutrino viscosity. A class of homogeneous models has been studied by Ellis *et al.*[14]. Heckmann *et al.*[15] have derived some more general models for incoherent matter distribution.

Banerjee *et al.*[11] have examined anisotropic cosmological model with viscous fluid in which they have found that the viscosity is more important in the initial epoch of the universe. Roy *et al.*[16] have obtained a cosmological model of cylindrical symmetry with two degrees of freedom where the gravitational field is Petrov Type ID with the degeneracy being in yz plane, assuming the energy-momentum tensor to be that of perfect fluid. Bali *et al.*[12] have investigated magnetized cylindrical symmetric universe in general relativity. Recently Bali *et al.*[13] have investigated magnetized stiff fluid cylindrically symmetric universe with two degrees of freedom in general relativity, where the magnetic field is due to an electric current produced along the x-axis. The distribution consists of an electrically neutral perfect fluid with an infinite electrical conductivity. Sudharshan *et al.*[17] have investigated the effect of bulk viscosity on the evolution of Friedmann models in the context of open thermodynamic systems which allow for particle creation. Tawfik *et al.*[18] have discussed the influences of bulk viscosity on the early universe, modeled by Friedmann-Robertson-Walker metric.

Cataldo *et al.*[15] have studied viscous cosmologies in scalar-tensor theories for Kasner type metrics and concluded that in the Brans-Dicke theory it is not possible to describe the growth of entropy[19].

Motivated by the works of the above scholars we are presenting here the exact solutions of Einstein's Field equations for a viscous fluid cosmological model with two degrees of freedom in general relativity. This paper is organized as follows: The metric and the field equations are presented in Section 2. In Section 3, we deal with the solution of the field equations in presence of viscous fluid. In sections 4, some physical and geometrical properties are discussed. In section 5 solution in absence of viscosity is presented. Finally, we have given the concluding remarks in the section 6.

2 The Metric and Field Equations :

We consider the cylindrically symmetric metric with two degrees of freedom in the form

$$ds^2 = A^2(dx^2 - dt^2) + C^2 dz^2 + (B^2 + D^2) dy^2 + 2CD\, dydz \tag{1}$$

Where, the metric potentials A, B, C and D are functions of cosmic time t only.

The energy momentum tensor for viscous fluid distribution is given by Landau and Lifshitz as[19]

$$T_i^j = (\varepsilon + p)v_i v^j + p\, g_i^j - \eta(v_{i\,;}^j + v_{;\,i}^j + v^j v^l v_{i\,;l} + v_i v^l v_{;\,l}^j) - (\zeta - \frac{2}{3}\eta)\theta(g_i^j + v_i v^j) \tag{2}$$

together with

$$v_i v^i = -1 \tag{3}$$

In equations (2), p is the isotropic pressure, ε the density, η and ζ are coefficients of viscosity and ; indicates covariant differentiation. Here v^i is the four velocity vector satisfying the relation (3). We assume the coordinates to be comoving, so that

$$v^1 = 0 = v^2 = v^3, v^4 = \frac{1}{A} \quad (4)$$

The Einstein's field equations are

$$R_i^j - \frac{1}{2} R g_i^j = -8\pi T_i^j \text{ (where c=1,G=1)} \quad (5)$$

For the line-element (1) equations (5) lead to the following system of equations:

$$\frac{1}{A^2}\left(-\frac{B_{44}}{B} - \frac{C_{44}}{C} - \frac{A_4 B_4}{AB} - \frac{A_4 C_4}{AC} + \frac{B_4 C_4}{BC}\right) + \frac{1}{4}\left[\frac{CD_4 - DC_4}{ABC}\right]^2 = -8\pi\left[p - (\zeta - \frac{2}{3}\eta)\theta\right] \quad (6)$$

$$\frac{1}{A^2}\left(\frac{A_{44}}{A} + \frac{B_{44}}{B} - \frac{A_4^{\,2}}{A^2}\right) + \left(\frac{DC}{2A^2B^2}\right)\left(\frac{D}{C}\right)_{44} - \left(\frac{DC}{2A^2B^2}\right)\left(\frac{D}{C}\right)_4\left(\frac{B_4}{B} - \frac{3C_4}{C}\right) + \frac{3}{4}\left[\frac{CD_4 - DC_4}{ABC}\right]^2 = -8\pi\left[p - (\zeta - \frac{2}{3}\eta)\theta - \eta\left(\frac{2C_4}{AC} + \frac{D(DC_4 - CD_4)}{AB^2C}\right)\right] \quad (7)$$

$$\frac{1}{A^2}\left(\frac{A_{44}}{A} + \frac{C_{44}}{C} - \frac{A_4^{\,2}}{A^2}\right) - \left(\frac{DC}{2A^2B^2}\right)\left(\frac{D}{C}\right)_{44} - \left(\frac{DC}{2A^2B^2}\right)\left(\frac{D}{C}\right)_4\left(\frac{B_4}{B} - \frac{3C_4}{C}\right) - \frac{1}{4}\left[\frac{CD_4 - DC_4}{ABC}\right]^2 = -8\pi\left[p - (\zeta - \frac{2}{3}\eta)\theta - \eta\left(\frac{2B_4}{AB} + \frac{D(DC_4 - CD_4)}{AB^2C}\right)\right] \quad (8)$$

$$\frac{1}{A^2}\left(\frac{A_4 B_4}{AB} + \frac{A_4 C_4}{AC} + \frac{B_4 C_4}{BC}\right) - \frac{1}{4}\left[\frac{CD_4 - DC_4}{ABC}\right]^2 = 8\pi\varepsilon \quad (9)$$

$$-\left(\frac{C^2}{2A^2B^2}\right)\left(\frac{D}{C}\right)_{44} + \left(\frac{D}{C}\right)_4\left(\frac{B_4}{B} - \frac{3C_4}{C}\right)\frac{C^2}{2A^2B^2} = -8\pi\eta\left[\frac{CD_4 - DC_4}{AB^2}\right] \quad (10)$$

where the sub indice 4 in A, B, C and D denotes ordinary differentiation with respect to t.

3 Solutions of the Field Equations :

The field equations (6) - (10) are a system of five non-linear differential equations with six unknown parameters A, B, C,D, ε and p. We need one additional condition to obtain determinate solution of the system. We assume that

$$A = m \text{ (constant)} \tag{11}$$

Equation (10) together with (11) leads to

$$\left(\frac{B}{C^3}\right) = \left(\frac{D}{C}\right)_4 Ne^{kt} \tag{12}$$

From equations (7), (8) and (11), we have

$$\left(\frac{B_{44}}{B} + \frac{C_{44}}{C}\right) + \frac{1}{2}\left[\frac{CD_4 - DC_4}{BC}\right]^2 =$$

$$-16\pi m^2\left[p - (\zeta - \frac{2}{3}\eta)\theta\right] + 16\pi\eta m\left(\frac{B_4}{B} + \frac{C_4}{C}\right) \tag{13}$$

Equations (6) and (13) lead to

$$\left(\frac{B_{44}}{B} - \frac{C_{44}}{C} + \frac{2B_4C_4}{BC}\right) = -k\left(\frac{B_4}{B} + \frac{C_4}{C}\right) \tag{14}$$

where $k = 16\pi\eta m$ (15)

Equations (6) , (8) , (11) and (12) lead to

$$\left(\frac{B_{44}}{B} + \frac{B_4C_4}{BC} - k\frac{B_4}{B}\right) = \frac{-e^{2kt}}{N^2C^4} \tag{16}$$

where k is given by the equation (15).
Equation (14) leads to

$$(CB)_4 = -Me^{-kt} \tag{17}$$

where M is a constant of integration.
Equation (17) on integration leads to

$$CB = \alpha e^{-kt} \tag{18}$$

Without any loss of generality the constant of integration M can be taken as less than zero and

$$\alpha = \frac{-M}{k} \tag{19}$$

where $\alpha \rangle 0$. Now using equation (18) in (16), we have

$$e^{2kt}\left(\frac{C_4}{C}\right)_4 = \frac{\beta}{C^4} \tag{20}$$

and

$$\beta = \frac{1}{2N^2}. \tag{21}$$

Let us assume that

$$C = e^{\chi}, \tag{22}$$

Which leads to

$$\frac{C_4}{C} = \chi_4 \tag{23}$$

and

$$\left(\frac{C_4}{C}\right)_4 = \chi_{44} \tag{24}$$

Using equations (22) and (24) in equation (20), we have

$$e^{(2kt+4\chi)}\chi_{44} = \beta. \tag{25}$$

Let us assume that

$$e^{(2kt+4\chi)} = \xi, \tag{26}$$

Which leads to

$$(2k + 4\chi_4) = \frac{\xi_4}{\xi}, \tag{27}$$

and

$$\chi_{44} = \frac{1}{4}\left(\frac{\xi_{44}}{\xi} - \frac{\xi_4^2}{\xi^2}\right). \qquad (28)$$

With the help of equations (26) to (28), equation (25) leads to

$$\left(\xi_{44} - \frac{\xi_4^2}{\xi^2}\right) = 4\beta\,, \qquad (29)$$

Which leads to

$$\frac{d(f^2)}{d\xi} - \left(\frac{2}{\xi}\right) f^2 = 8\beta \qquad (30)$$

Where

$$\xi_4 = f(\xi) \qquad (31)$$

From equation (30) we have

$$\frac{d\xi}{dt} = \sqrt{P\xi^2 - 8\beta\xi}\,, \qquad (32)$$

Where P is the constant of integration. Equation (32) on integration leads to

$$\xi = \left(\frac{4\beta}{P}\right)[1 + \cosh(t\sqrt{P} + Q)] \qquad (33)$$

Where Q is the constant of integration.
Now using equations (26) and (22) in equation (33), we get

$$C = \left(\frac{4\beta + 4\beta\cosh(t\sqrt{P} + Q)}{P}\right)^{\frac{1}{4}} e^{\frac{-kt}{2}} \qquad (34)$$

Using equation (34) in (18), we have

$$B = \left(\frac{P}{4\beta + 4\beta\cosh(t\sqrt{P} + Q)}\right)^{\frac{1}{4}} \alpha\, e^{\frac{-kt}{2}} \qquad (35)$$

Equation (12) with the help of (34) and (35), leads to

$$\left(\frac{D}{C}\right)_4 = \left(\frac{\alpha P}{4N\beta[1 + \cosh(t\sqrt{P} + Q)]}\right) \qquad (36)$$

Which on integration gives

$$\left(\frac{D}{C}\right) = \frac{\alpha\sqrt{P}}{2}[\coth(t\sqrt{P} + Q) - \operatorname{cos}ech(t\sqrt{P} + Q)] + \delta \qquad (37)$$

Where is δ the constant of integration.
Using equation (34) in (37), we have

$$D = \left(\frac{4\beta + 4\beta\cosh(t\sqrt{P} + Q)}{P}\right)^{\frac{1}{4}} \left[\frac{\alpha\sqrt{P}}{2}[\coth(t\sqrt{P} + Q) - \operatorname{cos}ech(t\sqrt{P} + Q)] + \delta\right] e^{\frac{-kt}{2}}\,. \qquad (38)$$

Consequently the line element (1) reduces to the form

$$ds^2 = m^2\left(dX^2 - \frac{dT^2}{P}\right) + \frac{2\sqrt{\beta + \beta\cosh T}}{\sqrt{P}} e^{\frac{-k(T-Q)}{\sqrt{P}}} dZ^2 + \frac{\sqrt{P}}{2\sqrt{\beta + \beta\cosh T}}\,.$$

$$\left[\alpha^2 + \frac{2\delta}{\sqrt{P}}\sqrt{\beta + \beta\cosh T} + \alpha\sqrt{\beta + \beta\cosh T}\,(\coth T - \operatorname{cos}echT)\right] e^{\frac{-k(T-Q)}{\sqrt{P}}} dY^2 + \frac{2}{\sqrt{P}}\sqrt{\beta + \beta\cosh T}\,.$$

$$\left[2\delta+\alpha\sqrt{P}\,(\coth T-\cos echT)\right]e^{\frac{-k(T-Q)}{\sqrt{P}}}\,dYdZ \tag{39}$$

Where we have applied the transformation of coordinates as $t\sqrt{P}+Q=T, x=X,$ $y=Y, z=Z.$

In the absence of viscosity *i.e.* when k$\rightarrow$ 0, the metric reduces to the form

$$ds^2=m^2\left(dX^2-\frac{dT^2}{P}\right)+\frac{2\sqrt{\beta+\beta\cosh T}}{\sqrt{P}}\cdot\sqrt{P}\,dZ^2+\frac{\sqrt{P}}{2\sqrt{\beta+\beta\cosh T}}.$$

$$\left[\frac{1}{m^2}+\frac{2}{\sqrt{P}}\sqrt{\beta+\beta\cosh T}+\frac{1}{m}\sqrt{\beta+\beta\cosh T}\,(\coth T-\cos echT)\right]dY^2+\frac{2\beta}{\sqrt{P}}\sqrt{\beta+\beta\cosh T}\,.$$
$$\left[2\delta+\frac{\sqrt{P}}{m}(\coth T-\cos echT)\right]dYdZ \tag{40}$$

where $k=16\pi\,\eta\,m$ and $m=-16\pi\eta$

The average scale for our model is given by

$$R^3=m\sqrt{\alpha}\,e^{-\frac{(Q-T)}{\sqrt{P}}} \tag{41}$$

Hence it is evident that the average scale factor increases when T<Q, becomes constant when T=Q and decreases as T<Q.

4. Some physical and geometrical features:

The pressure and density for the model (39) are given by

$$8\pi p=\frac{1}{m^2}\left[\frac{P+4P\beta-P\cosh T}{16(1+\cosh T)}-\frac{3k^2}{4}\right]$$
$$-\frac{8\pi k}{m^2}(m\varsigma+2k) \tag{42}$$

$$8\pi\varepsilon=\frac{1}{m^2}\left[\frac{P+4P\beta-P\cosh T}{16(1+\cosh T)}-\frac{k^2}{4}\right] \tag{43}$$

The model has to satisfy the strong energy conditions given by Ellis as[14]

(i) $(\varepsilon + p)\rangle\, 0$ and (ii) $(\varepsilon + 3p)\rangle\, 0$.

These conditions leads to

$$\left[\frac{P(1-4\beta-\cosh T)}{2(1+\cosh T)}\right]\rangle\, k^2+32\pi k\,(m\varsigma+2k) \tag{44}$$

and

$$\left[\frac{P(1-4\beta-\cosh T)}{8(1+\cosh T)}\right]\rangle\, k^2+12\pi k\,(m\varsigma+2k) \tag{45}$$

The rate of change of velocity vector v_i defines some important quantities of physical interest associated with the fluid flow. These are called kinematical parameters.

Decomposition of $v_{i;j}$ gives

$$v_{i;j} = \sigma_{ij} + \omega_{ij} + \frac{1}{3}\theta h_{ij} - \dot{v}_i v_{j}, \tag{46}$$

Where $\dot{v}_i$ is acceleration vector, h_{ij} the projection vector, σ_{ij} the shear tensor and ω_{ij} the velocity tensor given by

$$\dot{v}_i = v_{i;j} v^j, \tag{47}$$

$$h_{ij} = g_{ij} + v_i v_j, \tag{48}$$

$$\sigma_{ij} = \sigma_{ji} = v_{i;j} - \omega_{ij} + \frac{1}{3}\theta h_{ij} + \dot{v}_i v_j, \tag{49}$$

$$\omega_{ij} = -\omega_{ji} = \frac{1}{2}\left[(v_{i;j} - v_{j;i}) - (\dot{v}_i v_j + \dot{v}_j v_i)\right] \tag{50}$$

In the above, semi-colon stands for covariant differentiation.

The scalar of expansion θ for the flow vector $v^i_{,j}$ is given by

$$\theta = v^i_{;j}$$

The shear (σ) and rotation (ω) are defined by

$$\sigma^2 = \frac{1}{2}(\sigma_{ij}\sigma^{ij}), \tag{51}$$

and

$$\omega^2 = \frac{1}{2}(\omega_{ij}\omega^{ij}). \tag{52}$$

The expansion in the model is given by

$$\theta = \frac{16\pi\eta m}{M}. \tag{53}$$

The rotation ω is identically zero. Clearly $v^i_{;j} v^j = 0$ so that the lines of flow are geodetic.

The non-vanishing components of shear tensor (σ_{ij}) calculated for the flow vector v^i are given by

$$\sigma_{11} = \frac{m}{k}, \tag{54}$$

$$\sigma_{22} = \frac{\sqrt{\beta}[3\sqrt{P}\sinh T - 2k - 2k\sinh T]}{6m\sqrt{P}\sqrt{1+\cosh T}} e^{\frac{-k(T-Q)}{\sqrt{P}}} \tag{55}$$

$$\sigma_{33} = \frac{\sqrt{\beta+\beta\cosh T}}{2m\sqrt{P}}\left[2\delta + \alpha\sqrt{P}(\cosh T - \operatorname{cosech} T)\right]\left[\frac{\sqrt{P}\sinh T}{2(1+\cosh T)} + \sqrt{P}\operatorname{cosech} T - k\right] - 2\delta\sqrt{P}\Big]$$
$$. e^{\frac{-k(T-Q)}{\sqrt{P}}} + \frac{k}{3m}\left[\frac{\alpha^2\sqrt{P}}{2\sqrt{\beta+\beta\cosh T}} + \frac{\alpha\sqrt{P}}{2}(\cosh T - \operatorname{cosech} T) + \delta\right] e^{\frac{-k(T-Q)}{\sqrt{P}}}, \tag{56}$$

$$\sigma_{23} = \frac{\sqrt{\beta+\beta\cosh T}}{6m\sqrt{P}}\left[2\delta + \alpha\sqrt{P}(\cosh T - \operatorname{cosech} T)\right].\left[\frac{3\sqrt{P}\sinh T}{2(1+\cosh T)} + 3\sqrt{P}\operatorname{cosech} T + 5k\right] e^{\frac{-k(T-Q)}{\sqrt{P}}} -$$
$$\left[\frac{\delta}{m}\sqrt{\beta+\beta\cosh T}\right] e^{\frac{-k(T-Q)}{\sqrt{P}}} \tag{57}$$

The non- vanishing components of conformal curvature tensor are

$$C_{12}^{12} = -C_{34}^{34} = \frac{1}{48m^2}\left[\frac{(4P - 16P\beta - \cosh T - 6k\sqrt{P}\sinh T)}{(1+\cosh T)}\right], \tag{58}$$

$$C_{13}^{13} = -C_{24}^{24} = \left[\frac{(-6P + 8P\beta - \cosh T + 6k\sqrt{P}\sinh T)}{48m^2(1+\cosh T)}\right], \tag{59}$$

$$C_{14}^{14} = -C_{23}^{23} = \left[\frac{(P + 4P\beta + \cosh T)}{24m^2(1+\cosh T)}\right], \tag{60}$$

$$C_{12}^{13} = C_{24}^{34} = \frac{-k\sqrt{P\beta}}{4m^2\sqrt{1+\cosh T}} \tag{61}$$

The model in general represents shearing and non-rotating universe. The reality conditions are satisfied. The space-time is non-degenerate Petrov Type I in general. However, if P=0, then space-time reduces to Petrov type D. If $\beta = 0$, the space-time is conformally flat.

Since $\lim_{T\to\infty} \frac{\sigma}{\theta} \neq 0$ \, hence the model does not approach isotropy for large values of T.

The Hubble parameter (H) is given by

$$H = \frac{R_4}{R} = \frac{-k}{3} \tag{62}$$

5. *Behaviour in the absence of viscosity :*

In the absence of viscosity *i.e.* when $k \to 0$, then the pressure and density for the model (40) are given by

$$8\pi\, p = \frac{1}{m^2}\left[\frac{(P + 4P\beta - P\cosh T)}{16(1+\cosh T)}\right] \tag{63}$$

$$8\pi\,\varepsilon = \frac{1}{m^2}\left[\frac{(P + 4P\beta - P\cosh T)}{16(1+\cosh T)}\right] \tag{64}$$

The reality conditions $(\varepsilon + p) > 0$ and $(\varepsilon + 3p) > 0$ lead to

$$(\cosh T) < (1 + 4\beta). \tag{65}$$

The non-vanishing components of conformal curvature tensor for the model (16) are

$$C_{12}^{12} = -C_{34}^{34} = \left[\frac{(4P - 16P\beta - \cosh T)}{48m^2(1+\cosh T)}\right], \tag{66}$$

$$C_{13}^{13} = -C_{24}^{24} = \left[\frac{(-6P + 8P\beta - \cosh T)}{48m^2(1+\cosh T)}\right], \tag{67}$$

$$C_{14}^{14} = -C_{23}^{23} = \left[\frac{(P + 4P\beta + \cosh T)}{24m^2(1+\cosh T)}\right], \tag{68}$$

In absence of viscosity the components of shear tensor σ_{ij} are given by

$$\sigma_{11} = 0, \tag{69}$$

$$\sigma_{22} = \frac{\sqrt{\beta}[3\sqrt{P}\sinh T\,]}{6m\sqrt{P}\sqrt{1+\cosh T}} \tag{70}$$

$$\sigma_{33} = \frac{\sqrt{\beta+\beta\cosh T}}{2m\sqrt{P}}\left[2\delta + \frac{\sqrt{P}}{m}(\cosh T - \cos echT)\right].\left[\frac{\sqrt{P}\sinh T}{2(1+\cosh T)} + \sqrt{P}\cos echT\right]$$
$$-\delta\,\frac{\sqrt{\beta+\beta\cosh T}}{m} \tag{71}$$

$$\sigma_{23} = \frac{\sqrt{\beta+\beta\cosh T}}{6m\sqrt{P}}\left[2\delta + \frac{\sqrt{P}}{m}(\cosh T - \cos echT)\right].$$
$$.\left[\frac{3\sqrt{P}\sinh T}{2(1+\cosh T)} + 3\sqrt{P}\cos echT\right] - \left[\frac{\delta}{m}\sqrt{\beta+\beta\cosh T}\right] \tag{72}$$

In the absence of viscosity, we get the stiff fluid case,
When $T \to 0$, then

$$8\pi\, p = 8\pi\,\varepsilon = \frac{P\beta}{4m^2} \tag{73}$$

In the absence of viscosity, the average scale factor becomes

$$R^3 = m\sqrt{\alpha} \tag{74}$$

which is a constant.

In the absence of viscosity the model (40) represents shearing but non-rotating universe and the space-time is non-degenerate Petrov Type I. If P = 0 the space-time is conformally flat. Since $\lim_{T\to\infty}\frac{\sigma}{\theta} \neq 0$, hence the model does not approach isotropy for large values of T in the absence of viscosity. In the absence of viscosity, expansion in the model stops.

5. Discussion

The model obtained in this paper is of considerable interest and may be useful in general relativity. The model in general represents sheering and non-rotating universe. The behaviour of the model (15) is Petrov Type I non-degenerate. It reduces to Petrov Type D space-time for P = 0, and conformally flat for $\beta = 0$. The role of viscosity is to retard the expansion in the model. In absence of viscosity, expansion in the model stops.

It is remarkable to mention that in the absence of viscosity, we get the stiff fluid case.

Since $v^i_{;j}v^j = 0$, therefore the lines of flow are geodetic. In the model we find that the deceleration parameter q is always negative as in the case of the De-Sitter universe.

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