

On studying antibiotic in the bloodstream by the method of dynamical model

R. SOPHIAPORCHELVI¹ and R. VANITHA²

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Abstract

Primarily the antibiotics-the group drugs intended to inhibit or destroy bacteria, should be used only against bacterial infection. The antibiotic Norfloxacin is prescribed for treating bacterial infections, including urinary tract infections, prostatitis, and gonorrhea infections of the urethra or cervix. This antibiotic works to kill bacteria by interfering with specific enzymes, which prevents the bacteria from multiplying. When taking an antibiotic, it is important to keep the amount of the bloodstream fairly constant. If the amount gets too low, they can begin to regrow. If the amount gets too high, it could cause other Complications. In this paper, a dynamical model has been constructed to find the amount of the Norfloxacin in the blood stream.

Key word: Antibiotic, Half-life period of the drug, dosage of the drug, Discrete Dynamical system.

1. Introduction

A dynamical system is simply a system that changes over time. When time is measured in discrete units, the system is called a discrete dynamical system. Discrete dynamics is the study of change. In particular, it shows how to translate real world situations into the language of mathematics. With the increase in computational ability and the recent interest in chaos, discrete dynamics has emerged as an important area of mathematical study⁵. Dynamical systems are “easy to [model] and hard to solve” as

given by Meerschaert⁴ in 1999. In this paper, one dynamical model is formulated and graphical approaches to analyze the dynamical system and to calculate the amount of antibiotic in the Bloodstream are presented².

2. Preliminaries and Notations :

A discrete dynamical system is a sequence of numbers $\{a_n | n=0,1,2,\dots\}$ defined by a relation of the form $a_{n+1} = f(a_n)$, where f is some real-valued function.

The variable a_n is generically referred to as the state of the system. In simpler terms, a discrete dynamical system is one in which the state of the system is determined by the previous state. An **affine discrete dynamical system** is a sequence of numbers $\{a_n | n=0,1,2,\dots\}$ described by a relation of the form $a_{n+1} = ba_n + m$, where $b \neq 0$.

Central to the analysis of the long – term behavior of any dynamical system are *equilibrium values*. A number a is called an **equilibrium value** for the dynamical system $a_{n+1} = f(a_n)$ if $a_n = a$ for all n , whenever $a_0 = a$. The equilibrium value ‘ a ’ is stable or attracting if there is a number ϵ such that $\lim a_n = a$, whenever $|a_0 - a| < \epsilon$. The study of equilibrium values and limits associated with discrete dynamical systems is important to understand the nature and behavior of such systems. An equilibrium value has the property that if it is used as the initial value, then all successive values associated with the system are equal to the equilibrium value. In fact, that is the basis for the definition of the equilibrium value¹.

3. Description of the Model :

Primarily the antibiotics-the group drugs intended to inhibit or destroy bacteria, should be used only against bacterial infection. **Antibiotics** are used to treat infections caused by bacteria. Bacteria are microscopic organisms, some of which may cause illness. The word bacteria is the plural of bacterium. Such illnesses as sypilis, tuberculosis, salmonella, and some forms of meningitis are caused by bacteria. Some bacteria are harmless, while others are good for us. Before bacteria

can multiply and cause symptoms, the body’s immune system can usually destroy them. We have special white blood cells that attack harmful bacteria. Even if symptoms do occur, our immune system can usually cope and fight off the infection. There are occasions, however, when it is all too much and antibiotics have become essential to be used.

The first antibiotic was Penicillin. Such penicillin-related antibiotics as ampicillin, amoxicillin and benzyl penicillin are widely used today to treat a variety of infections and moreover these antibiotics have been used for a long time. There are several different types of modern antibiotics and they are only available with a doctor’s prescription in industrialized countries as in⁸. When taking an antibiotic, it is important to keep the amount of the blood-stream fairly constant. If the amount gets too low, they can begin to regrow. If the amount gets too high, it could cause other complications.

Norfloxacin is prescribed for treating bacterial infections, including urinary tract infections, prostatitis, and gonorrhea infections of the urethra or cervix. This antibiotic works to kill bacteria by interfering with specific enzymes, which prevents the bacteria from multiplying. Suppose the half-life of the drug is 3 to 4 hours (meaning that half the Norfloxacin remains in the blood after each 3 to 4 hours period, (0.917%)) and again a dosage of 400mg is given at end of 12 hours in a day⁶. Let us examine what happens to the amount of Norfloxacin in the bloodstream in the long run.

STEP 1: Identifying the Problem :

Determine the relationship between the amount

of drug in the bloodstream and time.

STEP 2: Making Assumptions Variables :

a_n - amount of drug(mg) in the bloodstream after the day n , where $n = 0, 1, 2, \dots$,
 n – number of days.

Assumptions :

1. The patient is of normal size and health.
2. There are no other drugs being taken that will affect the prescribed drug.
3. There are no internal or external factors that will affect the drug absorption rate.
4. The patient always takes the prescribed dosage at the correct time.

STEP 3: Constructing the Model

Here, we have

a_{n+1} = amount of drug in the bloodstream in the future (mg)

a_n = amount of drug currently in the bloodstream after the day n (mg) (i.e., at the end of the day n). Define the change as follows:

Change = dose- loss in the system

Change = $2 \times 400 - 0.917a_n$

Therefore, Future = Present + Change

$a_{n+1} = a_n - 0.917a_n + 800 \Rightarrow a_{n+1} = 0.083a_n + 800$

Hence, a simple affine model for the system is

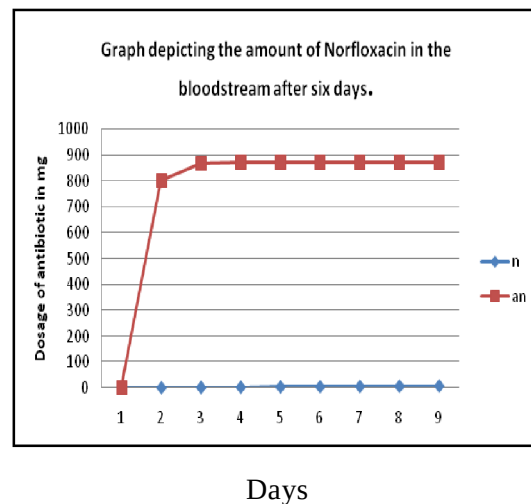
$$a_{n+1} = 0.083a_n + 800,$$

here a_n = the amount of Norfloxacin in the bloodstream at end of day n . Since the problem did not specify the initial dosage, a_0 , we need

to experiment with different values.

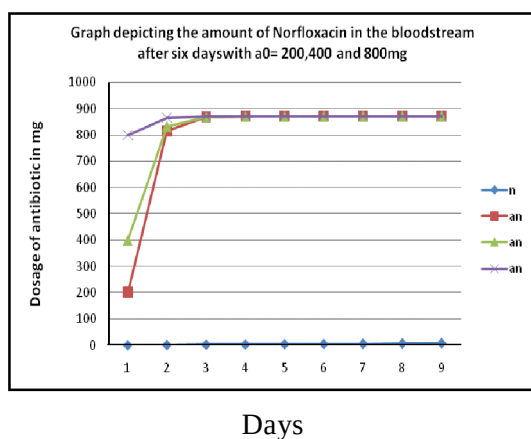
Using Excel we may obtain a_n values for $n = 0, 1, 2, \dots$ and creating a graph, we observe that with an initial dosage of 0 mg, the amount of Norfloxacin in the blood appears to approach 872 mg at the end of day 4 onwards⁷.

Number of days n	Amount of Norfloxacin in the bloodstream at the end of the day a_n
0	0
1	800
2	866.4
3	871.9112
4	872.3686
5	872.4066
6	872.4097



Now consider three starting values or initial doses: A: $a_0=200$; B: $b_0=400$; C: $c_0=600$. In Table 2: we compute the numerical solutions for each case.

	A	B	C
N	a_n	a_n	a_n
0	200	400	800
1	816.6	833.2	866.4
2	867.7778	869.1556	871.9112
3	872.0256	872.1399	872.3686
4	872.3781	872.3876	872.4066
5	872.4074	872.4082	872.4097
6	872.4098	872.4099	872.41
7	872.41	872.41	872.41
8	872.41	872.41	872.41



Specifically, we observe that with an initial dosage of 200 mg, 400 mg and 600 mg the amount of Norfloxacin in the blood stream appears to approach 872 mg at the end of day 3 onwards. Also note that no matter what the value of a_0 , the system appears to approach an equilibrium value. This shows that 872 mg is a *stable equilibrium*, which indicates the safe and effective level. It is concluded that

the amount of antibiotic-Norfloxacin by taking 400 mg twice in a day for three-four days will control the bacterial infection⁸.

Conclusion

The antibiotic-Norfloxacin might be taken by the adult (≥ 18 yrs) one hour or two hours after meals/milk. To treat uncomplicated urinary tract infections, doctors prescribed 400 mg Norfloxacin twice a day for three days. By constructing discrete dynamical system we conclude the same by obtained the stable equilibrium.

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