Ultra Scientist Vol. 25(1)A, 139-155 (2013).

Stochastic model to determine the optimal manpower reserve at four nodes in series

[1]R. ARULPAVAI and [2]R. ELANGOVAN

(Acceptance Date 25th March, 2013)

Abstract

The purpose of manpower planning is to best match future manpower demand and supply in the light of multiple objectives such as economic conditions, production/scale trends, people skills inventory, government regulations, as well as organization history and policies regarding personnel hearing, training, promotion, firing and retirement. One of the essential components of a manpower planning system is manpower forecasting, the process of anticipating the future size and nature of the manpower force. Stochastic models have been widely used in the study of manpower systems with different modes in series like training, promotion, placement, etc. There are many industries and organizations where the skilled personnel are to be recruited and they must be given prior training before employment. In human resource planning the training and induction of the right type of personnel is a pressing problem. In this paper, we consider the optimal solution for the manpower to be kept as reserve inventory at four different nodes in series. Two different models have been discussed. Numerical illustrations are provided using simulation studies.

Key words: Human Resource Planning, Reserve Inventory and Optimal Solution.

1. Introduction

The term "manpower planning" will be used as defined by James Walker[8], "Manpower planning refers to the rather complex task of forecasting and planning for the right numbers and the right kinds of people at the right places and the right times to perform activities that will benefit both the organization and the individuals in it", According to Grinold and Marshall (1977), manpower planning must be an ancient art, since manpower problems have existed for centuries. People, jobs, time and money are the basic ingredients of a manpower system. A decision maker must be aware of the interactions among these four ingredients in order to formulate and evaluate manpower policy. In the national level Human resources development is an important aspect of study and the manpower planning has to be done taking into consideration the dynamic of manpower availability and requirements. With advancement of civilization, science and technology, manpower is required to deal with complex problems of varying nature in different areas of real life, of course required with specialization and skill. All these associated problems of manpower requirements warrant a systematic approach to the study of manpower planning. The term manpower planning is also known as human resource planning listed in the recent literature for the detailed study refer to Bartholomew[1,2], Bartholomew and Forbes[3], McClean *et. al.* (1991).

The industrial and administrative environment and structure largely contribute to the socio-economic welfare of any nation. The availability of appropriate type of manpower is an important factor that contributes to the task force available for the completion of planned jobs and projects including industrial production. The rapid advancement of science and technology has thoroughly changed the conventional requirements of manpower to complete the jobs. It is in this context the human resource development has undergone a rational change over the years. Hence the methods of manpower planning have become more complex as it includes the determination of the equilibrium of manpower supply and demands, Mathematical and stochastic models have been applied to a large extend for the conceptualization of real life situations as appropriate mathematical models. Such a process enables the achievement of solutions, which would be precise, and advantageous. Some interesting results can also we seen in Susiganeshkumar and Elangovan[6], Elangovan *et. al*.[4], Susiganeshkumar and Elangovan[7]. There are many industries and organizations where the skilled personnel are to be recruited and they must be given prior training before employment. In human resource planning the training and induction of the right type of personnel is a pressing problem. Recently there has been considerable interest in developing stochastic models for describing optimal policies for item in inventory whose utility does not remain the same with the passage of time. Significant work has been done for describing optimal ordering policies for units with fixed life time. In this paper, we consider the optimal solution for the manpower to be kept as reserve inventory at four different nodes in series. This is based on the inventory model for finding the optimal size of reserve inventory at three successive stations in series[5].

This paper is organized as follows: in Section 2 assumptions and notations are discussed. The manpower reserve at four nodes in series and results are discussed in Section 3. Two different models have been discussed in Section 4 and Section 5. Numerical results using simulation studies is also highlighted. A summary of the results is shown in Section 6.

2. Assumptions :

(i) The manpower system comprises of four nodes namely the recruitment node, the training node, probation period employment node and permanent employment node.

(ii) The reserve of manpower inventory is at three points namely in between recruitment and training, training and probation period employment and similarly between probation period employment to permanent employment.

(iii) The delay in finding suitable candidates after training for employment is very costly.

2.1 Notations :

(i) The cost of excess candidates at the reserve points 1, 2 and 3 are denoted as h_1, h_2 and h_3 called holding costs.

(ii) d_1, d_2 and d_3 denote the cost arising due to the shortage of candidates or manpower at the two intermediate reserve points R_1, R_2 and R_3.

(iii) The breakdown occurs at the recruitment or entry point for a random duration τ, and has the p.d.f is $g(\tau)$ and c.d.f. is $G(\tau)$.

(iv) r_1 denotes the constant rate of training of personnel at the node R_2.

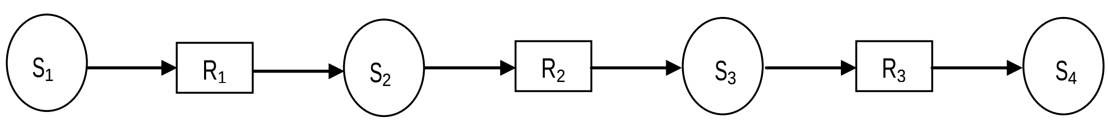
(v) r_2 denotes the constant rate of probation period employment of personnel at the node R_2.

(vi) r_3 denotes the constant rate of permanent employment of personnel at the node R_3.

(vii) μ_1, μ_2, μ_3 the mean interarrival time of breakdowns of system R_1, R_2 and R_3 respectively.

3. Manpower Reserve at four nodes in series:

We consider a manpower system in which the first node is the point of recruitment. This second node is the training of the selected before induction or employment. It may be noted that if there is any delay or breakdown at the first node namely the recruitment point then the training will be delayed and ultimately the shortage of trained personnel occurs. This in turn affects the running of the industry or organization. Similarly if there is any breakdown at the training node there will be again shortage of trained persons for employment. Therefore a reserve inventory of persons is to be maintained at three places namely in between first and second nodes, second and third nodes and third and forth nodes. The following configuration explains the conceptualized model.



S_1, S_2, S_3 and S_4 denote the four sections or systems devoted to different activities

S_1 = Recruitment division

S_2 = Training division

S_3 = Probation induction section

S_4 = Permanent induction section

R_1 = Reserve of persons at the first stage

R_2 = Reserve of persons at the second stage

R_3 = Reserve of persons at the third stage

The reserves are in terms of man hours.

In this model it is assumed that if the reserve of manpower is in excess then there is a cost of excess and similarly if the reserve of manpower is in shortage it involves the so called shortage cost. So the problem is to determine the optimal size of reserve at the two different points namely between recruitment and training and again in between training, probation period employment and finally probation period employment to permanent employment.

3.1 Results

In this model it can be seen that the total expected cost due to excess of manpower inventory and also shortage is

$$E(c) = h_1R_1 + h_2R_2 + h_3R_3 + \frac{d_1}{\mu_1}\int_{R_1/r_1}^{\infty}\left(\tau - \frac{R_1}{r_1}\right) g(\tau)\, d\tau + \frac{d_2}{\mu_1}\int_{\frac{R_1}{r_1}+\frac{R_2}{r_2}}^{\infty}\left(\tau - \left(\frac{R_1}{r_1}+\frac{R_2}{r_2}\right)\right) g(\tau)\, d\tau + \frac{d_3}{\mu_1}\int_{\frac{R_1}{r_1}+\frac{R_2}{r_2}+\frac{R_3}{r_3}}^{\infty}\left(\tau - \left(\frac{R_1}{r_1}+\frac{R_2}{r_2}+\frac{R_3}{r_3}\right)\right) g(\tau)\, d\tau \tag{1.1}$$

It may be observed that the first two terms of eqn. (1.1) represent the cost of excess of reserve of manpower, the third term is the cost of training section due to shortage of candidates and the fourth term is the cost due to the failure production due to non availability of manpower.

To optimal values of R_1, R_2 and R_3 can be obtained by solving the equations.

$$\frac{\partial E(c)}{\partial R_1} = 0 \tag{1.2}$$

$$\frac{\partial E(c)}{\partial R_2} = 0 \tag{1.3}$$

$$\frac{\partial E(c)}{\partial R_3} = 0 \tag{1.4}$$

On differentiating partially the above equations with respect to R_1 and R_2 and equating to zero we obtain,

$$h_1 - \frac{d_1}{\mu_1 r_1}\left[1 - G\left(\frac{R_1}{r_1}\right)\right] - \frac{d_2}{\mu_1 r_1}\left[1 - G\left(\frac{R_1}{r_1}+\frac{R_2}{r_2}\right)\right] - \frac{d_3}{\mu_1 r_1}\left[1 - G\left(\frac{R_1}{r_1}+\frac{R_2}{r_2}+\frac{R_3}{r_3}\right)\right] = 0 \tag{1.5}$$

$$h_2 - \frac{d_2}{\mu_1 r_2}\left[1 - G\left(\frac{R_1}{r_1}+\frac{R_2}{r_2}\right)\right] - \frac{d_3}{\mu_1 r_1}\left[1 - G\left(\frac{R_1}{r_1}+\frac{R_2}{r_2}+\frac{R_3}{r_3}\right)\right] = 0 \tag{1.6}$$

and

$$h_3 - \frac{d_3}{\mu_1 r_1}\left[1-G\left(\frac{R_1}{r_1}+\frac{R_2}{r_2}+\frac{R_3}{r_3}\right)\right] = 0 \quad (1.7)$$

It may be observed that Leibnitz rule for the differentiation of an integral namely

$$\int_{a(x)}^{b(x)} f(x,\ t)\ dt = b'(x)\ f\left[x,\ b(x)\right] - a'(x) f\left[x,\ a(x)\right] + \int_{a(x)}^{b(x)} \frac{d}{dx} f(x,\ t)\ dx \quad (1.8)$$

has been used, in differentiating eqn. (1.1) with respect to R_1, R_2 and R_3 respectively. Solving equations (1.5), (1.6) and (1.7), the optimal values of R_1, R_2 and R_3 are obtained. Rajagopal and Sathiyamoorthy (2003) have discussed similar such models in inventory management.

4. Model I :

Consider the case when G(.) has uniform distribution over [0, a]. The down time density of the recruitment is a constant and is independent of time, that is, the down time distribution is proportional to time.

The holding cost will arise when $a < (R_1/r_1 + R_2/r_2 + R_3/r_3)$.

When $a > (R_1/r_1 + R_2/r_2 + R_3/r_3)$, the equations (1.5), (1.6) and (1.7) present the following,

$$h_1 - \frac{d_1}{\mu_1 r_1}\left[\frac{a-(R_1/r_1)}{a}\right] - \frac{d_2}{\mu_1 r_1}\left[\frac{a-(R_1/r_1 + R_2/r_2)}{a}\right] - \frac{d_3}{\mu_1 r_1}\left[\frac{a-(R_1/r_1 + R_2/r_2 + R_3/r_3)}{a}\right] = 0 \quad (1.9)$$

$$h_2 - \frac{d_2}{\mu_1 r_2}\left[\frac{a-(R_1/r_1 + R_2/r_2)}{a}\right] - \frac{d_3}{\mu_1 r_1}\left[\frac{a-(R_1/r_1 + R_2/r_2 + R_3/r_3)}{a}\right] = 0 \quad (1.10)$$

and

$$h_3 - \frac{d_3}{\mu_1 r_1}\left[\frac{a-(R_1/r_1 + R_2/r_2 + R_3/r_3)}{a}\right] = 0 \quad (1.11)$$

On collecting the coefficients of R_1, R_2 and R_3 these three equations will be reduced to the three simultaneous, linear equations in R_1, R_2 and R_3 as follows.

Consider the eqn. (1.9) and it can be written as,

$$h_1 - \frac{d_1}{\mu_1 r_1} + \frac{d_1}{\mu_1 r_1^2 a} R_1 - \frac{d_2}{\mu_1 r_1} + \frac{d_2}{\mu_1 r_1^2 a} R_1 + \frac{d_2}{\mu_1 r_1 r_2\, a} R_2 - \frac{d_3}{\mu_1 r_1} + \frac{d_3}{\mu_1 r_1^2 a} R_1 + \frac{d_3}{\mu_1 r_1 r_2\, a} R_2 + \frac{d_3}{\mu_1 r_1 r_3\, a} R_3 = 0$$

$$h_1 - \frac{d_1 + d_2 + d_3}{\mu_1 r_1} + \frac{d_1 + d_2 + d_3}{\mu_1 r_1^2 a} R_1 + \frac{d_2 + d_3}{\mu_1 r_1 r_2 a} R_2 + \frac{d_3}{\mu_1 r_1 r_3 a} R_3 = 0 \quad (1.12)$$

Consider the eqn. (1.10) and it can be written as,

$$h_2 - \frac{d_2}{\mu_1 r_1} + \frac{d_2}{\mu_1 r_1 r_2 a} R_1 + \frac{d_2}{\mu_1 r_2^2 a} R_2 - \frac{d_3}{\mu_1 r_2} + \frac{d_2}{\mu_1 r_1 r_2 a} R_1 + \frac{d_3}{\mu_1 r_2^2 a} R_2$$

$$+ \frac{d_3}{\mu_1 r_2 r_3 a} R_3 = 0$$

$$h_2 - \frac{d_2 + d_3}{\mu_1 r_2} + \frac{d_2 + d_3}{\mu_1 r_1 r_2 a} R_1 + \frac{d_2 + d_3}{\mu_1 r_2^2 a} R_2 + \frac{d_3}{\mu_1 r_2 r_3 a} R_3 = 0 \quad (1.13)$$

Consider the eqn. (1.11) and it can be written as,

$$h_3 - \frac{d_3}{\mu_1 r_1} + \frac{d_3}{\mu_1 r_1 r_3 a} R_1 + \frac{d_3}{\mu_1 r_2 r_3 a} R_2 + \frac{d_3}{\mu_1 r_3^2 a} R_3 = 0 \quad (1.14)$$

Solving equ. (1.12), equ. (1.13) and equ. (1.14) we get solution. The obtained solutions are

$$\hat{R}_1 = \frac{a\ r_1 (d_1 - h_1 r_1 \mu_1 + h_2 r_2 \mu_1)}{d_1} \quad (1.15)$$

$$\hat{R}_2 = \frac{a\ r_2 (d_2 h_1 r_1 - d_1 h_2 r_2 - d_2 h_2 r_2 + d_1 h_3 r_3)\mu_1}{d_1 d_2} \quad (1.16)$$

$$\hat{R}_3 = \frac{a\ r_3 (-d_3 h_2 r_2 + d_2 h_3 r_3 + d_3 h_3 r_3)\mu_1}{d_2 d_3} \quad (1.17)$$

after simplification.

4.1 Numerical Example :

The Variation in $\hat{\boldsymbol{R}}_1$ consequent to the changes in h_1, keeping r_1=10, r_2=20, d_1=30, h_2=45, μ_1=1.0 and a=1.5 all are fixed, the simulated values are shown in Table 1.

Table 1. The Variation in $\hat{\boldsymbol{R}}_1$ consequent to the changes in h_1

h_1	10	20	30	40	50	60	70
$\hat{\boldsymbol{R}}_1$	415	365	315	265	215	165	115

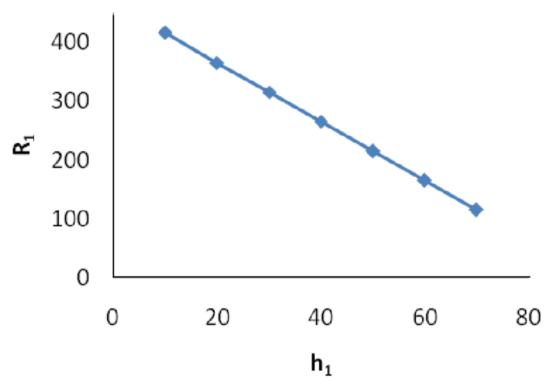


Fig. 1. The Variation in $\hat{\boldsymbol{R}}_1$ consequent to the changes in h_1

The Variation in $\hat{R}_1$ consequent to the changes in d_1, keeping r_1=10, r_2=20, h_1=10, h_2=45, μ_1=1.0 and a=1.5 all are fixed, the simulated values are shown in Table 2.

Table 2: The Variation in $\hat{R}_1$ consequent to the changes in d_1

d_1	50	55	60	65	70	75	80
$\hat{R}_1$	255	233	215	199	186	175	165

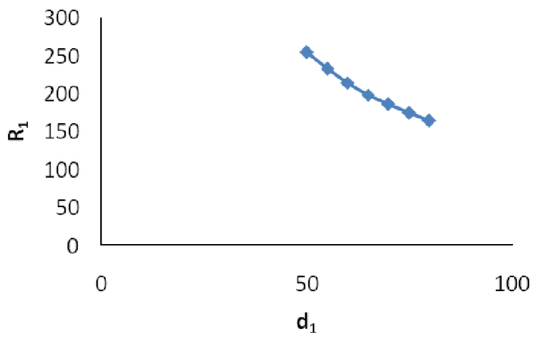


Fig. 2. The Variation in $\hat{R}_1$ consequent to the changes in d_1

The Variation in $\hat{R}_1$ consequent to the changes in μ_1, keeping r_1=10, r_2=20, h_1=10, h_2=45, d_1=50 and a=1.5 all are fixed, the simulated values are shown in Table 3.

Table 3. The Variation in $\hat{R}_1$ consequent to the changes in μ_1

μ_1	1.0	1.2	1.4	1.6	1.8	2.0	2.2
$\hat{R}_1$	255	303	351	399	447	495	543

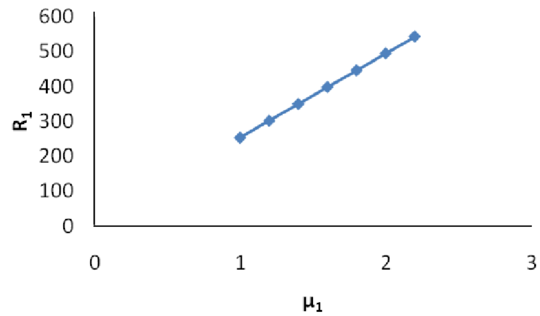


Fig. 3. The Variation in $\hat{R}_1$ consequent to the changes in μ_1

The Variation in $\hat{R}_2$ consequent to the changes in h_2, keeping r_1=10, r_2=20, r_3=15, d_1=50, d_2=40, h_1=10, h_3=80, μ_1=1.0 and a=1.5 all are fixed, the simulated values are shown in Table 4.

Table 4. The Variation in $\hat{R}_2$ consequent to the changes in h_2

h_2	20	21	22	23	24	25	26
$\hat{R}_2$	420	393	366	339	312	285	258

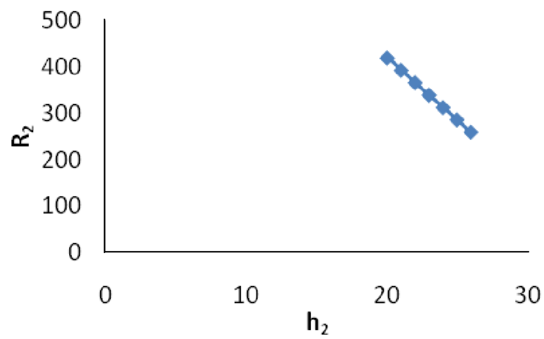


Fig. 4. The Variation in $\hat{R}_2$ consequent to the changes in h_2

The Variation in $\hat{R}_2$ consequent to the changes in d_2, keeping $r_1=10$, $r_2=20$, $r_3=15$, $d_1=50$, $h_1=10$, $h_2=20$, $h_3=80$, $\mu_1=1.0$ and $a=1.5$ all are fixed, the simulated values are shown in Table 5.

Table 5. The Variation in $\hat{R}_2$ consequent to the changes in d_2

d_2	40	45	50	55	60	65	70
$\hat{R}_2$	420	353	300	256	220	189	163

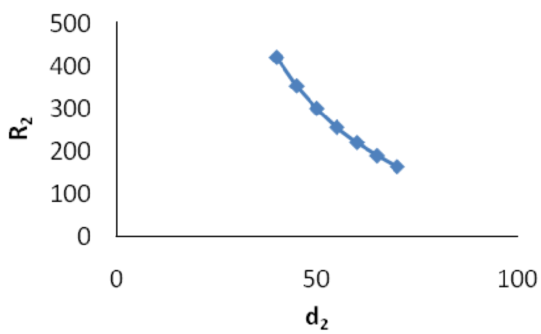


Fig. 5. The Variation in $\hat{R}_2$ consequent to the changes in d_2

The Variation in $\hat{R}_2$ consequent to the changes in μ_2, keeping $r_1=10$, $r_2=20$, $r_3=15$, $d_1=50$, $d_2=40$, $h_1=10$, $h_2=45$, $h_3=125$ and $a=1.5$ all are fixed, the simulated values are shown in Table 6.

Table 6. The Variation in $\hat{R}_2$ consequent to the changes in μ_1

μ_1	1.0	1.2	1.4	1.6	1.8	2.0	2.2
$\hat{R}_2$	251	302	352	402	452	503	553

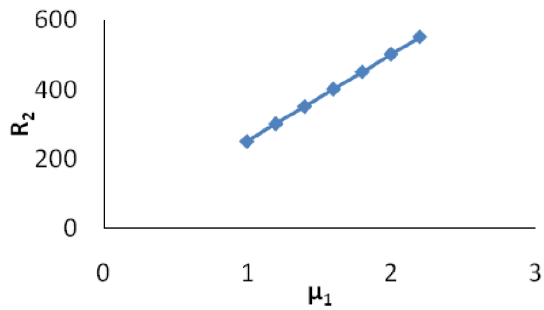


Fig. 6. The Variation in $\hat{R}_2$ consequent to the changes in μ_1

The Variation in $\hat{R}_3$ consequent to the changes in h_3, keeping $r_2=20$, $r_3=15$, $d_2=40$, $d_3=20$, $h_2=25$, $\mu_1=1.0$ and $a=1.5$ all are fixed, the simulated values are shown in Table 7.

Table 7. The Variation in $\hat{R}_3$ consequent to the changes in h_3

h_3	20	30	40	50	60	70	80
$\hat{R}_3$	281	534	788	1041	1294	1547	1800

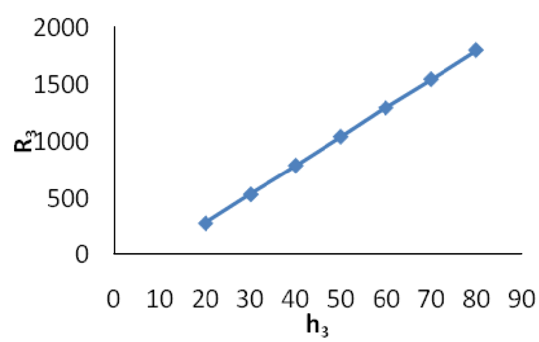


Fig. 7. The Variation in $\hat{R}_3$ consequent to the changes in h_3

The Variation in $\hat{R}_3$ consequent to the changes in d_3, keeping r_2=20, r_3=15, d_2=40, h_2=20, h_3=80, μ_1=1.0 and a=1.5 all are fixed, the simulated values are shown in Table 8.

Table 8. The Variation in $\hat{R}_3$ consequent to the changes in d_3

d_3	20	40	60	80	100	120	140
$\hat{R}_3$	1800	1125	900	788	720	675	643

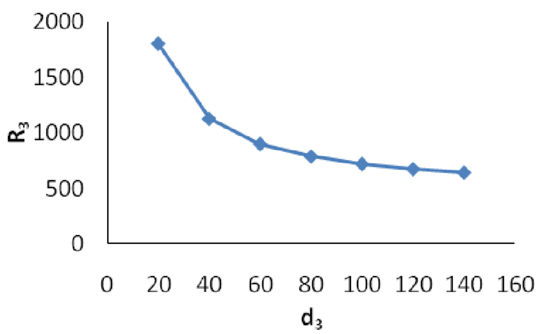


Fig. 8. The Variation in $\hat{R}_3$ consequent to the changes in d_3

The Variation in $\hat{R}_3$ consequent to the changes in μ_1, keeping r_2=20, r_3=15, d_2=40, d_3=60, h_2=45, h_3=75 and a=1.5 all are fixed, the simulated values are shown in Table 9.

Table 9. The Variation in $\hat{R}_3$ consequent to the changes in μ_1

μ_1	1.0	1.2	1.4	1.6	1.8	2.0	2.2
$\hat{R}_3$	548	658	768	878	987	1097	1207

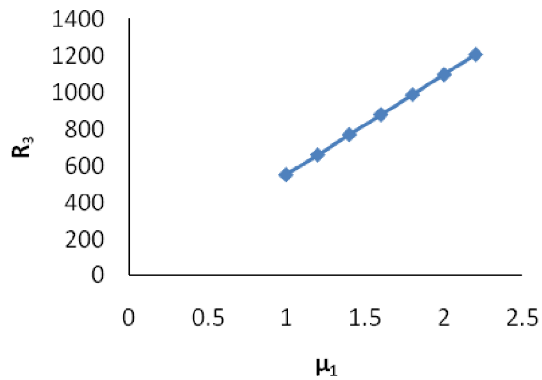


Fig. 9. The Variation in $\hat{R}_3$ consequent to the changes in μ_1

On the basis of the model I discussed the following conclusions are drawn using the numerical illustration.

(i) As the cost of manpower reserve between stages S_1, S_2, S_3 and S_4 namely h_1 increases a small reserve R_1 is suggested as an optimal one. This is indicated in Table 1 and Fig. 1.

(ii) If the cost of shortages at the first node namely d_1 increases, it is suggested that a larger manpower at R_1 is desirable as indicated in Table 2 and Fig. 2.

(iii) The interarrival time between breakdown μ_1 increase then a increased in a value of R_1 is suggested as in indicated in Table 3 and Fig. 3.

(iv) If the cost of manpower reserve at node 2 is higher namely as h_2 increases then a decrease in the value of R_2 is suggested as indicated in Table 4 and Fig. 4.

(v) If d_2, the cost of shortages at the second node increases a larger reserve namely R_2 is desirable as indicated in Table 5 and Fig. 5.

(vi) The interarrival time between breakdown μ_1 increase then a increased in a value of R_2 is suggested as in indicated in Table 6 and Fig. 6.

(vii) If the cost of manpower reserve at node 3 is higher namely as h_3 increases then a increase in the value of R_3 is suggested as indicated in Table 7 and Fig. 7.

(viii) If d_3, the cost of shortages at the third node increases a larger reserve namely R_3 is desirable as indicated in Table 8 and Fig. 8.

(ix) The interarrival time between breakdown μ_1 increase then a increased in a value of R_3 is suggested as in indicated in Table 9 and Fig. 9.

(x) If μ_1 the parameter of the interarrival times between the breakdowns of S_1, S_2, S_3 and S_4 increases then the interarrival times will be shorter since $E(\tau)=1/\mu_1$, decreases. Hence a larger reserve at R_1, R_2 and R_3 is suggested as indicated in Table 3, Table 6 and Table 9 and the Fig. 3, Fig. 6 and Fig. 9.

5. Model II :

In the above model an additional assumption that the down time of R_1 is a random variable which satisfies the so called Setting the Clock Back to Zero (SCBZ) property is introduced. This property has been due to Raja Rao and Talwalker (1990). Under this property the p.d.f. of the down time τ which is namely $g(\tau)$ is such that

$$\begin{aligned} g(\tau) &= \theta_1 e^{-\theta_1 \tau} \quad \text{if } \tau \le \tau_0 \\ &= \theta_2 e^{-\theta_2 \tau} e^{\tau_0(\theta_2-\theta_1)} \quad \text{if } \tau > \tau_0 \end{aligned}$$

when τ_0 is called the truncation point.

It can be shown that $\int_0^\infty g(\tau)d\tau = 1$,

and τ_0 is called the truncation point which itself is a random variable that follows exponential distribution with parameter λ. Now the cost function under this assumption is given as,

$$\begin{aligned} E(c) &= h_1R_1 + h_2R_2 + h_3R_3 \\ &+ \frac{d_1}{\mu_1}\left[\int_{R_1/r_1}^{\tau_0}\left(\tau - \frac{R_1}{r_1}\right)\theta_1 e^{-\theta_1\tau}\,d\tau\; p[\tau \le \tau_0] + e^{\tau_0(\theta_2-\theta_1)}\int_{\tau_0}^{\infty}\left(\tau - \frac{R_1}{r_1}\right)\theta_2 e^{-\theta_2\tau}\,d\tau\; p[\tau > \tau_0]\right] \\ &+ \frac{d_2}{\mu_2}\left[\int_{\frac{R_1}{r_1}+\frac{R_2}{r_2}}^{\tau_0}\left(\tau - \frac{R_1}{r_1} - \frac{R_2}{r_2}\right)\theta_1 e^{-\theta_1\tau}\,d\tau\; p[\tau \le \tau_0] + e^{\tau_0(\theta_2-\theta_1)}\int_{\tau_0}^{\infty}\left(\tau - \frac{R_1}{r_1} - \frac{R_2}{r_2}\right)\theta_2 e^{-\theta_2\tau}\,d\tau\; p[\tau > \tau_0]\right] \end{aligned}$$

$$+\frac{d_3}{\mu 3}\left[\begin{array}{l}\displaystyle\int_{\frac{R_1}{r_1}+\frac{R_2}{r_2}+\frac{R_3}{r_3}}^{\tau_0}\left(\tau-\frac{R_1}{r_1}-\frac{R_2}{r_2}-\frac{R_3}{r_3}\right)\theta_1 e^{-\theta_1\tau}\,d\tau\;p[\tau\le\tau_0]+\\ \displaystyle e^{\tau_0(\theta_2-\theta_1)}\int_{\tau_0}^{\infty}\left(\tau-\frac{R_1}{r_1}-\frac{R_2}{r_2}-\frac{R_3}{r_3}\right)\theta_2 e^{-\theta_2\tau}\,d\tau\;p[\tau>\tau_0]\end{array}\right]$$

$$E(c)=h_1R_1+h_2R_2+h_3R_3$$

$$+\frac{d_1}{\mu_1}\left[\theta_1\int_{R_1/r_1}^{\tau_0}\left(\tau-\frac{R_1}{r_1}\right)e^{-\theta_1\tau}e^{-\lambda\tau_0}\,d\tau+e^{\tau_0(\theta_2-\theta_1)}\,\theta_2\int_{\tau_0}^{\infty}\left(\tau-\frac{R_1}{r_1}\right)e^{-\theta_2\tau}\left[1-e^{-\lambda\tau_0}\right]d\tau\right]$$

$$+\frac{d_2}{\mu_2}\left[\theta_1\int_{\frac{R_1}{r_1}+\frac{R_2}{r_2}}^{\tau_0}\left(\tau-\frac{R_1}{r_1}-\frac{R_2}{r_2}\right)e^{-\theta_1\tau}\,e^{-\lambda\tau_0}\,d\tau+e^{\tau_0(\theta_2-\theta_1)}\,\theta_2\int_{\tau_0}^{\infty}\left(\tau-\frac{R_1}{r_1}-\frac{R_2}{r_2}\right)e^{-\theta_2\tau}\left[1-e^{-\lambda\tau_0}\right]d\tau\right]$$

$$+\frac{d_3}{\mu_3}\left[\begin{array}{l}\displaystyle\theta_1\int_{\frac{R_1}{r_1}+\frac{R_2}{r_2}+\frac{R_3}{r_3}}^{\tau_0}\left(\tau-\frac{R_1}{r_1}-\frac{R_2}{r_2}-\frac{R_3}{r_3}\right)e^{-\theta_1\tau}\,e^{-\lambda\tau_0}\,d\tau\\ \displaystyle+e^{\tau_0(\theta_2-\theta_1)}\,\theta_2\int_{\tau_0}^{\infty}\left(\tau-\frac{R_1}{r_1}-\frac{R_2}{r_2}-\frac{R_3}{r_3}\right)e^{-\theta_2\tau}\left[1-e^{-\lambda\tau_0}\right]d\tau\end{array}\right]$$

$$\frac{\partial E(c)}{\partial R_1}=h_1+\frac{d_1}{\mu_1}\left\{0-\frac{1}{r_1}(0)+\theta_1\int_{\frac{R_1}{r_1}}^{\tau_0}\left(-\frac{1}{r_1}\right)e^{-\theta_1\tau}e^{-\lambda\tau_0}d\tau\right\}+$$

$$\frac{d_1}{\mu_1}e^{\tau_0(\theta_2-\theta_1)}\theta_2\int_{\tau_0}^{\infty}\left(-\frac{1}{r_1}\right)e^{-\theta_2\tau}\left[1-e^{-\lambda\tau_0}\right]d\tau+$$

$$\frac{d_2}{\mu_2}\left\{0-\frac{1}{r_1}(0)+\theta_1\int_{\frac{R_1}{r_1}+\frac{R_2}{r_2}}^{\tau_0}\left(-\frac{1}{r_1}\right)e^{-\theta_1\tau}e^{-\lambda\tau_0}d\tau\right\}+$$

$$e^{\tau_0(\theta_2-\theta_1)}\frac{d_2}{\mu_2}\left[\theta_2\int_{\tau_0}^{\infty}\left(-\frac{1}{r_1}\right)e^{-\theta_2\tau}\left[1-e^{-\lambda\tau_0}\right]d\tau\right]$$

$$\frac{d_3}{\mu_3}\left\{0-\frac{1}{r_1}(0)+\theta_1\int_{\frac{R_1}{r_1}+\frac{R_2}{r_2}+\frac{R_3}{r_3}}^{\tau_0}\left(-\frac{1}{r_1}\right)e^{-\theta_1\tau}e^{-\lambda\tau_0}d\tau\right\}+$$

$$e^{\tau_0(\theta_2-\theta_1)}\frac{d_2}{\mu_2}\left[\theta_2\int_{\tau_0}^{\infty}\left(-\frac{1}{r_1}\right)e^{-\theta_2\tau}\left[1-e^{-\lambda\tau_0}\right]d\tau\right]$$

$$=I_1+I_2+I_3+I_4+I_5+I_6$$

$$I_1=h_1+\frac{d_1}{\mu_1}\left[\theta_1\int_{\frac{R_1}{r_1}}^{\tau_0}\left(-\frac{1}{r_1}\right)\right]e^{-\theta_1\tau}e^{-\lambda\tau_0}d\tau$$

$$=h_1-\frac{d_1\theta_1}{\mu_1 r_1}\left[e^{-\lambda\tau_0}\int_{\frac{R_1}{r_1}}^{\tau_0}e^{-\theta_1\tau}d\tau\right]$$

$$=h_1-\frac{d_1\theta_1}{\mu_1 r_1}e^{-\lambda\tau_0}\left[\frac{e^{-\theta_1\tau}}{-\theta_1}\right]_{\frac{R_1}{r_1}}^{\tau_0}$$

$$=h_1+\frac{d_1}{\mu_1 r_1}e^{-\lambda\tau_0}\left[e^{-\theta_1\tau_0}-e^{-\theta_1\frac{R_1}{r_1}}\right]$$

$$=h_1+\frac{d_1}{\mu_1 r_1}\left[e^{-(\lambda+\theta_1)\tau_0}-e^{-\theta_1\frac{R_1}{r_1}-\lambda\tau_0}\right]$$

$$I_2=\frac{d_1}{\mu_1}e^{(\theta_2-\theta_1)\tau_0}\,\theta_2\int_{\tau_0}^{\infty}\left(-\frac{1}{r_1}\right)e^{-\theta_2\tau}\left[1-e^{-\lambda\tau_0}\right]d\tau$$

$$=-\frac{d_1\,\theta_2}{\mu_1 r_1}e^{(\theta_2-\theta_1)\tau_0}\left[1-e^{-\lambda\tau_0}\right]\int_{\tau_0}^{\infty}e^{-\theta_2\tau}\,d\tau$$

$$=-\frac{d_1\,\theta_2}{\mu_1 r_1}e^{(\theta_2-\theta_1)\tau_0}\left[1-e^{-\lambda\tau_0}\right]\left[\frac{e^{-\theta_2\tau}}{-\theta_2}\right]_{\tau_0}^{\infty}$$

$$=\frac{d_1}{\mu_1 r_1}e^{(\theta_2-\theta_1)\tau_0}\left[1-e^{-\lambda\tau_0}\right]\left(-e^{-\theta_2\tau_0}\right)$$

$$=-\frac{d_1}{\mu_1 r_1}\left[1-e^{-\lambda\tau_0}\right]\left(e^{-\theta_1\tau_0}\right)$$

$$I_1+I_2=h_1+\frac{d_1}{\mu_1 r_1}\left[-e^{-\theta_1\frac{R_1}{r_1}-\lambda\tau_0}-e^{-\theta_1\tau_0}\right]$$

$$=h_1-\frac{d_1}{\mu_1 r_1}\left[e^{-\theta_1\frac{R_1}{r_1}-\lambda\tau_0}+e^{-\theta_1\tau_0}\right]\tag{1.18}$$

$$I_3=\frac{d_2}{\mu_2}\left[\theta_1\int_{\frac{R_1}{r_1}+\frac{R_2}{r_2}}^{\tau_0}\left(-\frac{1}{r_1}\right)e^{-\theta_1\tau}e^{-\lambda\tau_0}d\tau\right]$$

$$=-\frac{d_2\theta_1}{\mu_2 r_1}e^{-\lambda\tau_0}\left[\int_{\frac{R_1}{r_1}+\frac{R_2}{r_2}}^{\tau_0}e^{-\theta_1\tau}\,d\tau\right]$$

$$= -\frac{d_2\theta_1}{\mu_2 r_1} e^{-\lambda\tau_0}\left[\frac{e^{-\theta_1\tau}}{-\theta_1}\right]_{\frac{R_1}{r_1}+\frac{R_2}{r_2}}^{\tau_0}$$

$$= \frac{d_2}{\mu_2 r_1} e^{-\lambda\tau_0}\left[e^{-\theta_1\tau_0} - e^{-\theta_1\left(\frac{R_1}{r_1}+\frac{R_2}{r_2}\right)}\right]$$

$$I_4 = \frac{d_2}{\mu_2} e^{(\theta_2-\theta_1)\tau_0}\,\theta_2 \int_{\tau_0}^{\infty}\left(-\frac{1}{r_1}\right) e^{-\theta_2\tau}\left[1 - e^{-\lambda\tau_0}\right] d\tau$$

$$= -\frac{d_2\,\theta_2}{\mu_2 r_1} e^{(\theta_2-\theta_1)\tau_0}\left[1 - e^{-\lambda\tau_0}\right]\int_{\tau_0}^{\infty} e^{-\theta_2\tau}\, d\tau$$

$$= -\frac{d_2\,\theta_2}{\mu_2 r_1} e^{(\theta_2-\theta_1)\tau_0}\left[1 - e^{-\lambda\tau_0}\right]\left[\frac{e^{-\theta_2\tau}}{-\theta_2}\right]_{\tau_0}^{\infty}$$

$$= \frac{d_2}{\mu_2 r_1} e^{(\theta_2-\theta_1)\tau_0}\left[1 - e^{-\lambda\tau_0}\right]\left(-e^{-\theta_2\tau_0}\right)$$

$$= -\frac{d_2}{\mu_2 r_1}\left[1 - e^{-\lambda\tau_0}\right]\left(e^{-\theta_1\tau_0}\right)$$

$$I_3 + I_4 = \frac{d_2}{\mu_2 r_1} e^{-\lambda\tau_0}\left[e^{-\theta_1\tau_0} - e^{-\theta_1\left(\frac{R_1}{r_1}+\frac{R_2}{r_2}\right)}\right] - \frac{d_2}{\mu_2 r_1}\left[1 - e^{-\lambda\tau_0}\right]\left(e^{-\theta_1\tau_0}\right)$$

$$= \frac{d_2}{\mu_2 r_1}\left[e^{-(\lambda+\theta_1)\tau_0} - e^{-\lambda\tau_0-\theta_1\left(\frac{R_1}{r_1}+\frac{R_2}{r_2}\right)} - e^{-\theta_1\tau_0} + e^{-(\lambda+\theta_1)\tau_0}\right] \quad (1.19)$$

$$I_5 = \frac{d_3}{\mu_3}\left[\theta_1 \int_{\frac{R_1}{r_1}+\frac{R_2}{r_2}+\frac{R_3}{r_3}}^{\tau_0}\left(-\frac{1}{r_1}\right) e^{-\theta_1\tau} e^{-\lambda\tau_0} d\tau\right]$$

$$= \frac{d_3}{\mu_3}\theta_1\left(-\frac{1}{r_1}\right) e^{-\lambda\tau_0}\left[\int_{\frac{R_1}{r_1}+\frac{R_2}{r_2}+\frac{R_3}{r_3}}^{\tau_0} e^{-\theta_1\tau}\, d\tau\right]$$

$$= \frac{d_3}{\mu_3}\theta_1\left(-\frac{1}{r_1}\right) e^{-\lambda\tau_0}\left[\frac{e^{-\theta_1\tau}}{-\theta_1}\right]_{\frac{R_1}{r_1}+\frac{R_2}{r_2}+\frac{R_3}{r_3}}^{\tau_0}$$

$$= \frac{d_3}{\mu_3 r_1} e^{-\lambda\tau_0}\left[e^{-\theta_1\tau_0} - e^{-\theta_1\left(\frac{R_1}{r_1}+\frac{R_2}{r_2}+\frac{R_3}{r_3}\right)}\right]$$

$$I_6 = \frac{d_3}{\mu_3} e^{(\theta_2-\theta_1)\tau_0}\,\theta_2 \int_{\tau_0}^{\infty}\left(-\frac{1}{r_1}\right) e^{-\theta_2\tau}\left[1 - e^{-\lambda\tau_0}\right] d\tau$$

$$= \frac{d_3}{\mu_3} e^{(\theta_2-\theta_1)\tau_0}\,\theta_2\left(-\frac{1}{r_1}\right)\left[1 - e^{-\lambda\tau_0}\right]\int_{\tau_0}^{\infty} e^{-\theta_2\tau}\, d\tau$$

$$= \frac{d_3}{\mu_3} e^{(\theta_2-\theta_1)\tau_0}\,\theta_2\left(-\frac{1}{r_1}\right)\left[1 - e^{-\lambda\tau_0}\right]\left[\frac{e^{-\theta_2\tau}}{-\theta_2}\right]_{\tau_0}^{\infty}$$

$$= \frac{d_3}{\mu_3 r_1} e^{(\theta_2-\theta_1)\tau_0}\left[1 - e^{-\lambda\tau_0}\right]\left[0 - e^{-\theta_2\tau_0}\right]$$

$$I_5 + I_6 = \frac{d_3}{\mu_3 r_1}\left[e^{-(\lambda+\theta_1)\tau_0} - e^{-\lambda\tau_0-\theta_1\left(\frac{R_1}{r_1}+\frac{R_2}{r_2}+\frac{R_3}{r_3}\right)} - e^{-\theta_1\tau_0} + e^{-(\lambda+\theta_1)\tau_0}\right] \quad (1.20)$$

$$\frac{\partial E(c)}{\partial R_1} =$$

$$h_1 - \frac{d_1}{\mu_1 r_1}\left[e^{-\theta_1\frac{R_1}{r_1}-\lambda\tau_0} + e^{-\theta_1\tau_0}\right] +$$

$$\frac{d_2}{\mu_2 r_1}\left[e^{-(\lambda+\theta_1)\tau_0} - e^{-\lambda\tau_0-\theta_1\left(\frac{R_1}{r_1}+\frac{R_2}{r_2}\right)} \quad e^{-\theta_1\tau_0} + e^{-(\lambda+\theta_1)\tau_0}\right] + \frac{d_3}{\mu_3 r_1}\left[e^{-(\lambda+\theta_1)\tau_0} - e^{-\lambda\tau_0-\theta_1\left(\frac{R_1}{r_1}+\frac{R_2}{r_2}+\frac{R_3}{r_3}\right)} - e^{-\theta_1\tau_0} + e^{-(\lambda+\theta_1)\tau_0}\right] = 0 \tag{1.21}$$

$$\frac{\partial E(c)}{\partial R_2} = h_2 + \frac{d_2}{\mu_2 r_2}\left[e^{-(\lambda+\theta_1)\tau_0} - e^{-\lambda\tau_0-\theta_1\left(\frac{R_1}{r_1}+\frac{R_2}{r_2}\right)} - e^{-\theta_1\tau_0} + e^{-(\lambda+\theta_1)\tau_0}\right] + \frac{d_3}{\mu_3 r_1}\left[e^{-(\lambda+\theta_1)\tau_0} - e^{-\lambda\tau_0-\theta_1\left(\frac{R_1}{r_1}+\frac{R_2}{r_2}+\frac{R_3}{r_3}\right)} - e^{-\theta_1\tau_0} + e^{-(\lambda+\theta_1)\tau_0}\right] = 0 \tag{1.22}$$

$$\frac{\partial E(c)}{\partial R_3} = h_3 + \frac{d_3}{\mu_3 r_1}\left[e^{-(\lambda+\theta_1)\tau_0} - e^{-\lambda\tau_0-\theta_1\left(\frac{R_1}{r_1}+\frac{R_2}{r_2}+\frac{R_3}{r_3}\right)} - e^{-\theta_1\tau_0} + e^{-(\lambda+\theta_1)\tau_0}\right] = 0 \tag{1.23}$$

Solving eqn. (1.21), eqn. (1.22) and eqn. (1.23) we get the eqn. (1.24), eqn. (1.25) and eqn. (1.26) after simplification.

$$\widehat{R}_1 = \frac{\mathrm{Log}\left[-\frac{e^{-\theta_1\left(\frac{R_2}{r_2}+\frac{R_3}{r_3}-\tau_0\right)}\left(d_3 r_2 \mu_1 \mu_2 + e^{\frac{R_3\theta_1}{r_3}}\left(d_2 r_1 \mu_1 + e^{\frac{R_2\theta_1}{r_2}} d_1 r_2 \mu_2\right)\mu_3\right)}{\mu_1\left((-2+e^{\lambda\tau_0})d_3 r_2 \mu_2 + r_1\left((-2+e^{\lambda\tau_0})d_2 - e^{(\lambda+\theta_1)\tau_0} h_1 r_2 \mu_2\right)\mu_3\right)}\right] r_1}{\theta_1} \tag{1.24}$$

$$\widehat{R}_2 = \frac{\mathrm{Log}\left[-\frac{e^{-\theta_1\left(\frac{R_1}{r_1}+\frac{R_3}{r_3}-\tau_0\right)}\left(d_3 r_2 \mu_2 + e^{\frac{R_3\theta_1}{r_3}} d_2 r_1 \mu_3\right)}{(-2+e^{\lambda\tau_0})d_3 r_2 \mu_2 + r_1\left((-2+e^{\lambda\tau_0})d_2 - e^{(\lambda+\theta_1)\tau_0} h_2 r_2 \mu_2\right)\mu_3}\right] r_2}{\theta_1} \tag{1.25}$$

$$\widehat{R}_2 = -\frac{r_3\left(r_2 R_1 \theta_1 + r_1\left(R_2\theta_1 + r_2\left(\mathrm{Log}\left[-e^{-\theta_1\tau_0} + 2e^{-(\lambda+\theta_1)\tau_0} + \frac{h_3 r_1 \mu_3}{d_3}\right] + \lambda\tau_0\right)\right)\right)}{r_1 r_2 \theta_1} \tag{1.26}$$

Given the values of the parameters and the costs the expressions for $\hat{R}_1$, $\hat{R}_2$ and $\hat{R}_3$ can be evaluated numerically using simulation.

5.1 Numerical Illustration :

Using computer routines by fixing $r_1=30$, $r_2=40$, $r_3=50$, $R_2=30$, $R_3=40$, $\mu_1=1$, $\mu_2=2$, $\mu_3=3$, $d_1=200$, $d_2=300$, $d_3=400$, $\theta_1=1$, $\tau_0=1$ and $\lambda=0.5$, we find the optimal values of R_1 as shown in Table 10.

Table 10. The Variation in $\hat{R}_1$ consequent to the changes in h_1

h_1	1.0	1.1	1.2	1.3	1.4	1.5	1.6
$\hat{R}_1$	27	24	22	21	19	18	16

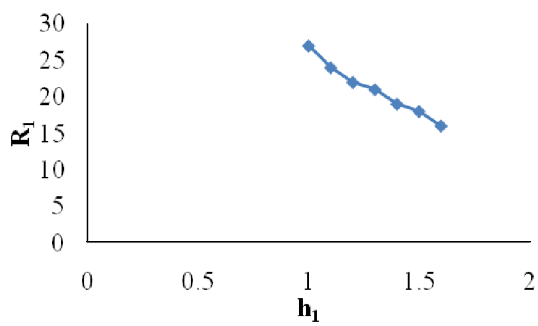


Fig. 10. The Variation in $\hat{R}_1$ consequent to the changes in h_1

Using computer routines by fixing $r_1=30$, $r_2=40$, $r_3=50$, $R_2=415$, $R_3=420$, $\mu_1=1$, $\mu_2=2$, $\mu_3=3$, $h_2=1$, $d_2=300$, $d_3=400$, $\theta_1=1$, $\tau_0=1$ and $\lambda=0.5$, we find the optimal values of R_1 as shown in Table 11.

Table 11. The Variation in $\hat{R}_1$ consequent to the changes in h_1

d_1	100	110	120	130	140	150	160
$\hat{R}_1$	6	9	12	14	16	18	20

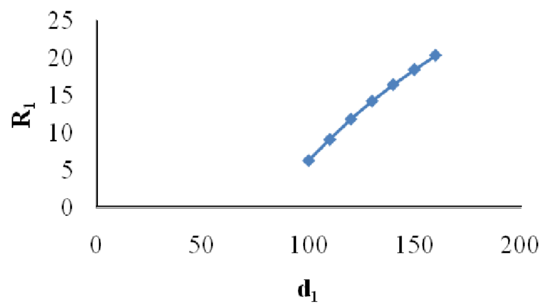


Fig. 11. The Variation in $\hat{R}_1$ consequent to the changes in d_1

Using computer routines by fixing $r_1=10$, $r_2=20$, $r_3=30$, $R_2=100$, $R_3=120$, $\mu_2=3$, $\mu_3=3$, $h_1=1$, $d_1=500$, $d_2=600$, $d_3=700$, $d_3=400$, $\theta_1=1.5$, $\tau_0=2.5$ and $\lambda=0.5$, we find the optimal values of R_1 as shown in Table 12.

Table 12. The Variation in $\hat{R}_1$ consequent to the changes in μ_1

μ_1	1	2	3	4	5	6	7
$\hat{R}_1$	21	16	14	12	10	9	8

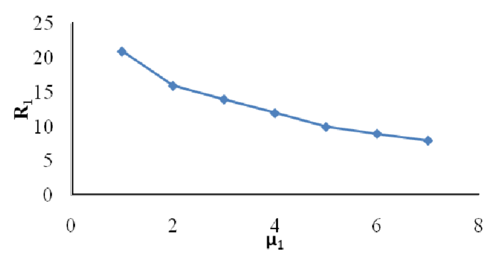


Fig. 12. The Variation in $\hat{R}_1$ consequent to the changes in μ_1

The simultaneous estimation of the reserve of manpower namely $\hat{R}_1$, $\hat{R}_2$ and $\hat{R}_3$ for the fixed values of all the other parameters has been found out and are shown in Table 10, Table 11 and Table 12, Fig. 10, Fig. 11 and Fig. 12. The simultaneous variations in $\hat{R}_1$, consequent to the changes in each one of the parameter keeping the others fixed is also possible.

6. Conclusion

From the above conclusion based on Model I and Model II it can be undershoot that as the cost of manpower reserve between stages S_1, S_2, S_3 and S_4 namely h_1 increases a small reserve R_1 is suggested as an optimal one. If the cost of shortages at the first node namely d_1 increases, it is suggested that a larger manpower at R_1 is desirable. If the cost of manpower reserve at node 2 is higher namely as h_2 increases then a increase in the value of R_2 is suggested. If d_2, the cost of shortages at the second node increases a small reserve namely R_2 is desirable. If μ_1 the parameter of the interarrival times between the breakdowns of S_1 increases then the interarrival times will be shorter since $E(\tau)=1/\mu_1$, decreases. Hence a larger reserve at R_2 is suggested. The simultaneous variations in $\hat{R}_2$ and $\hat{R}_3$ consequent to the changes in each one of the parameter keeping the others fixed is also possible. Based on Model II, The simultaneous estimation of the reserve of manpower namely, $\hat{R}_1$, $\hat{R}_2$ and $\hat{R}_3$ for the fixed values of all the other parameters has been found out. The simultaneous variations in $\hat{R}_1$, $\hat{R}_2$ and $\hat{R}_3$ consequent to the changes in each one of the parameter keeping the others fixed is also possible. The models developed in this paper will be used to facilitate the adequate provision of manpower for the industry at various levels. As such, it should provide an invaluable tool for the economic development of the province. The models developed in this paper also provide a tool for assessing the manpower profile and predicting future manpower development on an industry wide basis.

Suggestions for Future Research :

These are many areas of an organization or industry in which the application of stochastic models is quite necessary. It would be very much useful in every sector of human activity. First of all it is imperative to identify those areas of human activity where the demand for manpower and supply are at disequilibrium. Especially in the area of specialist skill, it becomes necessary to identify where the disequilibrium exists and also were there is interruption in the work schedule due to shortage of manpower. The identification of such areas, the type of problems involved and the conversion of a real life situation into a mathematical model are essential to develop Human Resource Management which will yield profits not only to the management but also to the society itself.

Acknowledgment

The authors very much thankful to Mr. P. Rajesh, Programmer, Engineering Mathematics Section, Faculty of Engineering and Technology, Annamalai University for his valuable help in the computer simulation work.

References

1. Bartholomew, D. J., *The Stochastic Model for Social Processes*. 3rd ed., John Wiley and Sons, New York (1982).
2. Bartholomew, D. J., A. F. Forbes, and S. I. McClean, *Statistical Techniques for Manpower Planning*. Wiley, Chichester (1991).
3. Bartholomew, D.J. and A.F. Forbes, *Statistical Techniques for Manpower Planning*. John Wiley and Sons, Chichester (1979).
4. Elangovan, R., R. Sathiyamoorthi and G. Arivazhagan, *A Stochastic Model for Estimation of time to stop Enrolement for Recruitment*. *Ultra Scientist* Vol. *22*, No. *1(M)*, pp. 277-284 (2010).
5. Grinold, R.C., *Manpower Planning with Uncertain Requirements*, *Operation Research*, Vol. *24*, No. *3*, pp. 387-400 (1976).
6. Susiganeshkumar, E., and R. Elangovan, *Predicting the growth of manpower for the Tamilnadu Software Industry (India)*. *Int. Journal of Physical Sciences*, *Vol. 21(2)M*, pp. 377-392 (2009).
7. Susiganeshkumar, E., and R. Elangovan, *Estimation of Expected Time to Recruitment under Two Sources of Depletion on Manpower*. *Bulletin of Pure and Applied Sciences*, Vol. *31* E, No. *1*, pp. 73-84 (2012).
8. Walker, J.W., *Trends in Manpower Management Research*, *Business Horizons*, pp. 36-46 (1968).