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Unsteady flow of a dusty viscous fluid through porous medium in a rectangular channel with time dependent pressure gradient

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Abstract

The unsteady flow of a dusty viscous incompressible fluid through porous medium in a long rectangular channel under the influence of time dependent pressure gradient has been studied. The solution of governing equations of motion is obtained by the application of Finite Fourier cosine transform and Laplace transform to study the behaviour of the flow of fluid and the dust particles through porous medium. The particular cases when the pressure gradient is (i) an absolute constant, (ii) periodic function of time, (iii) an exponentially decreasing function of time and (iv) $Cte^{-\lambda t}$, have been discussed in detail.

Introduction

The study of fluids having uniform distribution of solid spherical particles is of interest in a wide range of areas of technical importance. These areas include fluidization (flow through packed beds), flow in rocket tubes, where small carbon or metallic fuel particles are present, environmental pollution, the process by which rain drops are formed by the coalescence of small droplets, which might be considered as solid particles for the purpose of examining their movement prior to coalescence, combustion and more recently, blood flow in capillaries.

Considerable work has already been done on such models of dusty fluid flow[13]. Saffman[19] has discussed the stability of laminar flow of a dusty gas. The basic theory of multiphase flow is given by Soo[21]. Michael and Miller[17] and Liu[14] studied the flow produced by the motion of an infinite plate in a dusty gas occupying the semi-infinite space above it. Michael[16] studied the steady motion of a sphere in a dusty gas. Healy[10] proposed a different set of perturbed equations and studied the flow past a cylinder and a flat plate with position normal to the approach flow. Healy and Yang[11] obtained an exact solution for the problem using the technique of Laplace transform. Vimala[26] has discussed the flow of a dusty gas between two oscillating plates. Gupta and Gupta[8] studied the flow of a dusty gas through a channel with an arbitrary time varying pressure gradient. Later on the large number of dusty viscous fluid flow problems have been investigated by many researchers such as: Gupta[9]; Srivastava[24]; Sanyal and Dasgupta[20]; Gireesha and Bagewadi[5,6]; Gireesha, Venkatesh and Bagewadi[7] and Elangovan and Ratchagar[4] *etc*. through different type of channel under the influence of time dependent pressure gradient.

Singh, Lal and Sharma[22]; Bhatnagar and Bhardwaj[3]; Mal and Sengupta[15]; Mishra and Bhola[18]; Varshney and Singh[25]; Kumar, Jha and Shrivastava[12]; Singh, Singh and Jha[23]; Agrawal, Agrawal and Varshney[2] and Agrawal and Singh[1] *etc*. have discussed the unsteady flow of dusty fluid through porous medium in the channels of various cross-sections under the arbitrary time varying pressure gradient.

The aim of present paper is to consider the flow of viscous incompressible fluid with embedded non-conducting small identical spherical particles through porous medium in a long rectangular channel under the influence of a time varying pressure gradient, taking the fluid and dust particles to be initially at rest. The particular cases when (i) the pressure gradient is an absolute constant, (ii) the pressure gradient is a periodic function of time, (iii) the pressure gradient is an exponentially decreasing function of time and (iv) the pressure gradient is, have also been discussed in detail.

Equations of the Problem:

Using the rectangular Cartesian coordinate system, the walls of the channel are taken to be the planes $x = \pm a$ and $y = \pm b$. The fluid and dust particle velocities $u(x,y,t)$ and $v(x,y,t)$ respectively, are in z-direction which is considered along the axis of the channel. Taking the number of density of small non-conducting dust particles to be constant throughout the motion, the appropriate momentum equations of motion of dusty fluid through porous medium are:

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \upsilon\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + \frac{K_0 N_0}{\rho}(v-u) - \frac{\upsilon}{K}u \tag{1}$$

$$\frac{\partial v}{\partial t} = \frac{K_0}{M}(u-v) - \frac{k'}{\rho}\frac{\partial p}{\partial z} \tag{2}$$

where u and v denote the velocities of fluid and dust particle respectively; p is the fluid pressure; M, the mass of a particle; K_0, the Stokes resistance coefficient, which for spherical particle of radius r is $6\pi\mu r$, μ being the viscosity of the fluid; N_0, the number density of the particle; t, the time; ρ, ρ_p and $\overline{\rho}_p$ are density of the fluid, mass density of the particle and material density of the particle respectively; $\upsilon = \mu/\rho$, the kinematics viscosity of the fluid; K, the permeability of porous medium and $k' = \rho/\rho_p$.

Introducing the non-dimensional quantities:

$$x^* = \frac{x}{a}, \quad y^* = \frac{y}{a}, \quad z^* = \frac{z}{a}, \quad p^* = \frac{pa^2}{\rho\upsilon^2},$$

$$t^* = \frac{t\upsilon}{a^2}, \quad u^* = \frac{ua}{\upsilon}, \quad v^* = \frac{va}{\upsilon}, \quad K^* = \frac{K}{a^2}$$

eqns. (1) and (2) become (dropping the stars)

$$\frac{\partial u}{\partial t}=-\frac{\partial p}{\partial z}+\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+\beta(v-u)-\frac{1}{K}u \quad (3)$$

$$\frac{\partial v}{\partial t}=\gamma'(u-v)-k'Q\frac{\partial p}{\partial z} \quad (4)$$

where

$$\beta=\frac{K_0'}{\gamma}=\frac{N_0K_0a^2}{\rho\upsilon},\quad K_0'=\frac{N_0M}{\rho},\ \gamma=\frac{M\upsilon}{K_0a^2},$$

$$\gamma'=\frac{1}{\gamma},\ and\ Q=\frac{\upsilon}{a}$$

Initially, the fluid and particles are at rest. The fluid flow through porous medium takes place under the influence of time dependent pressure gradient with no-slip boundary conditions. From symmetric consideration, the flow in region $x\ge 0, y\ge 0$ is considered. Accordingly, the boundary conditions are:

$$\left.\begin{array}{ll} t>0 & \left.\begin{array}{l} u(1,y,t)=0 \\ v(1,y,t)=0 \end{array}\right\} 0\le y\le c \\ & \dfrac{\partial u}{\partial x}=0,\ \dfrac{\partial v}{\partial x}=0\ \ at\ x=0 \end{array}\right\} \quad (5)$$

and

$$\left.\begin{array}{l} \left.\begin{array}{l} u(x,c,t)=0 \\ v(x,c,t)=0 \end{array}\right\} 0\le x\le 1 \\ \dfrac{\partial u}{\partial y}=0,\ \dfrac{\partial v}{\partial y}=0\ \ at\ y=0 \end{array}\right\} \quad (6)$$

where $c=\dfrac{b}{a}$

Solution of the Problem:

For solving the problem, we choose the finite Fourier cosine transform defined as

$$\bar{u}(m,y,t)=\int_0^1 u(x,y,t)\cos q_m x dx \quad (7)$$

$$\bar{\bar{u}}(x,n,t)=\int_0^c u(x,y,t)\cos q_n y dy \quad (8)$$

where

$$q_m=\frac{2m+1}{2}\pi,\quad q_n=\frac{2n+1}{2c}\pi$$

It can be shown that the inversion formulae for the finite cosine transforms defined by (7) and (8) are given by

$$u(x,y,t)=2\sum_{m=0}^{\infty}\bar{u}(m,y,t)\cos q_m x \quad (9)$$

and

$$u(x,y,t)=\frac{2}{c}\sum_{n=0}^{\infty}\bar{\bar{u}}(x,n,t)\cos q_n y \quad (10)$$

respectively.

Multiplying eqns. (3) and (4) by $\cos q_m x.\cos q_n y$ and then integrating twice within the limits 0 to 1 and 0 to c and using the boundary conditions (5) and (6), we get

$$\frac{\partial U}{\partial t}=\frac{(-1)^{m+n}}{q_mq_n}f(t)-(q_m^2+q_n^2)U+\beta(V-U)-\frac{1}{K}U \quad (11)$$

$$\frac{\partial V}{\partial t}=\gamma'(U-V)-\frac{(-1)^{m+n}k'Q}{q_mq_n}f(t) \quad (12)$$

where

$$U=\int_0^1\int_0^c u(x,y,t)\cos q_m x.\cos q_n y dx dy$$

$$V=\int_0^1\int_0^c v(x,y,t)\cos q_m x.\cos q_n y dx dy$$

and

$$-\frac{\partial p}{\partial z} = f(t)$$

Again, Applying Laplace transforms to eqns. (11) and (12) under the transformed initial conditions:

$$U = 0, \quad V = 0 \quad at \quad t = 0$$

we get

$$s\overline{U} = \frac{(-1)^{m+n}}{q_m q_n}\overline{f}(s) - (q_m^2 + q_n^2)\overline{U} + \beta(\overline{V} - \overline{U}) - \frac{1}{K}\overline{U} \tag{13}$$

$$s\overline{V} = \gamma'(\overline{U} - \overline{V}) - \frac{(-1)^{m+n}k'Q}{q_m q_n}\overline{f}(s) \tag{14}$$

where $\overline{U}, \overline{V}$ and $\overline{f}(s)$ are Laplace transforms of the respective quantities.

Solving eqns. (13) and (14), we get

$$\overline{U} = \frac{(-1)^{m+n}}{q_m q_n}\frac{(s + \gamma' - k'Q\beta)\overline{f}(s)}{(s - \alpha_1)(s - \alpha_2)} \tag{15}$$

$$\overline{V} = \frac{(-1)^{m+n}}{q_m q_n}\left[\frac{\gamma'(s + \gamma' - k'Q\beta)}{(s + \gamma')(s - \alpha_1)(s - \alpha_2)} - \frac{k'Q}{(s + \gamma')}\right]\overline{f}(s) \tag{16}$$

where

$$\alpha_1 = -\frac{1}{2}\left[\left(\gamma' + \beta + \frac{1}{K} + q_m^2 + q_n^2\right) + \left\{\left(\gamma' + \beta + \frac{1}{K} + q_m^2 + q_n^2\right)^2 - 4\gamma'\left(q_m^2 + q_n^2 + \frac{1}{K}\right)\right\}^{1/2}\right]$$

and

$$\alpha_2 = -\frac{1}{2}\left[\left(\gamma' + \beta + \frac{1}{K} + q_m^2 + q_n^2\right) - \left\{\left(\gamma' + \beta + \frac{1}{K} + q_m^2 + q_n^2\right)^2 - 4\gamma'\left(q_m^2 + q_n^2 + \frac{1}{K}\right)\right\}^{1/2}\right]$$

are the roots of the equation.

$$s^2 + \left(\gamma' + q_m^2 + q_n^2 + \beta + \frac{1}{K}\right)s + \gamma'\left(q_m^2 + q_n^2 + \frac{1}{K}\right) = 0$$

Now to obtain u and v we may invert the Laplace transform by convolution theorem and then applying the inversion formulae for the finite cosine transforms, we get

$$u = \frac{4}{c}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-1)^{m+n}}{q_m q_n}\left[\int_0^t f(t - \eta)\left\{A_1 e^{\alpha_1 \eta} + A_2 e^{\alpha_2 \eta}\right\}d\eta\right]\cos q_m x \cos q_n y \tag{17}$$

and

$$v = \frac{4}{c}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-1)^{m+n}}{q_m q_n}\left[\int_0^t f(t - \eta)\left\{\gamma'\left(B_1 e^{\alpha_1 \eta} + B_2 e^{\alpha_2 \eta}\right) + B_3 e^{-\gamma' \eta}\right\}d\eta\right]\cos q_m x \cos q_n y \tag{18}$$

where

$$A_1 = \frac{(\alpha_1 + \gamma' - k'Q\beta)}{(\alpha_1 - \alpha_2)}, \qquad A_2 = -\frac{(\alpha_2 + \gamma' - k'Q\beta)}{(\alpha_1 - \alpha_2)},$$

$$B_1 = \frac{A_1}{(\alpha_1 + \gamma')} \qquad B_2 = \frac{A_2}{(\alpha_2 + \gamma')},$$

$$B_3 = -\left\{1 + \frac{\gamma' \beta}{(\alpha_1 + \gamma')(\alpha_2 + \gamma')}\right\} k' Q$$

Particular Cases:

(i) When the Pressure Gradient is Constant:

Substituting $f(t) = C$ (where C is an absolute constant) in the above equations and on simplifying, we get velocities of the fluid and the dust particles

$$u = \frac{4C}{c} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{(-1)^{m+n}}{q_m q_n} \left[\frac{(\gamma' - k' Q \beta)}{\alpha_1 \alpha_2} + \frac{A_1}{\alpha_1} e^{\alpha_1 t} + \frac{A_2}{\alpha_2} e^{\alpha_2 t} \right] \cos q_m x . \cos q_n y \tag{19}$$

$$v = \frac{4C}{c} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{(-1)^{m+n}}{q_m q_n} \left[\left(\frac{\gamma' - k' Q \beta}{\alpha_1 \alpha_2} - \frac{k' Q}{\gamma'} \right) + \frac{\gamma' B_1}{\alpha_1} e^{\alpha_1 t} + \frac{\gamma' B_2}{\alpha_2} e^{\alpha_2 t} - \frac{k' Q B_3}{\gamma'} e^{-\gamma' t} \right] \cos q_m x . \cos q_n y \tag{20}$$

(ii) When the Pressure Gradient is Periodic Function of Time:

Substituting $f(t) = C \sin \omega t$ (where C and ω are constants) in the above equations and on simplifying, we get velocities of the fluid and dust particles

$$u = \frac{4C}{c} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{(-1)^{m+n}}{q_m q_n} \left[\frac{\omega A_1}{(\alpha_1^2 + \omega^2)} e^{\alpha_1 t} + \frac{\omega A_2}{(\alpha_2^2 + \omega^2)} e^{\alpha_2 t} + \left\{ \frac{\omega^2 + (\gamma' - k' Q \beta)^2}{(\alpha_1^2 + \omega^2)(\alpha_2^2 + \omega^2)} \right\}^{1/2} (\sin(\omega t - \psi_1)) \right] \cos q_m x . \cos q_n y \tag{21}$$

and

$$v = \frac{4C}{c} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{(-1)^{m+n}}{q_m q_n} \left[\frac{\gamma' \omega B_1}{(\alpha_1^2 + \omega^2)} e^{\alpha_1 t} + \frac{\gamma' \omega B_2}{(\alpha_2^2 + \omega^2)} e^{\alpha_2 t} + \frac{\omega B_3}{(\gamma'^2 + \omega^2)} e^{-\gamma' t} + \gamma' \left\{ \frac{\omega^2 + (\gamma' - k' Q \beta)^2}{(\alpha_1^2 + \omega^2)(\alpha_2^2 + \omega^2)(\gamma'^2 + \omega^2)} \right\}^{1/2} \sin(\omega t - \psi_2) - \frac{k' Q}{(\gamma'^2 + \omega^2)^{1/2}} \sin(\omega t - \psi_3) \right] \times \cos q_m x \cos q_n y \tag{22}$$

respectively.

where

$$\psi_1 = \tan^{-1}\left(-\frac{\omega}{\alpha_1}\right) + \tan^{-1}\left(-\frac{\omega}{\alpha_2}\right) - \tan^{-1}\left(-\frac{\omega}{\left(\gamma^{'} - k^{'}Q\beta\right)}\right),$$

$$\psi_2 = \tan^{-1}\left(-\frac{\omega}{\alpha_1}\right) + \tan^{-1}\left(-\frac{\omega}{\alpha_2}\right) + \tan^{-1}\left(\frac{\omega}{\gamma^{'}}\right) - \tan^{-1}\left(-\frac{\omega}{\left(\gamma^{'} - k^{'}Q\beta\right)}\right),$$

$$\psi_3 = \tan^{-1}\left(\frac{\omega}{\gamma^{'}}\right).$$

(iii) When the Pressure Gradient is Exponentially Decreasing Function of Time:

Substituting $f(t) = Ce^{-\lambda t}$ (where C and λ are an constants) in the above equations and on simplifying, we get velocities of the fluid and dust particles

$$u = \frac{4C}{c}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-1)^{m+n}}{q_m q_n}\left[\frac{\gamma^{'} - k^{'}Q\beta - \lambda}{(\alpha_1+\lambda)(\alpha_2+\lambda)}e^{-\lambda t} + \frac{A_1}{(\alpha_1+\lambda)}e^{\alpha_1 t} + \frac{A_2}{(\alpha_2+\lambda)}e^{\alpha_2 t}\right]\cos q_m x.\cos q_n y \tag{23}$$

and

$$v = \frac{4C}{c}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-1)^{m+n}}{q_m q_n}\left[\left\{\frac{\gamma^{'}\left(\gamma^{'} - k^{'}Q\beta - \lambda\right)}{\left(\gamma^{'}-\lambda\right)(\alpha_1+\lambda)(\alpha_2+\lambda)} - \frac{k^{'}Q}{\left(\gamma^{'}-\lambda\right)}\right\}e^{-\lambda t} + \frac{\gamma^{'}B_1}{(\alpha_1+\lambda)}e^{\alpha_1 t}\right.$$
$$\left. + \frac{\gamma^{'}B_2}{(\alpha_2+\lambda)}e^{\alpha_2 t} - \frac{k^{'}QB_3}{\left(\gamma^{'}-\lambda\right)}e^{-\gamma^{'} t}\right]\cos q_m x.\cos q_n y \tag{24}$$

respectively.

(iv) When $f(t) = Cte^{-\lambda t}$

The velocities of the fluid and the dust particles are

$$u = \frac{4C}{c}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-1)^{m+n}}{q_m q_n}\left[\frac{A_1}{(\alpha_1+\lambda)^2}e^{\alpha_1 t} + \frac{A_2}{(\alpha_2+\lambda)^2}e^{\alpha_2 t} + \left\{\frac{\gamma^{'} - k^{'}Q\beta - \lambda}{(\alpha_1+\lambda)(\alpha_2+\lambda)}t\right.\right.$$
$$\left.\left. + \frac{(2\lambda+\alpha_1+\alpha_2)(\gamma^{'} - k^{'}Q\beta) + (\alpha_1\alpha_2 - \lambda^2)}{(\alpha_1+\lambda)^2(\alpha_2+\lambda)^2}\right\}e^{-\lambda t}\right]\cos q_m x.\cos q_n y \tag{25}$$

and

$$v=\frac{4C}{c}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-1)^{m+n}}{q_m q_n}\left[\frac{\gamma' B_1}{(\alpha_1+\lambda)^2}e^{\alpha_1 t}+\frac{\gamma' B_2}{(\alpha_2+\lambda)^2}e^{\alpha_2 t}+\frac{B_3}{(\gamma'-\lambda)^2}e^{-\gamma' t}\right.$$
$$\left.+\left\{\frac{(\gamma'-k'Q\beta-\lambda)\gamma' t}{(\alpha_1+\lambda)(\alpha_2+\lambda)(\gamma'-\lambda)}+\frac{(\gamma'-k'Q\beta)D}{\alpha_1\alpha_2\lambda}-\frac{k'Q\{(\gamma'-\lambda)t-1\}}{(\gamma'-\lambda)}\right\}e^{-\lambda t}\right]\cos q_m x.\cos q_n y \qquad (26)$$

where

$$D=\Big[\big\{(\alpha_1-\alpha_2)(\alpha_1+\gamma')(\alpha_2+\gamma')(\alpha_1+\lambda)^2(\alpha_2+\lambda)^2(\gamma'-\lambda)^2(\gamma'-k'Q\beta)$$
$$+\alpha_2\gamma'\lambda^2(\gamma'-k'Q\beta+\alpha_1)(\alpha_2+\gamma')(\alpha_2+\lambda)^2(\gamma'-\lambda)^2-\alpha_1\gamma'\lambda^2$$
$$\times(\gamma'-k'Q\beta+\alpha_2)(\alpha_1+\gamma')(\alpha_1+\lambda)^2(\gamma'-\lambda)^2+\alpha_1\alpha_2\lambda^2 k'Q\beta$$
$$\times(\alpha_1-\alpha_2)(\alpha_1+\lambda)^2(\alpha_2+\lambda)^2+\alpha_1\alpha_2\gamma'(k'Q\beta-\gamma'+\lambda)(\alpha_1-\alpha_2)$$
$$\times(\alpha_1+\gamma')(\alpha_2+\gamma')(\alpha_1+\lambda)(\alpha_2+\lambda)(\gamma'-\lambda)\big\}:\big\{(\gamma'-k'Q\beta)$$
$$\times(\alpha_1-\alpha_2)(\alpha_1+\gamma')(\alpha_2+\gamma')(\alpha_1+\lambda)^2(\alpha_2+\lambda)^2(\gamma'-\lambda)^2\big\}\Big]$$

Discussion

In the particular case (iv), if we put $\lambda = 0$, the velocities of the fluid and the dust particles can be obtained for a pressure gradient, which is linearly dependent on time.

If $K = \infty$, all the velocity expressions for fluid and dust particles can be obtained in the absence of porous medium under the influence of various pressure gradients and also if $k' = 0$, the results are in agreement with those of Gupta and Gupta[8].

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