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SHORT COMMUNICATION

Temperature in the prism involving I-function of two variables

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Abstract

The aim of this paper is to obtain the temperatures in the prism involving I-function function of two variables.

1. Introduction

The I–function of two variables introduced by Sharma & Mishra[1], will be defined and represented as follows:

$$I\left[\begin{matrix}x\\y\end{matrix}\right] = I^{0,\,n\,:\,m_1,\,n_1\,:\,m_2,\,n_2}_{p_i,\,q_i:\,r\,:\,p_{i'},\,q_{i'}:\,r'\,:\,p_{i''},\,q_{i''}:\,r''}\left[\begin{matrix}x\\y\end{matrix}\right|\begin{matrix}[(a_j;\alpha_j,A_j)_{1,n}],[(a_{ji};\alpha_{ji},A_{ji})_{n+1,p_i}]\\ [(b_{ji};\beta_{ji},B_{ji})_{1,q_i}]\end{matrix}$$

$$\left.\begin{matrix}:[(c_j;\gamma_j)_{1,n_1}],[(c_{ji'};\gamma_{ji'})_{n_1+1,p_{i'}}];[(e_j;E_j)_{1,n_2}],[(e_{ji''};E_{ji''})_{n_2+1,p_{i''}}]\\ :[(d_j;\delta_j)_{1,m_1}],[(d_{ji'};\delta_{ji'})_{m_1+1,q_{i'}}];[(f_j;F_j)_{1,m_2}],[(f_{ji''};F_{ji''})_{m_2+1,q_{i''}}]\end{matrix}\right]$$

$$= \frac{1}{(2\pi\omega)^2}\int_{L_1}\int_{L_2}\phi_1(\xi,\eta)\,\theta_2(\xi)\,\theta_3(\eta)x^{\xi}y^{\eta}\,d\xi\,d\eta, \tag{1}$$

where

$$\phi_1(\xi,\eta)=\frac{\prod_{j=1}^{n}\Gamma(1-a_j+\alpha_j\xi+A_j\eta)}{\sum_{i'=1}^{r}[\prod_{j=n+1}^{p_i}\Gamma(a_{ji}-\alpha_{ji}\xi-A_{ji}\eta)\prod_{j=1}^{q_i}\Gamma(1-b_{ji}+\beta_{ji}\xi+B_{ji}\eta)}$$

$$\theta_2(\xi)=\frac{\prod_{j=1}^{m_1}\Gamma(d_j-\delta_j\xi)\prod_{j=1}^{n_1}\Gamma(1-c_j+\gamma_j\xi)}{\sum_{i'=1}^{r'}[\prod_{j=m_1+1}^{q_{i'}}\Gamma(1-d_{ji'}+\delta_{ji'}\xi)\prod_{j=1n_1+1}^{p_{i'}}\Gamma(c_{ji'}-\gamma_{ji'}\xi)},$$

$$\theta_3(\eta)=\frac{\prod_{j=1}^{m_2}\Gamma(f_j-F_j\eta)\prod_{j=1}^{n_2}\Gamma(1-e_j+E_j\eta)}{\sum_{i''=1}^{r''}[\prod_{j=m_2+1}^{q_{i''}}\Gamma(1-f_{ji''}+F_{ji''}\eta)\prod_{j=1n_2+1}^{p_{i''}}\Gamma(e_{ji''}-E_{ji''}\eta)]},$$

x and y are not equal to zero, and an empty product is interpreted as unity p_i, $p_{i'}$, $p_{i''}$, q_i, $q_{i'}$, $q_{i''}$, n, n_1, n_2, n_j and m_k are non negative integers such that $p_i \geq n \geq 0$, $p_{i'} \geq n_1 \geq 0$, $p_{i''} \geq n_2 \geq 0$, $q_i > 0$, $q_{i'} > 0$, $q_{i''} > 0$, (i = 1, …, r; i´ = 1, …, r´; i´´ = 1, …, r´´; k = 1, 2) also all the A's, α's, B's, β's, γ's, δ's, E's and F's are assumed to be positive quantities for standardization purpose; the definition of I-function of two variables given above will however, have a meaning even if some of these quantities are zero. The contour L_1 is in the ξ–plane and runs from – ω∞ to + ω∞, with loops, if necessary, to ensure that the poles of $\Gamma(d_j-\delta_j\xi)$ (j = 1, ..., m_1) lie to the right, and the poles of $\Gamma(1-c_j+\gamma_j\xi)$ (j = 1, ..., n_1), $\Gamma(1-a_j+\alpha_j\xi+A_j\eta)$ (j = 1, ..., n) to the left of the contour[2,3].

The contour L_2 is in the η–plane and runs from–ω∞ to+ ω∞, with loops, if necessary, to ensure that the poles of $\Gamma(f_j-F_j\eta)$ (j=1,......, n_2) lie to the right, and the poles of $\Gamma(1-e_j+E_j\eta)$ (j = 1, ..., m_2), $\Gamma(1-a_j+\alpha_j\xi+A_j\eta)$ (j = 1, ..., n) to the left of the contour. Also contour[4]. Also

$$R=\sum_{j=1}^{p_i}\alpha_{ji}+\sum_{j=1}^{p_i''}\gamma_{ji}'-\sum_{j=1}^{q_i}\beta_{ji}-\sum_{j=1}^{q_i''}\delta_{ji}'<0,$$

$$S=\sum_{j=1}^{p_i}A_{ji}+\sum_{j=1}^{p_i''}E_{ji}''-\sum_{j=1}^{q_i}B_{ji}-\sum_{j=1}^{q_i''}F_{ji}''<0,$$

$$U=\sum_{j=n+1}^{p_i}\alpha_{ji}-\sum_{j=1}^{q_i}\beta_{ji}+\sum_{j=1}^{m_1}\delta_j-\sum_{j=m_1+1}^{q_i'}\delta_{ji}'+\sum_{j=1}^{n_1}\gamma_j-\sum_{j=n_1+1}^{p_i'}\gamma_{ji}'>0, \tag{2}$$

$$V=-\sum_{j=n+1}^{p_i}A_{ji}-\sum_{j=1}^{q_i}B_{ji}-\sum_{j=1}^{m_2}F_j-\sum_{j=m_2+1}^{q_i''}F_{ji''}+\sum_{j=1}^{n_2}E_j-\sum_{j=n_2+1}^{p_i''}E_{ji''}>0, \tag{3}$$

and | arg x | < ½ Uπ, | arg y | < ½ Vπ. (4)

2. *Result Required:*

The following results are required in our present investigation:

From Gradshteyn [2, p.372]:

$$\int^{L}(\sin\pi x/L)^{\omega-1}\sin n\pi x/L\,dx=\frac{L\sin\tfrac{1}{2}n\pi\,\Gamma(\omega)}{2^{\omega-1}\,\Gamma\{\tfrac{1}{2}(\omega\pm n+1)\}}, \tag{5}$$

where n is any integer and ω > 0.

3. *Formulation of the Problem:*

All four faces of an infinitely long rectangular prism, formed by the planes x = 0, x = a, y = 0 and y = b, are kept at temperature zero. Let the initial temperature distribution be f(x, y), and derive this expression for the temperature u(x, y, t) in the prism is given by [3, p.131] as follows:

$$u(x,y,t)=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}B_{mn}\exp\left[-\pi^2kt\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)\right]\sin\frac{n\pi x}{a}\sin\frac{m\pi y}{b}, \tag{6}$$

where

$$B_{mn}=\frac{4}{ab}\int_0^b\int_0^a f(x,y)\,\sin\frac{m\pi y}{b}\sin\frac{n\pi x}{a}\,dxdy. \tag{7}$$

4. Solution in terms of I-function:

Consider

$$f(x, y) = (\sin\frac{n\pi x}{a})^{\omega-1}(\sin\frac{m\pi y}{b})^{\delta-1} I\left[{}^{\xi(\sin\frac{n\pi x}{a})^{\lambda}(\sin\frac{m\pi y}{b})^{\mu}}_{\eta}\right], \tag{8}$$

where $I[{}^{\xi}_{\eta}]$ is the I-function of two variables, defined in (1).

Combining (8) and (7), making use of the definition of I-function of two variables as given in (1), changing the order of integration, after using the integral (5), we arrive at

$$B_{mn} = 2^{4-\omega-\delta} \sin\frac{m\pi}{2} \sin\frac{n\pi}{2} I^{0,n_1;m_2,n_2+2;m_3,n_3}_{p_i,q_i:r;p_i'+2,q_i'+4:r';p_i'',q_i'':r''}$$

$$\left[{}^{\xi 2^{-\lambda-\mu}}_{\eta}\Big|{}^{\ldots\ldots,\ldots\ldots:(1-\omega,\lambda),(1-\delta,\mu),\ldots\ldots:\ldots\ldots,\ldots\ldots}_{\ldots\ldots,\ldots\ldots:\ldots\ldots,(1/2-\omega/2\pm n/2,\lambda/2)\,(\frac{1}{2}-\frac{\delta}{2}\pm\frac{m}{2},\frac{\mu}{2}):\ldots\ldots,\ldots\ldots}\right], \tag{9}$$

provided that $\lambda \geq 0, \mu \geq 0, \mathrm{Re}(\omega) > 0, Re(\delta) > 0$ and $U > 0, V > 0, |\arg\xi| < \frac{1}{2}U\pi$, where U and V are given in (2) and (3).

Putting the value of B_{mn} from (9) in (6), we get following required solution of the problem:

$$u(x,y,t) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} 2^{4-\omega-\delta} \exp\left[-\pi^2 kt\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)\right]$$

$$\sin\frac{n\pi x}{a}\sin\frac{m\pi y}{b} I^{0,n_1;m_2,n_2+2;m_3,n_3}_{p_i,q_i:r;p_i'+2,q_i'+4:r';p_i'',q_i'':r''}$$

$$\left[{}^{\xi 2^{-\lambda-\mu}}_{\eta}\Big|{}^{\ldots\ldots,\ldots\ldots:(1-\omega,\lambda),(1-\delta,\mu),\ldots\ldots:\ldots\ldots,\ldots\ldots}_{\ldots\ldots,\ldots\ldots:\ldots\ldots,(1/2-\omega/2\pm n/2,\lambda/2)\,(\frac{1}{2}-\frac{\delta}{2}\pm\frac{m}{2},\frac{\mu}{2}):\ldots\ldots,\ldots\ldots}\right],$$

$$\sin\frac{m\pi}{2}\sin\frac{n\pi}{2}. \tag{10}$$

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