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Bianchi type III string cosmological model with bulk viscosity and with out time depending Λ term

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Abstract

We have investigated Bianchi type III string cosmological model with bulk viscosity and without time depending Λ term. To obtain a determinate solution, it is assumed that coefficient of bulk viscosity is proportional to shear scalar $\xi \propto \theta$. The physical and geometrical features are also discussed.

Key words: Bianchi type III, Cosmic string, Bulk viscosity, space time.

1 Introduction

The expansion of the Universe started many billions of years ago from a very hot, very small state. From that hot, small state, it evolved into the Universe today. The Big Bang is extrapolating between knowledge of particle physics today and projections from the mathematical model of an expanding universe in general relativity. The Einstein equations give us a mathematical model for describing how fast the Universe would expanding at what size and time, given the energy density of matter and radiation at that time. In recent years there has been considerable interest in string cosmology.

Cosmic string might be found during the phase transition in early universe[7,8]. The general cosmic strings initiated by Letelier [9,10] and Satchel[14]. The string cosmology with magnetic field are discussed by Chakraborty[5] Tikekar and Patel[16], Maharaj[11] Patel and Maharaj[12]. Bianchi types I and IX string cosmological model in general relativity is discussed by Bali *et.al.*[1-4]. Pradhan and Panday[13] has shown that Bianchi type I models with Bulk viscosity in cosmological theory based on lyra geometry. Wang[18-21] has discussed Bianchi type I and III cosmological model for a cloud string with bulk viscosity. Recently Vachana Singh[17] investigated Bianchi III Universe with viscous fluid. Adahav, Ugale, Kale and Bhende (2007) discussed Bianchi type III Anisotropic Cosmological models with varying Λ. Soni and Shrimali[15] investigated Shear free Bianchi Type III string cosmological model with bulk viscosity and time dependent Λ term.

2 *Field equation :*

Bianchi III consmological model as a gravitational field, given by metric

$$ds^2 = -dt^2 + A^2dx^2 + B^2(e^{-2\alpha x}dy^2 + dz^2) \quad (1)$$

Where A and B are function of t and α is constant.

The influence of the viscous fluid in the evolution of the universe is perform by means of its energy momentum tensor, which acts as the source of gravitational field. The energy momentum tensor for a cloud of string dust with a bulk viscous fluid of string given by Letlier [].

$$T^{\nu}_{\mu} = \rho u_{\mu}u^{\nu} - \lambda x_{\mu}x^{\nu} - \xi u^{l};\ l(g^{\nu}_{\mu} + u_{\mu}u^{\nu}) \quad (2)$$

Where

$$u^{\mu}u_{\mu} = -x^{\mu}x_{\mu} = -1, u^{\mu}x_{\mu} = 0 \quad (3)$$

ρ is proper energy density for a cloud string with particle attached to them, λ is string tensor density, u^{μ} is velocity of particle, x^{μ} is unit space like vector representing the direction of string. If particle density of configration is denoted by ρ_p then we have

$$\rho = \rho_p + \lambda \quad (4)$$

The Einstein's field equation without time dependent cosmological constant ∧

$$G^{\nu}_{\mu} = R^{\nu}_{\mu} - \frac{1}{2}\delta^{\nu}_{\mu}R = -8\pi G T^{\nu}_{\mu} \quad (5)$$

Where R^{ν}_{μ} is Ricci tensor and $R = g^{\mu\nu}R_{\mu\nu}$ is the Ricci scalar.

For the line element (1) and the field equation (5) can be written as

$$2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} = 8\pi G\xi\theta \quad (6)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = 8\pi G\xi\theta \quad (7)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{\alpha^2}{A^2} = 8\pi G(\lambda + \xi\theta) \quad (8)$$

$$2\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} - \frac{\alpha^2}{A^2} = 8\pi G\rho \quad (9)$$

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = 0 \quad (10)$$

Where dot represents ordinary diferentiation with respect to *t*.

The particle density ρ_p is given by

$$8\pi\rho_p = \frac{\dot{B}^2}{B^2} + 2\frac{\dot{A}\dot{B}}{AB} \quad (11)$$

The scalar expansion θ and shear σ is given by

$$\theta = \frac{\dot{A}}{A} + 2\frac{\dot{B}}{B} \tag{12}$$

$$\sigma^2 = \frac{1}{3}\left[\frac{\dot{A}^2}{A^2} + \frac{\dot{B}^2}{B^2} - 2\frac{\dot{A}\dot{B}}{AB}\right] \tag{13}$$

From equation (10), we have

$$A=\mu B \tag{14}$$

Where μ is constant of integration.

From equation (14), without loss of gravity we have to take

μ=1 so that,

$$A=B \tag{15}$$

In order to obtain the more general solution, we assume

$$\xi = k\theta \tag{16}$$

$$G = a/H \tag{17}$$

Where a and k are the positive constant and H is Hubble parameter, define by

$$\theta = 3H \tag{18}$$

Substituing equation (15) in (12), we have

$$\theta = 3\frac{\dot{B}}{B} \tag{19}$$

By using equation (16), (17), (18) and(19) in equation(6), we get

$$2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} = 8\pi G\xi\theta = 72\pi ak\frac{\dot{B}}{B} \tag{20}$$

$$2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} = l\frac{\dot{B}}{B} \tag{21}$$

Where

$$l = 72\pi ak \tag{22}$$

Multipling $\frac{B}{\dot{B}}$ in equation (21), we have

$$\frac{\ddot{B}}{\dot{B}} = \frac{1}{2}\left(l - \frac{\dot{B}}{B}\right) \tag{23}$$

Integrating equation (23), we get

$$\dot{B} = m_1 e^{\frac{lt}{2}} B^{-\frac{1}{2}} \tag{24}$$

Again integrating , we obtain

$$B = \left[\frac{3}{2}\left(\tfrac{2}{l}e^{\frac{lt}{2}}m_1 + m_2\right)\right]^{\frac{2}{3}} \tag{25}$$

Where m_1 and m_2 are constant of integration. Thus

$$A = \left[\frac{3}{2}\left(\tfrac{2}{l}e^{\frac{lt}{2}}m_1 + m_2\right)\right]^{\frac{2}{3}} \tag{26}$$

Therefore equation (1) reduced to

$$ds^2 = -dt^2 + \left[\frac{3}{2}\left(\tfrac{2}{l}e^{\frac{lt}{2}}m_1 + m_2\right)\right]^{\frac{4}{3}} (dx^2 + e^{-2\alpha x}dy^2 + dz^2) \tag{27}$$

$$\Rightarrow \frac{\dot{B}}{B} = m_1 e^{\frac{lt}{2}}\left[\frac{3}{2}\left(\tfrac{2}{l}e^{\frac{lt}{2}}m_1 + m_2\right)\right]^{-1} \tag{28}$$

For the model of equation (27), the other physical and geometrical parameter. The expressions for the energy density ρ, the string tension density λ, the cofficient of bulk viscosity ξ, the scalar expansion θ and the shear scalar σ^2 are

$$\rho = 3m^2{}_1 e^{lt}\left[\frac{3}{2}\left(\tfrac{2}{l}e^{\frac{lt}{2}}m_1 + m_2\right)\right]^{-2} \qquad (29)$$

$$\lambda = -\alpha^2\left[\frac{3}{2}\left(\tfrac{2}{l}e^{\frac{lt}{2}}m_1 + m_2\right)\right]^{-\frac{4}{3}} \qquad (30)$$

$$\xi = 3km_1 e^{\frac{lt}{2}}\left[\frac{3}{2}\left(\tfrac{2}{l}e^{\frac{lt}{2}}m_1 + m_2\right)\right]^{-1} \qquad (31)$$

$$\theta = 3m_1 e^{\frac{lt}{2}}\left[\frac{3}{2}\left(\tfrac{2}{l}e^{\frac{lt}{2}}m_1 + m_2\right)\right]^{-1} \qquad (32)$$

$$\sigma^2 = 0$$
$$\Rightarrow \sigma = 0 \qquad (33)$$

3 Conclusion

It is seen that $\rho\to\infty$ when $t\to\infty$, but scalar expasion θ tends to finite and ρ tends to finite when $t\to 0$ due to presence of bulk viscosity. Hence model represets the shearing and non rotating expanding universe with the big bang start. Since $\lim_{t\to\infty}\frac{\sigma}{\theta} = 0$, the model approches isotropy for large value of t.

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