

Analysis of Joule heating and Radiation on Hydromagnetic Peristaltic flow with porous medium through a coaxial asymmetric Vertical Inclined Tapered Channel

Sk. ABZAL^{*1}, S.VIJAYA KUMAR VARMA² and S. RAVI KUMAR³

¹Department of Mathematics, NBKR Institute of Science and Technology (Autonomous),
Vidyanagar, SPSR Nellore, Andhra Pradesh (India)

²Department of Mathematics, Sri Venkateswara University, Tirupati,
Andhra Pradesh (India)

³Department of Mathematics NBKR Institute of Science and Technology (Autonomous),
Vidyanagar, SPSR Nellore, Andhra Pradesh, India.
Corresponding author email: abbuoct23@gmail.com

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Abstract

The aim of the present attempt was to investigate an analysis of joule heating and radiation on hydromagnetic peristaltic flow with porous medium through a coaxial asymmetric vertical inclined tapered channel. The Mathematical modeling is investigated by utilizing long wavelength and low Reynolds number assumptions. Analytical solutions are obtained for the axial velocity, pressure gradient, temperature and heat transfer coefficient. The effects of Hartmann number, porosity parameter, volumetric flow rate, radiation parameter, non-uniform parameter, gravitational parameter, shift angle, Prandtl number, Brinkman number, heat source/sink parameter on axial velocity characteristics, pressure gradient characteristics, temperature characteristics and heat transfer coefficient characteristics are presented graphically and discussed in detail.

Key words: Joule heating, radiation parameter, inclined channel, tapered channel, MHD.

1. Introduction

The peristaltic flow of a fluid is very

important for medical diagnosis and it has many clinical applications. These applications are swallowing of food bolus through the esophagus,

the urine transport from a kidney to the bladder, the movement of chyme in the tract, the transport of lymph in the lymphatic vessels and the vasomotion of small blood vessel. The flow of non-Newtonian fluids also has benefit to the field environmental engineering, chemical and biomedical. Furthermore, the peristaltic pump is found in many applications of medicine, engineering and water waste. That is dialysis machine, a heart-lung machine, infusion pump, concrete pump, sewage sludge and etc. Magnetohydrodynamics deals with the study of motion of an electrically conducting fluid in presence of an applied magnetic field. When a conducting fluid moves through a magnetic field, electric currents are induced in the fluid. The magnetic field exerts a force known as the Lorentz force as a result the flow field is modified. Solid or fluid material moving in a magnetic field experiences an electromagnetic force. If the material is electrically conducting and a current path is available, electric current ensue. Alternatively, currents may be induced by change of the magnetic field with time and also study of flow through porous medium has received much attention in recent years because of its application in bio-physical and hydrological problem, industrial, particularly in nuclear, petroleum and chemical industries. The role played by porous medium in the study of the flow of blood and other fluids and electro-osmosis. This study is also useful to understand the mechanism of transfer heat from the deep interior of the earth to a shallow depth in the geothermal regions which are of vital importance in the present day grave power crisis. The study of behavior of fluid saturated porous media is known as Poromechanics.

LATHAM¹ made initial effort

regarding peristaltic mechanism of viscous fluids. The primary mathematical models of peristalsis obtained by a train of sinusoidal waves in an infinitely long symmetric channel or tube were introduced by Fung and Yih² and Shapiro *et al.*³. Peristaltic flow of a Newtonian fluid in an asymmetric channel was described by Mishra and Ramachandra Rao⁴. In another attempt, Influence of convective conditions in radiative peristaltic flow of pseudoplastic nanofluid in a tapered asymmetric channel has discussed by T. Hayat *et al.*⁵. F. M. Abbasi *et al.*⁶ presented a theoretical study on Numerical Analysis for Peristaltic Motion of MHD Eyring-Prandtl Fluid in an Inclined Symmetric Channel with Inclined Magnetic Field. Some pertinent studies on the present topic can be found from the list of Refs. Such as Kh. S. Mekheimer⁷, J.C. Misra, S. Maiti, G.C. Shit⁸, M.H. Haroun⁹, N.S. Akbar *et al.*¹⁰, B.C. Sarkar *et al.*¹¹, Ravikumar¹²⁻¹⁴.

In general heat transfer, play a vital role in MHD flows. When a fluid is at a different temperature from that of its surroundings, the thermal energy transfers from high-temperature region to low temperature region until the fluid and the surroundings attain thermal equilibrium. This is called heat transfer or heat exchange. Apart from this, there are many industrial, chemical engineering processes, Automotive Engineering, Thermal Insulations, Thermal Engineering of electronic devices and system, Material Processing, Power Plant Engineering, Bio-heat Transfer, Aerospace Technology, *etc.* Kabir, K. H. *et al.*¹⁵ presented a theoretical study on effects of stress work on MHD natural convection flow along a vertical wavy surface with Joule heating. T. Hayat *et al.*¹⁶

have conferred the characteristics of convective heat transfer in the MHD peristalsis of Carreau fluid with Joule heating. F.M. Abbasi *et al.*¹⁷ discussed on effects of inclined magnetic field and Joule heating in mixed convective peristaltic transport of non-Newtonian fluids. Recently, Ravi Kumar¹⁸ investigated on Analysis of Heat Transfer on MHD Peristaltic Blood Flow with Porous Medium through Coaxial Vertical Tapered Asymmetric Channel with Radiation – Blood Flow Study. Some more works on this topic can be seen in (S. Nadeem and Noreen Sher Akbar¹⁹, T. Hayat *et al.*²⁰, K. Vajravelu *et al.*²¹, S. Srinivas and M. Kothandapani²², T. Hayat *et al.*²³, .O. U. Mehmood *et al.*²⁴, Musharafa Saleem and Aun Haider²⁵, G. Radhakrishnamacharya, Ch. Srinivasulu²⁶, K. Vajravelu *et al.*²⁷, K. Ramesh, M. Devakar²⁸, M Kothandapani *et al.*²⁹ and Sk Abzal *et al.*³⁰).

2. Formulation of the problem :

The model simulates the peristaltic transport of a viscous fluid through an infinite two-dimensional asymmetric vertical tapered channel through the porous medium. Asymmetry in the flow is due to the propagation of peristaltic waves of different amplitudes and phase on the channel walls. We assume that the fluid is subject to a constant transverse magnetic field B_0 . The flow is generated by sinusoidal wave trains propagating with steady speed c along the tapered asymmetric channel walls³¹.

The geometry of the wall surface is defined as

$$Y = H_2 = b + m'X + d \sin \left[\frac{2\pi}{\lambda}(X - ct) \right] \quad (2.1)$$

$$Y = H_1 = -b - m'X - d \sin \left[\frac{2\pi}{\lambda}(X - ct) + \phi \right] \quad (2.2)$$

Where b is the half-width of the channel, d is the wave amplitude, c is the phase speed of the wave and m' ($m' \ll 1$) is the non-uniform parameter, λ is the wavelength, t is the time and X is the direction of wave propagation. The phase difference ϕ varies in the range $0 \leq \phi \leq \pi$, $\phi = 0$ corresponds to symmetric channel with waves out of phase and further b , d and ϕ satisfy the following conditions for the

divergent channel at the inlet $d \cos(\frac{\phi}{2}) \leq b$

It is assumed that the left wall of the channel is maintained at temperature T_0 while the right wall has temperature T_1 .

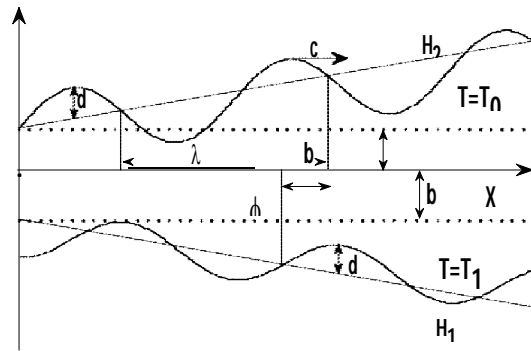


Fig. 1 Schematic diagram of the physical model

The equations governing the motion for the present problem prescribed by

The continuity equation is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (2.3)$$

The movement equations are

$$\rho \left[u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right] = - \frac{\partial p}{\partial x} + \mu \left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right] - \left[\sigma B_0^2 \right] (u + c) - \left[\frac{\mu}{k_1} \right] (u + c) + \rho g \sin \alpha \quad (2.4)$$

$$\rho \left[u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right] = - \frac{\partial p}{\partial y} + \mu \left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right] - \left[\sigma B_0^2 \right] v - \left[\frac{\mu}{k_1} \right] v - \rho g \cos \alpha \quad (2.5)$$

The energy equation is

$$\rho C_p \left[u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right] = k \left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right] + Q_0 + \sigma B_0^2 u^2 - \frac{\partial q}{\partial y} \quad (2.6)$$

u and v are the velocity components in the corresponding coordinates, k_1 is the permeability of the porous medium, ρ is the density of the fluid, p is the fluid pressure, k is the thermal conductivity, μ is the coefficient of the viscosity, Q_0 is the constant heat addition/absorption, C_p is the specific heat at constant pressure, σ is the electrical conductivity, g is the acceleration due to gravity and T is the temperature of the fluid.

The relative boundary conditions are

$$\bar{U} = 0, \quad \bar{T} = T_0 \quad \text{at} \quad \bar{Y} = \bar{H}_1$$

$$\bar{U} = 0, \quad \bar{T} = T_1 \quad \text{at} \quad \bar{Y} = \bar{H}_2$$

The radioactive heat flux (Cogley *et al.*³¹) is given by

$$\frac{\partial q}{\partial y} = 4\alpha^2 (T_0 - T_1) \quad (2.7)$$

here α is the mean radiation absorption coefficient.

Introducing a wave frame (x, y) moving with velocity c away from the fixed frame (X, Y) by the transformation

$$x = X - ct, \quad y = Y, \quad u = U - c, \quad v = V \quad \text{and} \quad p(x) = P(X, t) \quad (2.8)$$

Introducing the following non-dimensional quantities:

$$\bar{x} = \frac{x}{\lambda}, \quad \bar{y} = \frac{y}{b}, \quad \bar{t} = \frac{ct}{\lambda}, \quad \bar{u} = \frac{u}{c}, \quad \bar{v} = \frac{v}{c\delta}$$

$$h_1 = \frac{H_1}{b}, \quad h_2 = \frac{H_2}{b}, \quad p = \frac{H_2}{b} p = \frac{b^2 p}{c \lambda \mu} \theta = \frac{T - T_0}{T_1 - T_0} \delta = \frac{b}{\lambda}$$

$$\text{Re} = \frac{\rho c b}{\mu} N^2 = \frac{4\alpha^2 d_1^2}{k} \varepsilon = \frac{d}{b} \eta = \frac{\rho a_0^2 g}{\mu c}$$

$$\eta_1 = \frac{\rho a_0^3 g}{\lambda \mu c} \quad (2.9)$$

where $\varepsilon = \frac{d}{b}$ is the non-dimensional amplitude

of channel $\delta = \frac{b}{\lambda}$, is the wave number,

$k_1 = \frac{\lambda m'}{b}$ is the non-uniform parameter,

Re is the Reynolds number, M is the Hartman number $K = \frac{k}{b^2}$, Permeability parameter, Pr

is the Prandtl number, E_c is the Eckert number, γ is the heat source/sink parameter, $B_r (= E_c \text{Pr})$ is the Brinkman number, η and η_1 are gravitational parameters and N^2 is the radiation parameter.

3. Solution of the problem :

In view of the above transformations (2.8) and non-dimensional variables (2.9), equations

(2.3-2.6) are reduced to the following non-dimensional form after dropping the bars

$$\text{Re} \delta \left[u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right] = \left(-\frac{\partial p}{\partial x} + \delta^2 \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} - Au - A + \eta \sin \alpha \right) \quad (3.1)$$

$$\text{Re} \delta^3 \left[u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right] = \left(-\frac{\partial p}{\partial y} + \delta^4 \frac{\partial^2 v}{\partial x^2} + \delta^2 \frac{\partial^2 v}{\partial y^2} - M^2 \delta^2 v - \delta^2 \frac{1}{Da} v - \eta_1 \cos \alpha \right) \quad (3.2)$$

$$\text{Re} \left[\delta u \frac{\partial \theta}{\partial x} + v \delta \frac{\partial \theta}{\partial y} \right] = \frac{1}{\text{Pr}} \left[\delta^2 \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right] + \beta +$$

$$M^2 E u^2 + \frac{N^2 \theta}{P_r} \quad (3.3)$$

Applying long wave length approximation and neglecting the wave number along with low-Reynolds numbers. Equations (10-12) become

$$\frac{\partial^2 u}{\partial y^2} - Au = \frac{\partial p}{\partial x} + A - \eta \sin \alpha \quad (3.4)$$

$$\frac{\partial p}{\partial y} = 0 \quad (3.5)$$

$$\frac{1}{\text{Pr}} \left[\frac{\partial^2 \theta}{\partial y^2} \right] + \beta + M^2 E u^2 + \frac{N^2 \theta}{P_r} = 0 \quad (3.6)$$

$$\text{Where } A = \left(M^2 + \frac{1}{Da} \right)$$

The relative boundary conditions in dimensionless form are given by

$$u = -1, \quad \theta = 0 \quad \text{at } y = h_1 \quad (3.7)$$

$$u = -1, \quad \theta = 1 \quad \text{at } y = h_2 \quad (3.8)$$

Where

$$h_1 = -1 - k_1 x - \varepsilon \sin[2\pi(x-t) + \phi]$$

$$h_2 = 1 + k_1 x + \varepsilon \sin[2\pi(x-t)]$$

Where β is the non-dimensional slip velocity parameter.

The solutions of velocity and temperature with subject to boundary conditions (3.7) and (3.8) are given by

$$u = f_1 \text{Sin} h[\alpha y] + f_2 \text{Cos} h[\alpha y] + F \quad (3.9)$$

Where

$$\alpha = \sqrt{\left(M^2 + \frac{1}{Da} \right)}$$

$$F = \frac{-\frac{\partial p}{\partial x}}{M^2 + \frac{1}{Da}} - 1 + \frac{\eta \text{Sin} \alpha}{M^2 + \frac{1}{Da}}$$

$$f_1 = \left(\frac{(-1-F)}{\left(\frac{\text{Cosh}[\alpha h_2] - \text{Cosh}[\alpha h_1]}{\text{Sinh}[\alpha h_1] - \text{Sinh}[\alpha h_2]} \right) \text{Sinh}[\alpha h_1] + \text{Cosh}[\alpha h_1]} \right)$$

$$\left(\frac{\text{Cosh}[\alpha h_2] - \text{Cosh}[\alpha h_1]}{\text{Sinh}[\alpha h_1] - \text{Sinh}[\alpha h_2]} \right)$$

$$f_2 = \left(\frac{(-1-F)}{\left(\frac{\text{Cosh}[\alpha h_2] - \text{Cosh}[\alpha h_1]}{\text{Sinh}[\alpha h_1] - \text{Sinh}[\alpha h_2]} \right) \text{Sinh}[\alpha h_1] + \text{Cosh}[\alpha h_1]} \right)$$

$$\theta = f_9 \text{Cos}[N y] + f_8 \text{Sin}[N y] - \left(\frac{P_r \gamma}{N^2} \right) - \left(\frac{M^2 B_r f_3}{4\alpha^2 + N^2} e^{2\alpha y} \right)$$

$$- \left(\frac{M^2 B_r f_4}{4\alpha^2 + N^2} e^{-2\alpha y} \right) - \left(\frac{M^2 B_r f_5}{\alpha^2 + N^2} e^{\alpha y} \right)$$

$$-\left(\frac{M^2 B_r f_6}{\alpha^2 + N^2} e^{-\alpha y}\right) - \left(\frac{M^2 B_r f_7}{N^2}\right) \quad (3.10)$$

Where

$$f_3 = \left(\frac{f_1^2}{4} + \frac{f_2^2}{4} + \frac{f_1 f_2}{2}\right)$$

$$f_4 = \left(\frac{f_1^2}{4} + \frac{f_2^2}{4} - \frac{f_1 f_2}{2}\right) \quad f_5 = F(f_1 + f_2)$$

$$f_6 = F(f_2 - f_1) \quad f_7 = \left(\frac{-f_1^2}{2} + \frac{f_2^2}{2} + F^2\right)$$

$$f_8 = \left(\frac{-\cos[N h_1]}{\sin[N h_1] \cos[N h_2] - \sin[N h_2] \cos[N h_1]}\right) -$$

$$\left(\frac{\left(\frac{P_r \gamma}{N^2}\right)(\cos[N h_1] - \cos[N h_2])}{\sin[N h_1] \cos[N h_2] - \sin[N h_2] \cos[N h_1]}\right) -$$

$$\left(\frac{\frac{M^2 B_r f_3}{4 \alpha^2 + N^2} (e^{2 \alpha h_2} \cos[N h_1] - e^{2 \alpha h_1} \cos[N h_2])}{\sin[N h_1] \cos[N h_2] - \sin[N h_2] \cos[N h_1]}\right) -$$

$$\left(\frac{\frac{M^2 B_r f_4}{4 \alpha^2 + N^2} (e^{-2 \alpha h_2} \cos[N h_1] - e^{-2 \alpha h_1} \cos[N h_2])}{\sin[N h_1] \cos[N h_2] - \sin[N h_2] \cos[N h_1]}\right) -$$

$$\left(\frac{\frac{M^2 B_r f_5}{\alpha^2 + N^2} (e^{\alpha h_2} \cos[N h_1] - e^{\alpha h_1} \cos[N h_2])}{\sin[N h_1] \cos[N h_2] - \sin[N h_2] \cos[N h_1]}\right) -$$

$$\left(\frac{\frac{M^2 B_r f_6}{\alpha^2 + N^2} (e^{-\alpha h_2} \cos[N h_1] - e^{-\alpha h_1} \cos[N h_2])}{\sin[N h_1] \cos[N h_2] - \sin[N h_2] \cos[N h_1]}\right) -$$

$$\left(\frac{\frac{M^2 B_r f_7}{N^2} (\cos[N h_1] - \cos[N h_2])}{\sin[N h_1] \cos[N h_2] - \sin[N h_2] \cos[N h_1]}\right)$$

$$f_9 = \left(\frac{-f_8 \sin[N h_1] + \left(\frac{P_r \gamma}{N^2}\right) + \frac{M^2 B_r f_3}{4 \alpha^2 + N^2} (e^{2 \alpha h_1})}{\cos[N h_1]}\right) +$$

$$\left(\frac{\frac{M^2 B_r f_4}{4 \alpha^2 + N^2} (e^{-2 \alpha h_1}) + \frac{M^2 B_r f_5}{\alpha^2 + N^2} (e^{\alpha h_1})}{\cos[N h_1]}\right) +$$

$$\left(\frac{\frac{M^2 B_r f_6}{\alpha^2 + N^2} (e^{-\alpha h_1}) + \frac{M^2 B_r f_7}{N^2}}{\cos[N h_1]}\right)$$

The coefficients of the heat transfer Zh_1 and Zh_2 at the walls $y = h_1$ and $y = h_2$ respectively, are given by

$$Zh_1 = \theta_y h_{1x} \quad (3.11)$$

$$Zh_2 = \theta_y h_{2x} \quad (3.12)$$

The solutions of the coefficient of heat transfer at $y = h_1$ and $y = h_2$ are given by

$$Zh_1 = \theta_y h_{1x} =$$

$$\left(-N f_9 \sin[N y] + N f_8 \cos[N y] - \left(\frac{2 \alpha M^2 B_r f_3}{4 \alpha^2 + N^2} e^{2 \alpha y}\right) - \left(\frac{-2 \alpha M^2 B_r f_4}{4 \alpha^2 + N^2} e^{-2 \alpha y}\right) - \left(\frac{\alpha M^2 B_r f_5}{\alpha^2 + N^2} e^{\alpha y}\right) + \right.$$

$$\left(\frac{\alpha M^2 B_r f_6}{\alpha^2 + N^2} e^{-\alpha y} \right) * \left(-2\pi \varepsilon \cos [2\pi (x-t) + \phi] - k_1 \right) \quad (3.13)$$

$$Zh_2 = \theta_y h_{2x} =$$

$$\left(-N f_9 \sin[Ny] + N f_8 \cos[Ny] - \left(\frac{2\alpha M^2 B_r f_3}{4\alpha^2 + N^2} e^{2\alpha y} \right) - \right.$$

$$\left(\frac{-2\alpha M^2 B_r f_4}{4\alpha^2 + N^2} e^{-2\alpha y} \right) - \left(\frac{\alpha M^2 B_r f_5}{\alpha^2 + N^2} e^{\alpha y} \right) +$$

$$\left(\frac{\alpha M^2 B_r f_6}{\alpha^2 + N^2} e^{-\alpha y} \right) * (2\pi \varepsilon \cos[2\pi(-t+x)] + k_1) \quad (3.14)$$

The volumetric flow rate in the wave frame is defined by

$$q = \int_{h_1}^{h_2} u dy = \int_{h_1}^{h_2} (f_1 \sinh[\alpha y] + f_2 \cosh[\alpha y] + F) dy$$

$$\left(\frac{f_1}{\alpha} \right) (\cosh[\alpha h_2] - \cosh[\alpha h_1]) + \left(\frac{f_2}{\alpha} \right) (\sinh[\alpha h_2] - \sinh[\alpha h_1]) + F(h_2 - h_1) \quad (3.15)$$

The pressure gradient obtained from equation (3.15) can be expressed as

$$\frac{dp}{dx} = -A - A f_{12} + \eta \sin \alpha \quad (3.16)$$

Where

$$f_{10} = \left(\frac{(-1)}{\left(\frac{\cosh[\alpha h_2] - \cosh[\alpha h_1]}{\sinh[\alpha h_1] - \sinh[\alpha h_2]} \right) \sinh[\alpha h_1] + \cosh[\alpha h_1]} \right) *$$

$$\left(\frac{\cosh[\alpha h_2] - \cosh[\alpha h_1]}{\sinh[\alpha h_1] - \sinh[\alpha h_2]} \right)$$

$$f_{11} = \left(\frac{(-1)}{\left(\frac{\cosh[\alpha h_2] - \cosh[\alpha h_1]}{\sinh[\alpha h_1] - \sinh[\alpha h_2]} \right) \sinh[\alpha h_1] + \cosh[\alpha h_1]} \right)$$

$$f_{12} = \frac{q}{f_{13}} - \frac{\left(\frac{(\cosh[\alpha_1 h_2] - \cosh[\alpha_1 h_1])}{\alpha} \right) (f_{10})}{f_{13}} -$$

$$\frac{\left(\frac{(\sinh[\alpha_1 h_2] - \sinh[\alpha_1 h_1])}{\alpha} \right) (f_{10})}{f_{13}}$$

Where

$$f_{13} = \left(\frac{(\cosh[\alpha_1 h_2] - \cosh[\alpha_1 h_1])}{\alpha} \right) (f_{10}) +$$

$$\left(\frac{(\sinh[\alpha_1 h_2] - \sinh[\alpha_1 h_1])}{\alpha} \right) (f_{10}) + (h_2 - h_1)$$

The instantaneous flux $Q(x, t)$ in the laboratory frame is

$$Q = \int_{h_2}^{h_1} (u + 1) dy = q - h \quad (3.17)$$

The average volume flow rate over one wave period ($T = \lambda/c$) of the peristaltic wave is defined as

$$\bar{Q} = \frac{1}{T} \int_0^T Q dt = q + 1 + d \quad (3.18)$$

From the equations (3.16) and (3.18), the pressure gradient can be expressed as

$$\frac{dp}{dx} = -A - A f_{14} + \eta \sin \alpha \quad (3.19)$$

Where

$$f_{14} = \frac{(\bar{Q} - 1 - d)}{f_{13}} - \frac{\left(\frac{(\cosh[\alpha_1 h_2] - \cosh[\alpha_1 h_1])}{\alpha} \right) (f_{10})}{f_{13}} - \frac{\left(\frac{(\sinh[\alpha_1 h_2] - \sinh[\alpha_1 h_1])}{\alpha} \right) (f_{10})}{f_{13}}$$

4. Discussion of the problem

Fig. 2 illustrates the variation in axial velocity for different values of the Hartman number M ($M = 1, 1.5, 2$) with fixed $Da = 0.3$, $\phi = \pi/6$, $\alpha = \pi/6$, $\eta = 0.5$, $\varepsilon = 0.2$, $k_1 = 0.1$, $x = 0.6$, $t = 0.4$, $dp/dx = -0.5$. We observe from this figure that the axial velocity diminished with increasing the values of Hartman number M . Fig. 3 depicts the variation in axial velocity for the variation in the porous parameter Da ($Da = 0.1, 0.2, 0.3$) with $M = 1$, $\phi = \pi/6$, $\alpha = \pi/6$, $\eta = 0.5$, $\varepsilon = 0.2$, $k_1 = 0.1$, $x = 0.6$, $t = 0.4$, $dp/dx = -0.5$. It shows that the axial velocity increases when the porosity parameter increases. The variation in axial velocity with different values of gravitational parameter η ($\eta = 0.1, 0.5, 1$) is shown in Fig. 4 with fixed $M = 1$, $\phi = \pi/6$, $\alpha = \pi/6$, $\eta = 0.5$, $\varepsilon = 0.2$, $k_1 = 0.1$, $x = 0.6$, $t = 0.4$, $dp/dx = -0.5$. We observe that an increasing the values of gravitational parameter η enhances the axial velocity. Fig. 5 displays the effect of non-uniform parameter k_1 ($k_1 = 0.1, 0.2, 0.3$) on the fluid velocity with fixed $Da = 0.3$, $\phi = \pi/6$, $\alpha = \pi/6$, $M = 1$, $\eta = 0.5$, $\varepsilon = 0.2$, $x = 0.6$, $t = 0.4$, $dp/dx = -0.5$. It can be seen from this figure that an increase in the value

of non-uniform parameter k_1 results in a fluid velocity increases. Therefore, we conclude from these figures that the fluid velocity (See fig. 2-5) diminished when Hartmann number increased and also we notice that the fluid velocity enhances with increasing the values of Da , η and k_1 .

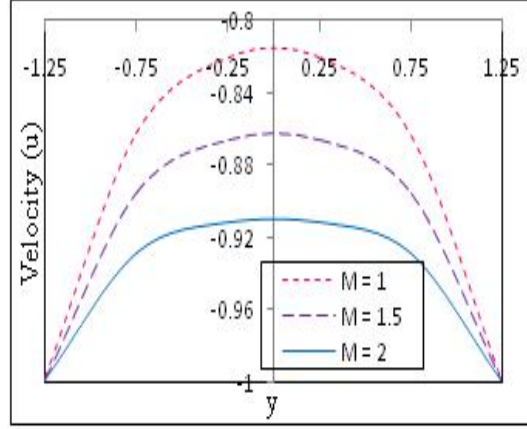


Figure (2): Velocity for different values of M with fixed $Da = 0.3$, $\phi = \pi/6$, $\alpha = \pi/6$, $\eta = 0.5$, $\varepsilon = 0.2$, $k_1 = 0.1$, $x = 0.6$, $t = 0.4$, $dp/dx = -0.5$.

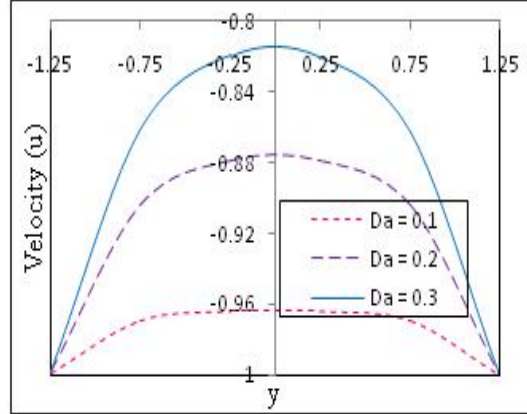


Figure (3): Velocity for different values of Da with fixed $M = 1$, $\phi = \pi/6$, $\alpha = \pi/6$, $\eta = 0.5$, $\varepsilon = 0.2$, $k_1 = 0.1$, $x = 0.6$, $t = 0.4$, $dp/dx = -0.5$.

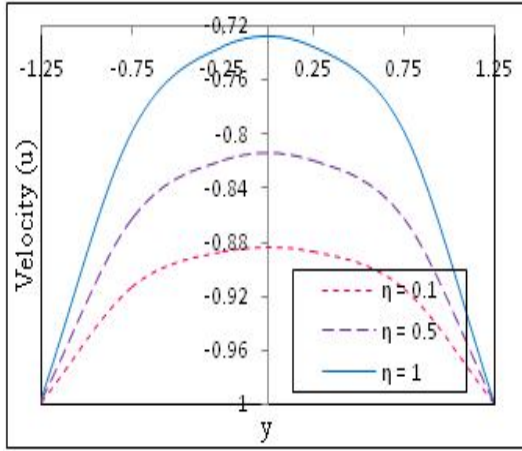


Figure (4): Velocity for different values of η with fixed $Da = 0.3$, $\phi = \pi/6$, $\alpha = \pi/6$, $M = 1$, $\varepsilon = 0.2$, $k_1 = 0.1$, $x = 0.6$, $t = 0.4$, $dp/dx = -0.5$.

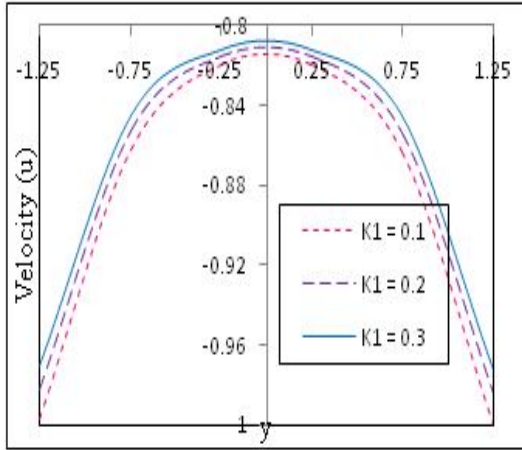


Figure (5): Velocity for different values of K_1 with fixed $Da = 0.3$, $\phi = \pi/6$, $\alpha = \pi/6$, $M = 1$, $\eta = 0.5$, $\varepsilon = 0.2$, $x = 0.6$, $t = 0.4$, $dp/dx = -0.5$.

Fig. 6 displays the effect of Hartmann number M on the pressure gradient with fixed other parameters $Da = 0.3$, $\phi = \pi/6$, $\alpha = \pi/6$, $\eta = 0.5$, $\varepsilon = 0.2$, $k_1 = 0.1$, $t = \pi/4$, $\mu = 0.2$, $d = 2$. It shows from this figure that the pressure gradient increases with increasing the values

of Hartmann number M ($M = 1, 1.5, 2$). Fig. 7 illustrates that the porous parameter Da on pressure gradient with fixed $M = 1$, $\phi = \pi/6$, $\alpha = \pi/6$, $\eta = 0.5$, $\varepsilon = 0.2$, $k_1 = 0.1$, $t = \pi/4$, $\mu = 0.2$, $d = 2$. We notice that the pressure gradient diminishes when increase in porosity parameter Da . Influence of gravitational parameter η on pressure gradient is shown in fig. 8. This figure indicates the pressure gradient enhances when an increase in gravitational parameter η ($\eta = 0.1, 0.5, 1$) with fixed other parameters. Fig. 9 displays the effect of volumetric flow rate \bar{Q} ($\bar{Q} = 0.2, 0.4, 0.6$) on pressure gradient with fixed $M = 1$, $\phi = \pi/6$, $\alpha = \pi/6$, $Da = 0.3$, $\eta = 0.5$, $\varepsilon = 0.2$, $k_1 = 0.1$, $t = \pi/4$, $d = 2$. It shows from this figure that an increase in volumetric flow rate μ results in pressure gradient diminished. Effect of non-uniform parameter k_1 on pressure gradient is presented in Fig.10. We observe that the pressure gradient decreases with increase in non-uniform parameter k_1 ($k_1 = 0.1, 0.2, 0.3$) with fixed $M = 1$, $\phi = \pi/6$, $\alpha = \pi/6$, $Da = 0.3$, $\eta = 0.5$, $\varepsilon = 0.2$, $\mu = 0.2$, $t = \pi/4$, $d = 2$. An important result presented in Fig. 11. We notice from this figure that the pressure gradient decreases in the narrow part of the channel $x \in [0.5, 1]$ and here we require a lesser amount of pressure gradient to pass the flow in an asymmetric channel to the values of increase in ϕ . The pressure gradient increases in the wider part of the channel $x \in [0, 0.5]$ with increasing the values of ϕ . In the wider part of the channel, the flow cannot pass easily. Therefore, it required large pressure gradient to maintain the same flux to pass in the wider part of the channel.

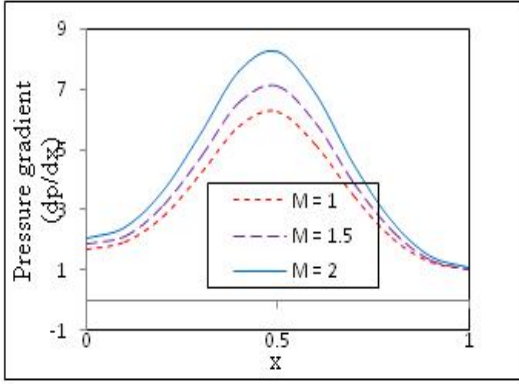


Figure (6): Pressure gradient (dp/dx) for different values of M with fixed $Da = 0.3$, $\phi = \pi/6$, $\alpha = \pi/6$, $\eta = 0.5$, $\varepsilon = 0.2$, $k_1 = 0.1$, $t = \pi/4$, $\mu = 0.2$, $d = 2$.

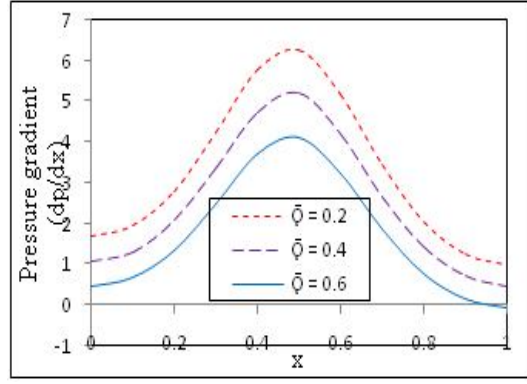


Figure (9): Pressure gradient (dp/dx) for different values of \bar{Q} with fixed $M = 1$, $\phi = \pi/6$, $\alpha = \pi/6$, $Da = 0.3$, $\eta = 0.5$, $\varepsilon = 0.2$, $k_1 = 0.1$, $t = \pi/4$, $d = 2$.

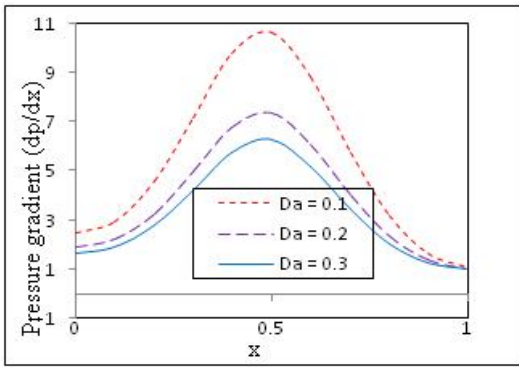


Figure (7): Pressure gradient (dp/dx) for different values of Da with fixed $M = 1$, $\phi = \pi/6$, $\alpha = \pi/6$, $\eta = 0.5$, $\varepsilon = 0.2$, $k_1 = 0.1$, $t = \pi/4$, $\mu = 0.2$, $d = 2$.

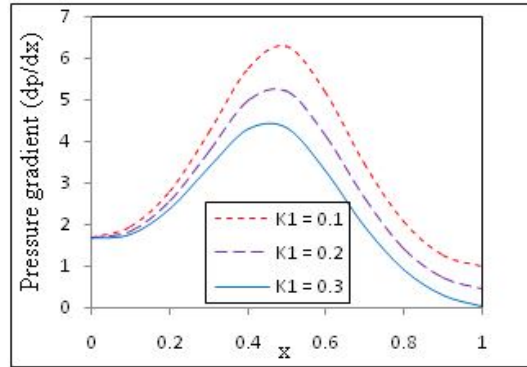


Figure (10): Pressure gradient (dp/dx) for different values of \bar{Q} with fixed $M = 1$, $\phi = \pi/6$, $\alpha = \pi/6$, $Da = 0.3$, $\eta = 0.5$, $\varepsilon = 0.2$, $\mu = 0.2$, $t = \pi/4$, $d = 2$.

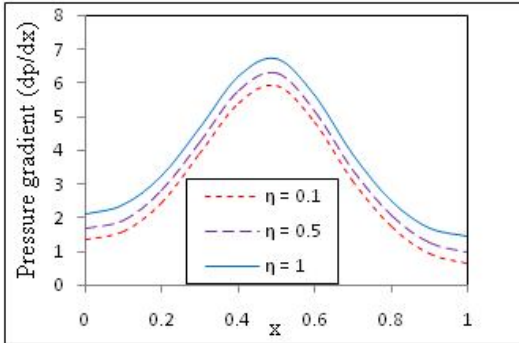


Figure (8): Pressure gradient (dp/dx) for different values of η with fixed $M = 1$, $\phi = \pi/6$, $\alpha = \pi/6$, $Da = 0.3$, $\varepsilon = 0.2$, $k_1 = 0.1$, $t = \pi/4$, $\mu = 0.2$, $d = 2$.

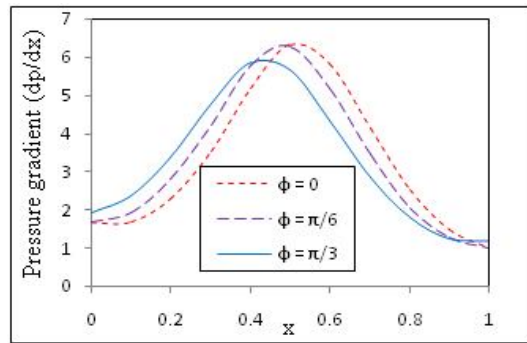


Figure (11): Pressure gradient (dp/dx) for different values of ϕ with fixed $M = 1$, $\alpha = \pi/6$, $Da = 0.3$, $\eta = 0.5$, $k_1 = 0.1$, $\varepsilon = 0.2$, $\mu = 0.2$, $t = \pi/4$, $d = 2$.

Influence of Hartmann number on temperature distribution (θ) is displayed in fig.12 with fixed $Da = 0.3$, $N = 0.5$, $Pr = 1$, $\gamma = 0.1$, $Br = 0.1$, $\phi = \pi/6$, $\alpha = \pi/6$, $\eta = 0.5$, $\varepsilon = 0.2$, $k_1 = 0.1$, $x = 0.6$, $t = 0.4$, $p = 0.5$. It was evident that the temperature distribution enhances when increase in Hartmann number M . Fig. 13 depicts the influence of porosity parameter Da on temperature distribution (θ) with fixed $M = 1.5$, $N = 0.5$, $Pr = 1$, $\gamma = 0.3$, $Br = 0.1$, $\phi = \pi/6$, $\alpha = \pi/6$, $\eta = 0.5$, $\varepsilon = 0.2$, $k_1 = 0.1$, $x = 0.6$, $t = 0.4$, $p = 0.5$. It has been inferred that the results in temperature of the fluid diminished as increase in porosity parameter Da ($Da = 0.1, 0.2, 0.3$) Fig. 14. Shows that the variation of radiation parameter on temperature distribution with $M = 1$, $Da = 0.3$, $Pr = 1$, $\gamma = 0.1$, $Br = 0.1$, $\phi = \pi/6$, $\alpha = \pi/6$, $\eta = 0.5$, $\varepsilon = 0.2$, $k_1 = 0.1$, $x = 0.6$, $t = 0.4$, $p = 0.5$. It could noted that the temperature distribution increases with increase in radiation parameter N ($N = 0.5, 0.7, 0.9$). Variation of Prandtl number Pr ($Pr = 1, 3, 5$) on temperature distribution has been plotted in Fig. 15. It was observe from graph that the temperature of the fluid enhances when increase in Prandtl number Pr with fixed other parameters. Fig.16. displays the effect of Brinkman number Br on temperature of the fluid (θ) with fixed $M = 1$, $Da = 0.3$, $N = 0.5$, $\gamma = 0.1$, $Pr = 1$, $\phi = \pi/6$, $\alpha = \pi/6$, $\eta = 0.5$, $\varepsilon = 0.2$, $k_1 = 0.1$, $x = 0.6$, $t = 0.4$, $p = 0.5$. Indeed, the temperature of the fluid increases with increase in Brinkman number Br ($Br = 0.1, 0.2, 0.3$). Fig. 17 depicts to examine the effect of gravitational parameter η on temperature of the fluid with fixed $M = 1$, $Da = 0.3$, $N = 0.5$, $\gamma = 0.1$, $Pr = 1$, $Br = 0.1$, $\phi = \pi/6$, $\alpha = \pi/6$, $\varepsilon = 0.2$, $k_1 = 0.1$, $x = 0.6$, $t = 0.4$, $p = 0.5$. This figure shows that the results in temperature of the fluid diminished as increase in gravitational

parameter η ($\eta = 0.1, 0.5, 1$). Fig. 18 shows to examine the effect of heat source/sink parameter γ ($\gamma = 0.1, 0.3, 0.5$) on temperature distribution with $M = 1$, $Da = 0.3$, $N = 0.5$, $\eta = 0.5$, $Pr = 1$, $Br = 0.1$, $\phi = \pi/6$, $\alpha = \pi/6$, $\varepsilon = 0.2$, $k_1 = 0.1$, $x = 0.6$, $t = 0.4$, $p = 0.5$. This figure shows that the temperature of the fluid enhanced with increasing the values of heat source/sink parameter γ .

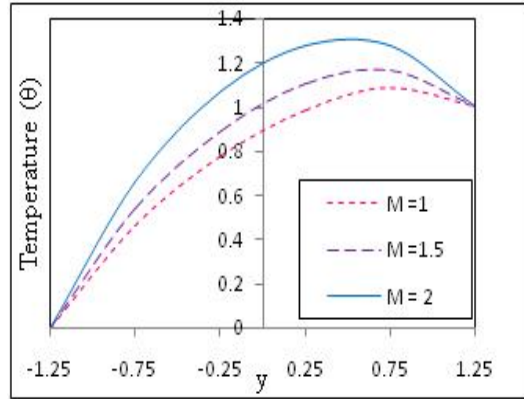


Figure (12): Temperature (θ) for different values of M with fixed $Da = 0.3$, $N = 0.5$, $Pr = 1$, $\gamma = 0.1$, $Br = 0.1$, $\phi = \pi/6$, $\alpha = \pi/6$, $\eta = 0.5$, $\varepsilon = 0.2$, $k_1 = 0.1$, $x = 0.6$, $t = 0.4$, $p = 0.5$.

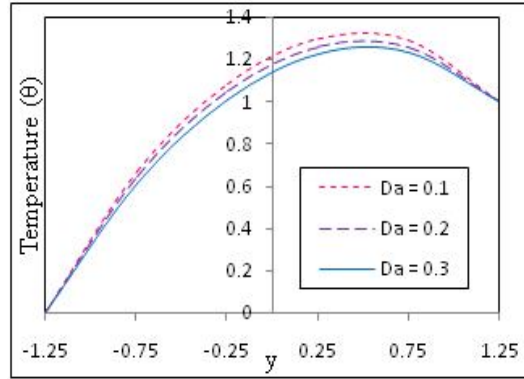


Figure (13): Temperature (θ) for different values of Da with fixed $M = 1.5$, $N = 0.5$, $Pr = 1$, $\gamma = 0.3$, $Br = 0.1$, $\phi = \pi/6$, $\alpha = \pi/6$, $\eta = 0.5$, $\varepsilon = 0.2$, $k_1 = 0.1$, $x = 0.6$, $t = 0.4$, $p = 0.5$.

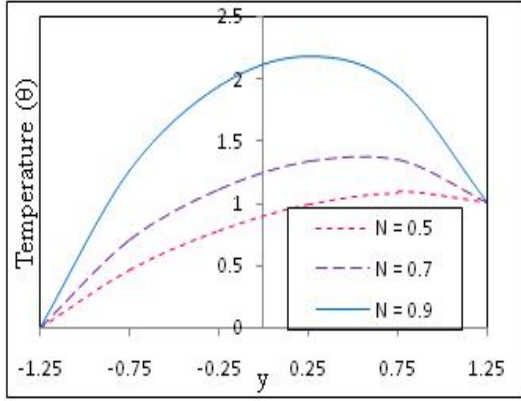


Figure (14): Temperature (θ) for different values of N with fixed $M = 1$, $Da = 0.3$, $Pr = 1$, $\gamma = 0.1$, $Br = 0.1$, $\phi = \pi/6$, $\alpha = \pi/6$, $\eta = 0.5$, $\varepsilon = 0.2$, $k_1 = 0.1$, $x = 0.6$, $t = 0.4$, $p = 0.5$.

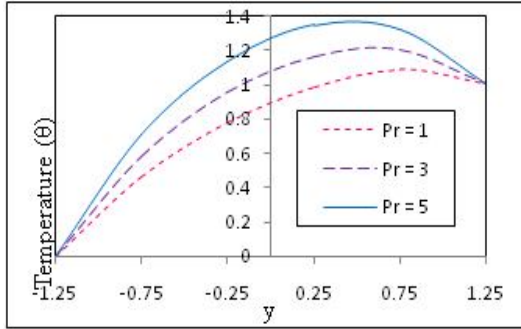


Figure (15): Temperature (θ) for different values of Pr with fixed $M = 1$, $Da = 0.3$, $N = 0.5$, $\gamma = 0.1$, $Br = 0.1$, $\phi = \pi/6$, $\alpha = \pi/6$, $\eta = 0.5$, $\varepsilon = 0.2$, $k_1 = 0.1$, $x = 0.6$, $t = 0.4$, $p = 0.5$.

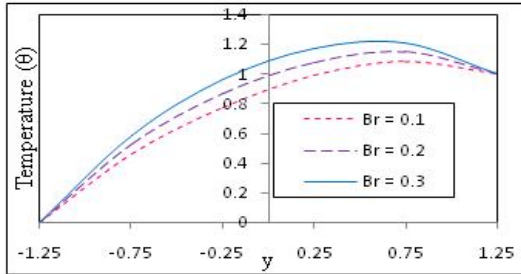


Figure (16): Temperature (θ) for different values of Br with fixed $M = 1$, $Da = 0.3$, $N = 0.5$, $\gamma = 0.1$, $Pr = 1$, $\phi = \pi/6$, $\alpha = \pi/6$, $\eta = 0.5$, $\varepsilon = 0.2$, $k_1 = 0.1$, $x = 0.6$, $t = 0.4$, $p = 0.5$.

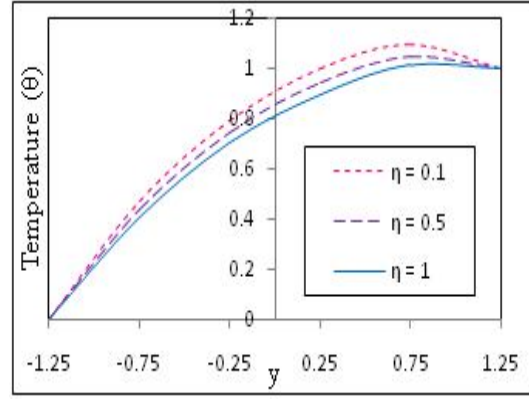


Figure (17): Temperature (θ) for different values of η with fixed $M = 1$, $Da = 0.3$, $N = 0.5$, $\gamma = 0.1$, $Pr = 1$, $Br = 0.1$, $\phi = \pi/6$, $\alpha = \pi/6$, $\varepsilon = 0.2$, $k_1 = 0.1$, $x = 0.6$, $t = 0.4$, $p = 0.5$.

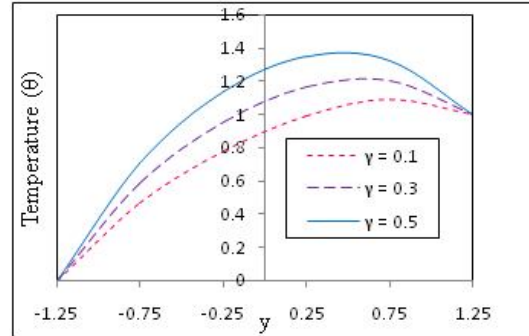


Figure (18): Temperature (θ) for different values of γ with fixed $M = 1$, $Da = 0.3$, $N = 0.5$, $\eta = 0.5$, $Pr = 1$, $Br = 0.1$, $\phi = \pi/6$, $\alpha = \pi/6$, $\varepsilon = 0.2$, $k_1 = 0.1$, $x = 0.6$, $t = 0.4$, $p = 0.5$.

Hartmann number M on heat transfer coefficient at the wall $y = h_1$ is presented in Fig. 19 with $Da = 0.3$, $N = 0.5$, $Pr = 1$, $\gamma = 0.1$, $Br = 0.1$, $\phi = \pi/6$, $\alpha = \pi/6$, $\eta = 0.5$, $\varepsilon = 0.2$, $k_1 = 0.1$, $t = 0.4$, $p = 0.5$. This figure reveals that the heat transfer coefficient increases in the portion of the inclined tapered channel $x \in [0, 0.05]$ and decreases in the other portion of the inclined tapered channel $x \in [0.05, 0.58]$ with increase in M ($M = 1, 1.5, 2$). Fig. 20 shows

the effect of porosity parameter Da on heat transfer coefficient at the wall $y = h_1$ with $M = 1$, $N = 0.5$, $Pr = 1$, $\gamma = 0.1$, $Br = 0.1$, $\phi = \pi/6$, $\alpha = \pi/6$, $\eta = 0.5$, $\varepsilon = 0.2$, $k_1 = 0.1$, $t = 0.4$, $p = 0.5$. We observe from this figure that the results in the heat transfer coefficient is not significant in the entire inclined tapered channel. This may be due to the porosity parameter. We observe that the heat transfer coefficient gradually increases in part of the channel $x \in [0.15, 0.5]$ and then decreases in the part of the channel $x \in [0.5, 0.9]$ with increase in porosity parameter Da . Fig. 21 depicts to examine the effect of radiation parameter on heat transfer coefficient at the wall $y = h_1$ with $M = 1$, $Da = 0.3$, $Pr = 1$, $\gamma = 0.1$, $Br = 0.1$, $\phi = \pi/6$, $\alpha = \pi/6$, $\eta = 0.5$, $\varepsilon = 0.2$, $k_1 = 0.1$, $t = 0.4$, $p = 0.5$. We notice that the heat transfer coefficient increases in the portion of the portion channel $x \in [0, 0.05] \cup [0.58, 1]$ and then decreases in the other portion of the channel $x \in [0.05, 0.58]$ with increase in N ($N = 0.5, 0.7, 0.9$). Fig. 21 reveals the heat transfer coefficient at the wall $y = h_1$ with Prandtl number Pr being fixed $M = 1$, $Da = 0.3$, $N = 0.5$, $\gamma = 0.1$, $Br = 0.1$, $\phi = \pi/6$, $\alpha = \pi/6$, $\eta = 0.5$, $\varepsilon = 0.2$, $k_1 = 0.1$, $t = 0.4$, $p = 0.5$. It can be notice that the heat transfer coefficient enhances in the portion of the channel $x \in [0, 0.05] \cup [0.58, 1]$ and then diminished in the other portion of the channel $x \in [0.05, 0.58]$ with increasing the values of Prandtl number Pr ($Pr = 1, 3, 5$). Fig. 23 illustrates the variation in heat transfer coefficient at the wall $y = h_1$ with Brinkman number Br being fixed $M = 1$, $Da = 0.3$, $N = 0.5$, $Pr = 1$, $\gamma = 0.1$, $\phi = \pi/6$, $\alpha = \pi/6$, $\eta = 0.5$, $\varepsilon = 0.2$, $k_1 = 0.1$, $t = 0.4$, $p = 0.5$. This figure indicates that the heat transfer coefficient enhances in the portion of the inclined tapered

channel $x \in [0, 0.05] \cup [0.58, 1]$ and then diminished in the rest of the inclined tapered channel $x \in [0.05, 0.58]$ when increase Brinkman number Br ($Br = 0.1, 0.2, 0.3$). Influence of gravitational parameter η on heat transfer coefficient at the wall $y = h_1$ is presented in fig. 24 with fixed $M = 1$, $Da = 0.3$, $N = 0.5$, $Pr = 1$, $Br = 0.1$, $\gamma = 0.1$, $\phi = \pi/6$, $\alpha = \pi/6$, $\varepsilon = 0.2$, $k_1 = 0.1$, $t = 0.4$, $p = 0.5$. It can be notice that the heat transfer coefficient enhances in the portion of the channel $x \in [0.05, 0.58]$ and then diminished in the portion of the $x \in [0.58, 1]$ with increase in gravitational parameter η . Fig. 25 reveals the variation in heat transfer coefficient at the wall $y = h_1$ with heat source/sink parameter γ ($\gamma = 0.1, 0.3, 0.5$) being fixed $M = 1$, $Da = 0.3$, $N = 0.5$, $Pr = 1$, $Br = 0.1$, $\eta = 0.5$, $\phi = \pi/6$, $\alpha = \pi/6$, $\varepsilon = 0.2$, $k_1 = 0.1$, $t = 0.4$, $p = 0.5$. We notice from this figure that the results in heat transfer coefficient enhances in the portion of the inclined tapered channel $x \in [0, 0.05] \cup [0.58, 1]$ and then diminished in the rest of the inclined tapered channel $x \in [0.05, 0.58]$ when increase in heat source/sink parameter γ .

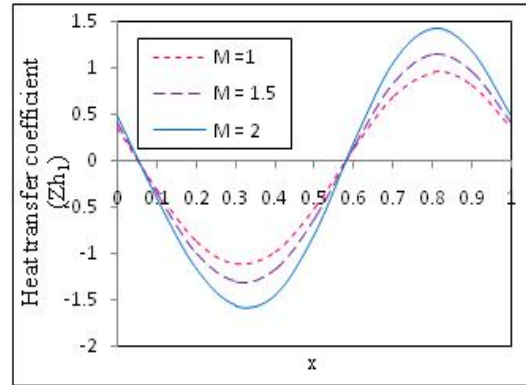


Figure (19): Heat transfer coefficient ($y = h_1$) for different values of M with fixed $Da = 0.3$, $N = 0.5$, $Pr = 1$, $\gamma = 0.1$, $Br = 0.1$, $\phi = \pi/6$, $\alpha = \pi/6$, $\eta = 0.5$, $\varepsilon = 0.2$, $k_1 = 0.1$, $t = 0.4$, $p = 0.5$.

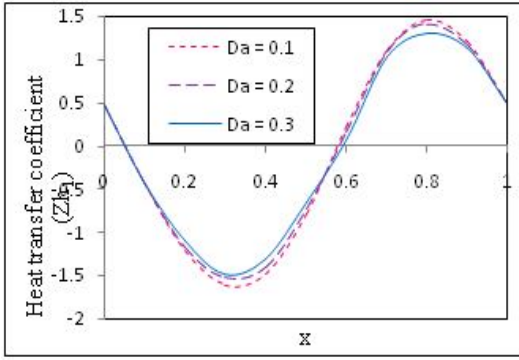


Figure (20): Heat transfer coefficient ($y = h_1$) for different values of Da with fixed $M=1, N=0.5, Pr=1, \gamma=0.1, Br=0.1, \phi=\pi/6, \alpha=\pi/6, \eta=0.5, \epsilon=0.2, k_1=0.1, t=0.4, p=0.5$.

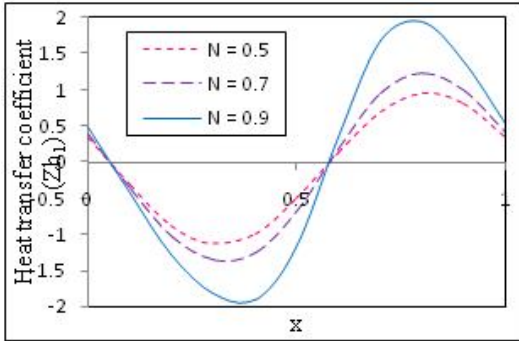


Figure (21): Heat transfer coefficient ($y = h_1$) for different values of N with fixed $M=1, Da=0.3, Pr=1, \gamma=0.1, Br=0.1, \phi=\pi/6, \alpha=\pi/6, \eta=0.5, \epsilon=0.2, k_1=0.1, t=0.4, p=0.5$.

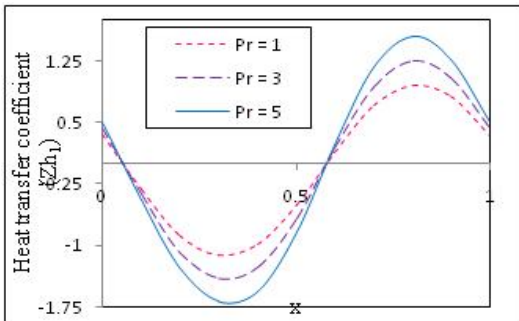


Figure (22): Heat transfer coefficient ($y = h_1$) for different values of Pr with fixed $M=1, Da=0.3, N=0.5, \gamma=0.1, Br=0.1, \phi=\pi/6, \alpha=\pi/6, \eta=0.5, \epsilon=0.2, k_1=0.1, t=0.4, p=0.5$.

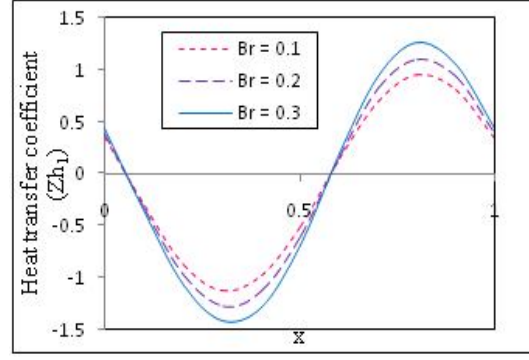


Figure (23): Heat transfer coefficient ($y = h_1$) for different values of Br with fixed $M=1, Da=0.3, N=0.5, Pr=1, \gamma=0.1, \phi=\pi/6, \alpha=\pi/6, \eta=0.5, \epsilon=0.2, k_1=0.1, t=0.4, p=0.5$.

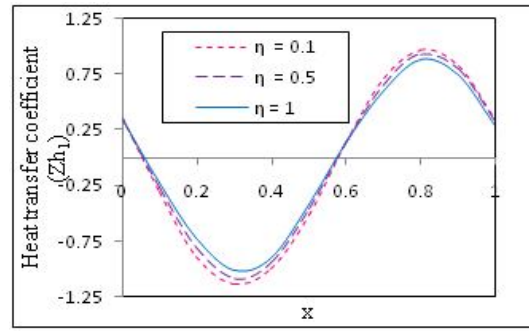


Figure (24): Heat transfer coefficient ($y = h_1$) for different values of η with fixed $M=1, Da=0.3, N=0.5, Pr=1, Br=0.1, \gamma=0.1, \phi=\pi/6, \alpha=\pi/6, \epsilon=0.2, k_1=0.1, t=0.4, p=0.5$.

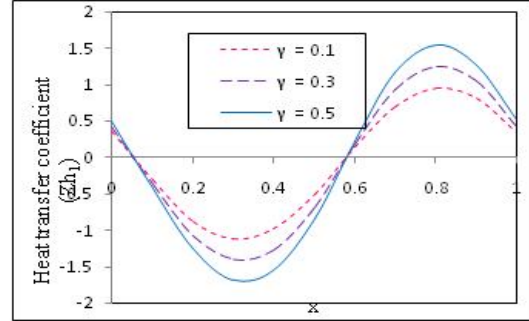


Figure (25): Heat transfer coefficient ($y = h_1$) for different values of γ with fixed $M=1, Da=0.3, N=0.5, Pr=1, Br=0.1, \eta=0.5, \phi=\pi/6, \alpha=\pi/6, \epsilon=0.2, k_1=0.1, t=0.4, p=0.5$.

Conclusions

In this paper, we have proposed a theoretical study of an analysis of joule heating and radiation on hydromagnetic peristaltic flow with porous medium through a coaxial asymmetric vertical inclined tapered channel. The study has paid a special attention to examining the effects of Hartmann number, porosity parameter, volumetric flow rate, radiation parameter, non-uniform parameter, gravitational parameter, shift angle, Prandtl number, Brinkman number, heat source/sink parameter on the flow characteristics. We have concluded the following key observations:

- (1) The axial velocity increases with increase in porosity parameter Da , gravitational parameter η and non-uniform parameter k_1 .
- (2) The axial velocity diminished with increase in Hartmann number M .
- (3) Pressure gradient enhances with an increase in Hartmann number M and gravitational parameter η .
- (4) Pressure gradient diminished with increase in porosity parameter Da , non-uniform parameter k_1 and volumetric flow rate \bar{Q} .
- (5) The temperature of the fluid enhances with an increase in Hartmann number M , radiation parameter N , Prandtl number Pr , Brinkman number Br and heat source/sink parameter γ .
- (6) The temperature of the fluid diminished with increase in porosity parameter Da and gravitational parameter η .
- (7) Heat transfer coefficient increases in the portion of the inclined tapered channel $x \in [0, 0.05] \cup [0.58, 1]$ and decreases in the other portion of the inclined tapered channel $x \in [0.05, 0.58]$ with increase in Hartmann number M , radiation parameter N , Prandtl number Pr , Brinkman number Br and heat source/sink parameter γ .
- (8) Heat transfer coefficient gradually increases in part of the channel $x \in [0.15, 0.5]$ and then decreases in the part of the channel $x \in [0.5, 0.9]$ with an increase in porosity parameter Da .
- (9) Heat transfer coefficient enhances in the portion of the channel $x \in [0.05, 0.58]$ and then diminished in the portion of the $x \in [0.58, 1]$ with an increase in gravitational parameter η .

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