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New Sort of Separation Axioms in Extended Bitopology

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Abstract

In this paper, we introduce and establish some new separation axioms using the $(1,2)^{*+}$ -α-open sets,$(1,2)^{*+}$-semi-open sets, $(1,2)^{*+}$-pre-open sets in extended bitopological ultra spaces called as $Ultra^+$-R_i, $Ultra^+$-Semi-R_i (i=0,1), $Ultra^+$-R_D, $Ultra^+$-R_T, and $Ultra^+$-R_{YS} spaces. We also investigate some of their basic properties and establish the relationship between them.

Keywords : $Ultra^+$-R_i, $Ultra^+$-Semi-R_i (i=0,1), $Ultra^+$-R_D, $Ultra^+$-R_T, and $Ultra^+$-R_{YS} spaces.

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1. Introduction

In 1963, Kelly[1] initiated the study of the bitopological space which is to be a set X equipped with two topologies τ_1 and τ_2 on X. Lellis Thivagar[2] introduced new bitopological notions of $\tau_{1,2}$-open sets and also proved that each (1, 2)*-α-open sets is (1, 2)*-semi open and (1, 2)*-pre-open but the converse of each is not true and also introduced the concept of Ultra space in bitopological spaces. Further the extended bitopological space is initiated and characterized their properties by LellisThivagar *et al.*[3]. Also in 1974, Dube *et al.*[8] introduced some more separation axioms R_Y, R_{YS} and R_D, which are weaker than R_0. In this paper, we introduce some new separation axioms by using $(1,2)^{*+}$-α-open sets, $(1,2)^{*+}$-semi-open sets,

$(1,2)^{*+}$-pre-open sets in extended bitopological ultra spaces and we develop some weak separation axioms of R_0. Further we derive its various properties and relation between other existing spaces. The most of the results in this paper can be extended to Digital Topology.

2. Preliminaries

Definition 2.1.[1]. A non-empty set X together with two topologies τ_1 and τ_2 is called a bitopologicalspaces and is denoted by (X, τ_1, τ_2).

Definition 2.2.[2]. A subset S of a bitopological space X is called $\tau_{1,2}$-open if $S = A \cup B$, where $A \in \tau_1$ and $B \in \tau_2$ and $\tau_{1,2}$-closed if S^C is $\tau_{1,2}$ open.

The family of all $\tau_{1,2}$-open (resp. $\tau_{1,2}$-closed) sets is denoted by $\tau_{1,2O}(X)$ (resp. $\tau_{1,2}O(X)$).

Definition 2.3.[3]. Let (X, τ_1, τ_2) be a bitopological space and $\tau_{1,2}O(X) \subset (\tau_{1,2})^+$. Then $(\tau_{1,2})^+$ will be termed a simple extension of $\tau_{1,2}O(X)$ if and only if there exists an $A \notin \tau_{1,2}O(X)$ such that $(\tau_{1,2})^+ = (\tau_{1,2})^+(A) = \{G_1 \cup (G_2 \cap A) : G_1, G_2 \in \tau_{1,2}O(X)\}$. We call $(X, (\tau_{1,2})^+(A))$ an extended bitopological space of (X, τ_1, τ_2) w.r.t A.

Throughout this paper $(X, (\tau_{1,2})^+(A))$, $(Y, (\tau_{1,2})^+(B))$ [or simply X, Y] denote the extended bitopological space on which no separation axioms are assumed unless explicitly stated.

Definition 2.4.[3] Let $(X, (\tau_{1,2})^+(A))$ be an extended bitopological space and $S \subseteq X$. Then $(\tau_{1,2})^+$ closure of S is defined as $(\tau_{1,2})^+cl(S) = \cap\{F : S \subseteq F \text{ and } F \text{ is } (\tau_{1,2})^+ \text{ closed}\}$ and $(\tau_{1,2})^+$ interior of S is defined as $(\tau_{1,2})^+ int(S) = \cup\{G : G \subseteq S \text{ and } G \text{ is } (\tau_{1,2})^+ \text{ open}\}$.

Theorem 2.5.[3] Let (X, τ_1, τ_2) be a bitopological space which is T_0, T_1or T_2 and $A \notin \tau_{1,2}O(X)$. Then $(X, (\tau_{1,2})^+(A))$ is T_0^+, T_1^+, T_2^+ respectively.

Definition 2.6.[3] Let $(X, (\tau_{1,2})^+(A))$ be an extended bitopological space. A subset S of X is called

(i) $(1, 2)^{*+}$-α-open if $S \subseteq (\tau_{1,2})^+ int((\tau_{1,2})^+ cl((\tau_{1,2})^+ int(S)))$

(ii) $(1, 2)^{*+}$-semi-open if $S \subseteq (\tau_{1,2})^+ cl((\tau_{1,2})^+ int(S))$

(iii) $(1,2)^{*+}$-pre-open if $S \subseteq (\tau_{1,2})^+ int((\tau_{1,2})^+ cl(S))$.

The collection of all $(1, 2)^{*+}$-α-open sets, $(1, 2)^{*+}$-semi-open sets and $(1, 2)^{*+}$-preopen sets of X are denoted by $(1, 2)^{*+}\alpha O(X,A)$, $(1, 2)^{*+}SO(X,A)$, $(1, 2)^{*+}PO(X,A)$ respectively.

Remark 2.7.[3] In an extended bitopological space $(X, (\tau_{1,2})^+)$, the collection $(1, 2)^{*+}\alpha O(X)$ need not form a topology. If $(1, 2)^{*+}\alpha O(X,A)$ is form a topology, then we call $(1, 2)^{*+}\alpha O(X,A)$ is a $(1, 2)^{*+}\alpha$-topology or an Ultra$^+$-space. Here $(1, 2)^{*+}\alpha cl(S)$[resp. $(1, 2)^{*+}scl(S)$ and $(1, 2)^{*+}pcl(S)$] is defined as the intersection of all $(1, 2)^{*+}$-α-closed [resp. $(1, 2)^{*+}$-semi closedand $(1, 2)^{*+}$-pre-closed] sets containing A.

Theorem 2.8.[3] Let X be an extended bitopological space . Then $S \subseteq X$ is a $(1, 2)^{*+}$-α-open if and only if S is a $(1, 2)^{*+}$-semi-open and a $(1, 2)^{*+}$-pre-open

Definition 2.9.[8] A topological space X is said to be R_{YS}-space if for $x, y \in X$, $cl(\{x\}) \notin cl(\{y\})$ implies $cl(\{x\}) \cap cl(\{y\}) = \phi$ or $\{x\}$ or $\{y\}$.

Definition 2.10.[8] A topological space Xis said to be R_D-space if for each $x \in X$, $cl(\{x\}) \cap ker(\{x\}) = \{x\}$ implies $D(\{x\}) = cl(\{x\}) - \{x\}$ is closed, where $ker(\{x\}) = \cap\{G \in \tau$ and $x \in G\}$.

Definition 2.11.[8] A topological space X is said to be R_T-space if for each $x \in X$, both $cl(\{x\}) - ker(\{x\})$ and $ker(\{x\}) - cl(\{x\})$ are degenerate. Degenerate set means a set which does not contains more than one element.

3. Ultra$^+$ -R_i, Ultra$^+$ semi-R_i(i=0,1) Spaces:

In this section, we introduce the notion of Ultra$^+$-R_i,Ultra$^+$semi-R_i(i=0,1) spaces by using $(1, 2)^{*+}\alpha$-open sets, $(1, 2)^{*+}$-semi-open sets respectively and derive the relationship between these spaces and other existing spaces.

Definition 3.1. An extended bitopological space X is called an

1. Ultra$^+$-R_0 (resp. Ultra$^+$semi-R_0)- space if $((1, 2)^{*+}scl(\{x\}) \subset U$(resp. $(1,2)^{*+}scl(\{x\})\subset$) whenever $x \in U \in (1, 2)^{*+}\alpha O(X)$ (resp. $x \in U \in (1,2)^{*+}SO(X)$).
2. Ultra$^+$-R_1(resp. Ultra$^+$semi-R_1)- space if for $x,y\in X$ such that $x \notin ((1, 2)^{*+} \alpha cl(\{y\})$ (resp$(1, 2)^{*+}scl(\{y\})$) there exists disjoint $(1, 2)^{*+}\alpha$-open(resp. $(1, 2)^{*+}$semi open) sets U and V in X such that $x \in U$ and $y \in V$.
3. Weakly Ultra$^+$-R_0-space(resp. Weakly Ultra$^+$semi-R_0 and Weakly Ultra$^+$pre-R_0-space) if $\cap_{x\in X} (1, 2)^{*+}\alpha cl(\{x\}) \neq \phi$ (resp. $\cap_{x\in X}(1, 2)^{*+}scl(\{x\})$ and $\cap_{x\in X}(1, 2)^{*+} pcl(\{x\} \neq \phi)$

Example 3.2. Let $X = \{a, b, c, d\}$, $\tau_1 = \{\varphi, X, \{b\}, \{a, b\}\}$, $\tau_2 = \{\varphi, X\}$. Then $\tau_{1,2}O(X)=\{\varphi,X, \{b\},\{a,b\}\}$. Let $A = \{a\} \notin \tau_{1,2}O(X)$. Then $(\tau_{1,2})^+(A) = \{\varphi,X, \{a\}, \{b\}, \{a, b\}\}$. Then $(1, 2)^{*+}\alpha O(X,A) = \{\varphi,X, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$ and $(1, 2)^{*+} \alpha cl(\{a\}) =\{a, c, d\} \subseteq \{a\} \in (1, 2)^{*+}\alpha O(X,A)$, for $a \in \{a\}$. Here X is weakly Ultra$^+$-R_0-space but not Ultra$^+$-R_0-space. $(1, 2)^{*+}SO(X,A)= \{\varphi, X, \{a\}, \{b\}, \{a, b\}, \{a,c\},\{a,d\},\{b,c\},\{b,d\}, \{a,b,c\}, \{a,b,d\},\{a,c,d\},\{b,c,d\}\}$ and $1, 2)^{*+} SC(X,A) = \{\varphi,X, \{a\},\{b\}, \{c\},\{d\},\{a,c\}, \{b,c\}, \{a,d\},\{b,d\},\{c,d\},\{a,c,d\}, \{b,c,d\}\}$. Here X is weakly Ultra$^+$-$R_0$-space and Ultra$^+$-semi-$R_1$-space but not Ultra$^+$-$R_0$ -space.

Remark 3.3. Let $X = \{a, b, c, d\}$, $\tau_1 = \{\varphi, X, \{b\}, \{c\}, \{b, c\}\}$, $\tau_2 = \{\varphi,X\}$. Then $\tau_{1,2}O(X)=\{\varphi,X, \{b\}, \{c\}, \{b, c\}\}$. Let $A = \{a\} \notin \tau_{1,2}O(X)$. Then $(\tau_{1,2})^+(A) = \{\varphi,X,\{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$. Then $(1, 2)^{*+}$

$\alpha O(X,A) = \{\varphi, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$ and $(1, 2)^{*+}SO(X,A) = \{\varphi, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$. Here X is Ultra$^+$ semi-R_0- space and weakly Ultra$^+$semi-R_0-space. This example shows that an Ultra$^+$semi-R_0-space need not be an Ultra$^+$-R_0 -space and also X is need not be an Ultra$^+$semi-R_1-space. But an Ultra$^+$-R_0-space is Ultra$^+$semi-R_0-space.

Proposition 3.4. If X is an Ultra$^+$-R_1-space, then it is Ultra$^+$-R_0-space.

Proof: Let X be Ultra$^+$-R_1-space, $U \in (1, 2)^{*+}\alpha O(X,A)$ and $x \in U$. For each $y \in X\setminus U$, $x \notin (1, 2)^{*+}\alpha cl(\{y\})$. Therefore, there exist $(1, 2)^{*+}$-α-open sets U_x, V_y in X such that $x \in U_x$ and $y \in V_y$ such that $U_x \cap V_y = \emptyset$. Let $A = \cup\{V_y : y \in X\setminus U\}$, then $X\setminus U \subset A$ and $x \notin A$ which is a $(1, 2)^{*+}$-α-open set so that $(1, 2)^{*+}\alpha cl(\{x\}) \subset X\setminus A \subset U$. Therefore, X is Ultra$^+$-$R_0$-space

Proposition 3.5. If X is an Ultra$^+$semi-R_1-space, then it is Ultra$^+$semi-R_0-space.

Theorem 3.6. An extended bitopological space X is an Ultra$^+$-T_2 space if and only if it is Ultra$^+$-T_1 space and Ultra$^+$-R_0 space.

Proof: If X is an Ultra$^+$-T_2 space, then it is Ultra$^+$-T_1space by [7] Remark 3.7 (i). We prove X is Ultra$^+$-R_1 space. If $x, y \in X$ such that $x \notin (1, 2)^{*+}\alpha cl(\{y\})$ then $x \notin y$. Therefore, there exist disjoint ((1, 2)$^{*+}$-α-open sets U, V in X such that $x \in U$ and $y \in V$. Hence, X is Ultra$^+$-R_1 space. Conversely, if X is Ultra$^+$-T_1 space and Ultra$^+$-R_1 space and $x, y \in X$ such that $x \notin (1, 2)^{*+}\alpha cl(\{y\})$ there exist disjoint $(1, 2)^{*+}$-α-open sets U, V in X such that $x \in U$ *and* $y \in V$. Since X is Ultra$^+$-T_1space, $(1, 2)^{*+}\alpha cl(\{y\}) = \{y\}$ by Theorem 3.10 [4]. Thus for $x \notin y$ and $V \in (1, 2)^{*+}\alpha O(X,A)$ such that $x \in U$ *and* $y \in V$, $U \cap V = \varphi$. Therefore, X is Ultra$^+$-T_2 space

Corollary 3.7. An extended bitopological space X is an Ultra$^+$semi-T_2-space if and only if it is Ultra$^+$semi-T_1 space and Ultra$^+$semi-R_1 space.

Proposition 3.8. Every weakly Ultra$^+$-R_0-space is weakly Ultra$^+$semi-R_0-space and weakly Ultra$^+$pre-R_0-space.

Proof: If X is weakly Ultra$^+$-R_0-space, $\cap_{x \in X}(1, 2)^{*+}\alpha cl(\{x\}) \neq \phi$. Therefore $\cap_{x \in X}(1, 2)^{*+}scl(\{x\}) \neq \phi$ and $\cap_{x \in X}(1, 2)^{*+}pcl(\{x\}) \neq \phi$.

Theorem 3.9. An extended bitopological space X is weakly Ultra$^+$-R_0 space if and only if $(1, 2)^{*+}\alpha ker(\{x\}) \notin X$ for each $x \in X$ where $(1, 2)^{*+}\alpha ker(\{x\}) = \cap\{U : x \in U \in (1, 2)^{*+}\alpha O(X,A)\}$.

Proof: Necessary : Suppose there is $x_0 \in X$ some with $(1, 2)^{*+}\alpha\, ker(\{x_0\}) = X$, then X is the only $(1, 2)^{*+}$-α-open set containing x_0. This implies that $x_0 \in (1, 2)^{*+}\alpha cl(\{x\})$ for every $x \in X$. Hence $\cap_{x \in X}(1, 2)^{*+}\alpha cl(\{x\}) \neq \phi$, which is a contradiction. Thus $(1,2)^{*+}\alpha\, ker(\{x\}) \neq X$ for each $x \in X$.

Sufficiency: Suppose X is not weakly Ultra$^+$-R_0-space, then choose some $x_0 \in \cap_{x\in X}$ $(1, 2)^{*+}\alpha$ cl({x}). So $x_0 \in (1, 2)^{*+}\alpha$ cl({x}) for each $x\in X$. This implies that every $(1, 2)^{*+}$-α-open set containing x_0 contains every point of X. Hence $(1, 2)^{*+}\alpha$ ker({x_0}) = X, which is a contradiction. Thus X is weakly Ultra$^+$-R_0 space

4. Some New Weak form of Spaces :

This section is to introduce and establish the properties of some new spaces which isweaker than the space Ro, such spaces are called as Ultra^+R_D, Ultra$^+R_T$and Ultra$^+R_{YS}$. We also discuss their relationship and the counter examples.

Definition 4.1. An extended bitopological space X is called an

i. Ultra^+R_D- space if for each $x\in X$, $(1, 2)^{*+}$ αcl({x}) ∩ $(1, 2)^{*+}\alpha$ ker({x}) = {x} implies that the $(1, 2)^{*+}$-α-derived set, $(1, 2)^{*+}\alpha$-D({x}) = $(1, 2)^{*+}\alpha$ cl({x}) - {x} is $(1, 2)^{*+}\alpha$–closed

ii. Ultra$^+$-R_T-space if for each $x\in X$ such that both $(1, 2)^{*+}$αcl({x}) – $(1, 2)^{*+}$αker({x}) and $(1, 2)^{*+}\alpha$ ker({x})–$(1, 2)^{*+}$αcl({x}) are degenerate set, where degenerateset means that a null set or a singleton set.

iii. Ultra$^+$-R_{YS}- space if for each $y\in X$, $(1, 2)^{*+}$ αcl({x}) ≠ $(1, 2)^{*+}$αcl({y}) impliesthat $(1, 2)^{*+}$ αcl({x})∩ ($(1, 2)^{*+}$αcl({y}) = φ or {x} or {y}.

Remark 4.2. Obviously every Ultra$^+$-R_0-space is Ultra$^+$-R_T-space, but converse is not true as it is shown by the following examples.

Example 4.3. Let X = {a, b, c}, τ_1 = {φ, X, {a}, {a, b}}, τ_2 = {φ,X, {b}}. Then $\tau_{1,2}$O(X) ={φ,X, {a}, {b}, {a, b}}. Let A = {b, c} ∉ $\tau_{1,2}$O(X). Then $(\tau_{1,2})^+$(A) = {φ,X, {a}, {b}, {a, b}, {b, c}}. Then $(1, 2)^{*+}$αO(X,A) = {φ,X, {a}, {b}, {a, b}, {b, c}} and here X is Ultra$^+$-R_T-space but not Ultra$^+$ -R_0 -space.

Proposition 4.4. Every Ultra$^+$-R_T-space is Ultra$^+$-R_D-space, but converse is not true as it is shown by the following example.

Proof: Let Xbe Ultra$^+$-R_T-space. Then both $(1, 2)^{*+}$ 'α ker({x}) − $(1, 2)^{*+}$ αcl({x}) and $(1, 2)^{*+}$αcl({x})– $(1, 2)^{*+}$ α ker ({x}) are degenerate. Now let <x> = $(1, 2)^{*+}$ αcl({x}∩ $(1, 2)^{*+}\alpha$ ker({x}). Then $(1, 2)^{*+}\alpha$ ker({x}) = <x> ∪ D and $(1, 2)^{*+}$αcl({x}) =<x> ∪ E, where D ⊈ $(1, 2)^{*+}$αcl({x} and E⊈$(1, 2)^{*+}\alpha$ ker({x}) and clearly D and E are degenerate sets. If <x>={x}, then $(1, 2)^{*+}$ αcl({x}) = E ∪ {x} and $(1, 2)^{*+}\alpha$ ker({x}) = D∪{x}. We prove that $(1, 2)^{*+}\alpha$-D({x}) = $(1, 2)^{*+}$ αcl({x})–{x} is$(1, 2)^{*+}$-α-closed. Let U be a $(1, 2)^{*+}$-α-open set containing $(1, 2)^{*+}\alpha$ ker({x}). Then (X– U) is$(1, 2)^{*+}$-α-closed set. Hence (X − U) ∩ $(1, 2)^{*+}$αcl({x}) = E or ϕ

Case(i). If (X – U) ∩ $(1, 2)^{*+}$αcl({x}) = E, then E is the intersection of two $(1, 2)^{*+}$-α- closed sets hence is also $(1, 2)^{*+}$-α-closed.

Case (ii) (X − U) ∩ $(1, 2)^{*+}$αcl({x})

$= \phi$, then $(1, 2)^{*+}\alpha cl(\{x\}) \subset U$, $E \subset U$. Since $E \not\subseteq (1, 2)^{*+}\alpha\ ker(\{x\})$, there is a $(1, 2)^{*+}$-α-open set V such that $x \in V$ and $E \not\subseteq V$. Then $(1, 2)^{*+}\alpha cl(\{x\}) \cap (X - V) = E$ is a $(1, 2)^{*+}$-α-closed set. Therefore, $(1, 2)^{*+}\alpha$- $D(\{x\})$ is $(1, 2)^{*+}$-α-closed set, whenever $<x> = \{x\}$ and hence X is Ultra^{+}-R_{D}.

Example 4.5. Let $X = \{a, b, c, d\}$, $\tau_1 = \{\varphi, X, \{a\}\}$, $\tau_2 = \{\varphi, X, \{a, b\}\}$. Then $\tau_{1,2}O(X) = \{\varphi, X, \{a\}, \{a,b\}\}$. Let $A = \{b\} \notin \tau_{1,2}O(X)$. Then $(\tau_{1,2})^{+}(A) = \{\varphi, X, \{a\}, \{b\}, \{a, b\}\}$. Then $(1, 2)^{*+}\alpha O(X,A) = \{\varphi, X, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$ and here X is Ultra^{+}- R_{D}-space but not Ultra^{+}-R_{T}-space.

Proposition 4.6. Every Ultra^{+}-R_{T}-space is Ultra^{+}-$R_{Y\,S}$-space, but converse is not true.

Proof: Let X be an Ultra^{+}-R_{T}-space and $x, y \in X$. If $(1, 2)^{*+}\alpha cl(\{x\}) \neq (1, 2)^{*+}\alpha cl(\{y\})$ and $(1, 2)^{*+}\alpha cl(\{x\}) \cap (1, 2)^{*+}\alpha cl(\{y\}) \neq \phi$. Hence we assume that there exits an element $a \in x$ such that $a \neq x, a \neq y$ and $a \in (1,2)^{*+}\alpha cl(\{x\}) \cap (1,2)^{*+}\alpha cl(\{y\})$, then $a \in (1, 2)^{*+}\alpha cl(\{x\})$ and $a \in (1, 2)^{*+}\alpha cl(\{y\})$ and hence $x, y \in (1, 2)^{*+}\alpha\ ker(\{a\})$. Since X is an Ultra^{+}-R_{T}-space, $(1, 2)^{*+}\alpha\ ker(\{a\}) = <a> \cup E$, where E is a degenerate set and $E \not\subseteq (1, 2)^{*+}\alpha cl(\{a\})$.

Example 4.7. Let $X = \{a, b, c, d\}$, $\tau_1 = \{\varphi, X, \{a\}, \{a, c\}\}$, $\tau_2 = \{\varphi, X, \{c\}, \{a, c\}\}$. Then $\tau_{1,2}O(X) = \{\varphi, X, \{a\}, \{c\}, \{a, c\}\}$. Let $A = \{d\} \notin \tau_{1,2}O(X)$. Then $(\tau_{1,2})^{+}(A) = \{\varphi, X, \{a\}, \{c\}, \{d\}, \{a, c\}, \{a, d\}, \{c, d\}, \{a, c, d\}\}$. Then $(1, 2)^{*+}\alpha O(X,A) = \{\varphi, X, \{a\}, \{c\}, \{d\}, \{a, c\}, \{a, d\}, \{c, d\}, \{a, c, d\}\}$ and here Xis Ultra^{+}-$R_{Y\,S}$-space but not Ultra^{+}-R_{T}-space.

Proposition 4.8. Every Ultra^{+}-$R_{Y\,S}$-space is Ultra^{+}-R_{D}-space, but converse need not be true

Proof: Let X be an Ultra^{+}-RY S-space. Here there are three cases to discuss.

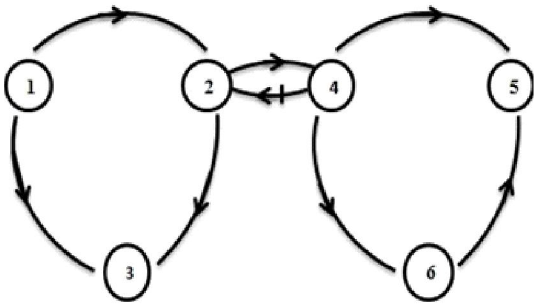
Case (i) Let $(1, 2)^{*+}\alpha cl(\{x\}) \cap (1, 2)^{*+}\alpha cl(\{y\}) = \{x\}$. Then $(1, 2)^{*+}\alpha$-$D(\{x\}) = \phi$ which is $(1, 2)^{*+}$-α-closed set.

Case (ii) Let $(1, 2)^{*+}\alpha cl(\{x\}) \cap (1, 2)^{*+}\alpha cl(\{y\}) = \{y\}$. Then $(1, 2)^{*+}\alpha$-$D(\{y\}) = \phi$.

Case (iii) Let $(1, 2)^{*+}\alpha cl(\{x\}) \cap (1, 2)^{*+}\alpha cl(\{y\}) = \phi$ and by hypothesis we have, $(1, 2)^{*+}\alpha cl(\{x\}) \cap (1, 2)^{*+}\alpha\ ker(\{x\}) = \{x\}$. Hence we get a contradiction. Therefore X is an Ultra^{+}-R_{D}-space.

Example 4.9. Let $X = \{a, b, c, d\}$, $\tau_1 = \{\varphi, X, \{a\}\}$, $\tau_2 = \{\varphi, X, \{a, b\}\}$. Then $\tau_{1,2}O(X) = \{\varphi, X, \{a\}, \{a,b\}\}$. Let $A = \{b\}\ \tau_{1,2}O(X)$. Then $(\tau_{1,2})^{+}(A) = \{\varphi, X, \{a\}, \{b\}, \{a, b\}\}$. Then $(1, 2)^{*+}\ \alpha O(X,A) = \{\varphi, X, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$ and here X is Ultra^{+}-R_{D}-space but not Ultra^{+} -$R_{Y\,S}$–space.

Remark 4.10. From the above theorems and examples we have the following diagram. We depict by arrow the implications between the separation axioms.



(1) Ultra$^+$-R_1,(2) Ultra$^+$-R_0, (3) Ultra$^+$semi-R_0, (4) Ultra$^+$-R_T, (5) Ultra$^+$-R_D, (6) Ultra$^+$-$R_{Y\,S}$.

Conclusion

In this paper, we have introduced new separation axioms in extended bitopologicalultra spaces and established their relationship between other existing Ultra spaces. Also we characterized the properties of such Ultra spaces and also included their necessary and sufficient conditions . We can also develop this separation axioms into other fields.

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