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# Fuzzy soft hyper K-subalgebra

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## Abstract

In this paper, the notion of fuzzy soft hyper K-subalgebra is given and the level subset, union, intersection & some basic results of fuzzy hyper K-subalgebra are studied. We also state a characterization for a fuzzy soft hyper K-subalgebra in terms of iths level hyper K-subalgebra.

*Key words:* Fuzzy soft hyper K-algebra, Levelset.

## 1. Introduction

The hyper algebraic structure theory was introduced in 1934 by F. Marty at the 8th congress of Scandinavian Mathematicians. Since the many researchers have worked on this area. Recently Y.B. June *et al.*,[8] applied the hyper structures to BCK- algebras and R.A. Boorzoei *et al.*,[4] defined the notion of a hyper K-algebra. In this paper, we introduce the concept of fuzzy soft hyper K-sub algebras and investigate some related properties. We state a characterization for a fuzzy soft hyper K-subalgebra in terms of its level hyper K-sub algebras.

## 2. *Preliminaries :*

In[6] Young Bae Jun with R.A. Boorzoei M.M. Zahedi & A. Hasanhani established the notion of hyper K-algebras as follows.

*Definition 2.1 :*

By a hyper K-algebra we mean a non-empty set H endowed with a hyper operation 'o' and a constant 0 satisfying the following axioms[1-3].

(HK1) (x o z) o (y o z) < x o y
(HK2) (x o y) o z = (x o z) o y
(HK3) x < x
(HK4) x < y & y < x == > x = y for x, y, z $\in$ H.

Where x < y is defined by 0 $\in$ x o y and for every A, B $\subseteq$ H , A $\subset$ B is defined by there exists a $\in$ A & there exists b $\in$ B such

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that $a < b$.

(HK5) $0 < x$ for all $x \in H$.

Then $(H, o, 0)$ is called a hyper K-algebra.

Let $(H, o, 0)$ be a hyper K-algebra and S be a subset of H containing 0. If S is a hyper K-algebra with respect to the hyper operation 'o' on H. we say that S is a hyper K-subalgebra of H.

*Definition 2.2 :*

A fuzzy set in a set X is a function $\mu : X \rightarrow [0, 1]$

*Definition 2.3 :*

For a fuzzy set $\mu$ in X and $\alpha \in [0, 1]$

Define $U(\mu; \alpha)$ to be the set $U(\mu; \alpha) = \{x \in X / \mu(x) \geq \alpha\}$ which is called a level set of $\mu$.

*Definition 2.4 :*

A fuzzy set $\mu$ in H is said to be a fuzzy hyper K-subalgebra of H if it satisfies the inequality $\inf_{z \in x o y} \mu(z) \geq \min\{\mu(x), \mu(y)\}$ for all $x, y \in H$.

*Proposition 2.5 :*

Let $\mu$ be a fuzzy hyper K-subalgebra of H. Then $\mu(0) \geq \mu(x)$ for all $x \in H$.

*Proof :*

Using (HK3) $0 \in x o x$ for all $x \in H$

Hence $\mu(0) \geq \inf_{z \in x o x} \mu(z) \geq \min\{\mu(x), \mu(x)\} = \mu(x)$.

*Example 2.6 :*

Let $H = \{0, a, b\}$ in which the hyper operation 'o' is given by the following table

| 'o' | 0 | a | b |
|---|---|---|---|
| 0 | {0} | {a} | {0} |
| a | {a} | {0, a} | {0,a} |
| b | {b} | {a, b} | {0, a, b} |

Define a fuzzy set $\mu : H \rightarrow [0, 1]$ by $\mu(0)=\mu(a)= \alpha_1 > \alpha_2 = \mu(b)$ Then $\mu$ is a fuzzy hyper K-subalgebra of H.

Molodt sov defined the notion of soft set in the following way. Let U be an initial universe and E be a set of paremeters. Let P(U) denote the power set of U. Let A be a non-empty subset of E. A pair (f, A) is called a soft set over U where f is a mapping given by $f: A \rightarrow P(U)$ For $\varepsilon \in A$, $f(\varepsilon)$ may be considered as the set of $\varepsilon$-approximate elements of the soft set (f, A). Clearly a soft set is not just a subset of U. Maji *et al*.[3] defined fuzzy soft set in the following way. A pair (f, A) is called a fuzzy soft set over U. where f is a mapping given by $f: A \rightarrow P(U)=I^U$ $I=[0, 1]$. In general for every $\varepsilon \in A$ $f(\varepsilon)=f_\varepsilon$ is a fuzzy set of U and is called a fuzzy value set of parameters. The set of all fuzzy soft sets over U with parameters from E is called a fuzzy soft class and it is denoted by FS(U, E).

*Definition 2.7 :*

A fuzzy soft set (f, A) over U is called a null fuzzy soft set denoted by $\varphi$ if $f(\varepsilon)$ is the null fuzzy set $\bar{0}$ of U where $\bar{0}(x)=0$ for all $x \in U$.

A fuzzy soft set (g, A) over U is called a whole fuzzy soft denoted by U if g(ε) is the whole fuzzy set $\bar{1}$ of U where $\bar{1}(x)=1$ for all $x \in U$ .

To solve decision making problems based on fuzzy soft sets Feng *et al*. introduced the following notion called t-level soft sets of fuzzy soft sets.

*Definition 2.8 :*

Let (f, A) be a fuzzy soft set over U. For each $t \in [0, 1]$ the set $(f, A)^t=(f^t, A)$ is called an t-level soft set of (f, A) where $f_\varepsilon^t=\{x \in U \mid f_\varepsilon^t(x) \geq t\}$ for all $\varepsilon \in A$. Clearly $(f, A)^t$ is a soft set over U.

*Definition 2.9 :*

Let (f, A) & (g, B) be two fuzzy soft sets over U. We say that (f, A) is a fuzzy soft subset of (g, B) and write $(f, A) \subseteq (g, B)$ if

i) $A \subseteq B$
ii) $f(\varepsilon) \subseteq g(\varepsilon)$ for any $\varepsilon \in A$

*Definition 2.10 :*

Let, (f, A) & (g, B) be two fuzzy soft sets over U. Then their extended intersection is a fuzzy soft set denoted by (h, C) where $C = A \cup B$

$$\text{and } h(\varepsilon) = \begin{cases} f_\varepsilon \text{ if } \varepsilon \in A - B \\ g_\varepsilon \text{ if } \varepsilon \in B - A \\ f_\varepsilon \cap g_\varepsilon \text{ if } \varepsilon \in A \cap B \end{cases} \text{ for all } \varepsilon \in C.$$

This is denoted by $(h, C)=(f, A) \tilde{\cap} (g, B)$

*Definition 2.11 :*

If (f, A) and (g, B) are two fuzzy soft sets over the same universe U then "(f, A) and (g, B)" is a fuzzy soft set denoted by $(f, A) \wedge (g, B)$ and it is defined by $(f, A)\wedge(g, B)=(h, A \times B)$ where $h(a, b) = f(a) \cap f(b)$ for all $(a, b) \in A \times B$.

Here $\cap$ is the operation of fuzzy intersection.

*Definition 2.12:*

Let (f, A) and (g, B) be two fuzzy soft sets over U. Then their extended union denoted by (h, C) where $C=A\cup B$

$$\text{and } h(\varepsilon) = \begin{cases} f_\varepsilon \text{ if } \varepsilon \in A - B \\ g_\varepsilon \text{ if } \varepsilon \in B - A \\ f_\varepsilon \cap g_\varepsilon \text{ if } \varepsilon \in A \cap B \end{cases} \text{ for all } \varepsilon \in C.$$

This is denoted by $(h, C)=(f, A) \tilde{\cup} (g, B)$

*Definition 2.13 :*

Let (f, A) and (g, B) be two fuzzy soft sets over a common universe U with $A \cap B \neq \phi$. Then their restricted intersection is a fuzzy soft set, $(h, A\cap B)$ denoted by $(f, A) \cap (g, B) = (h, A\cap B)$ where $h(\varepsilon)=f(\varepsilon)$ for all $\varepsilon \in A\cap B$.

*Definition 2.14 :*

Let (f, A) and (g, B) be two fuzzy soft sets over a common universe U with $A\cap B \neq \phi$ then their restricted union is denoted by $(f, A) \cup (g, B)$ & is defined as $(f, A) \cup (g, B)=(h, C)$ where $C=A\cap B$ and for all $\varepsilon \in C$, $h(\varepsilon) = f(\varepsilon) \cup g(\varepsilon)$

*3. Fuzzy soft set over a hyper K-algebra :*

*Definition 3.1 :*

Let (f, A) be a soft set over a hyper K-algebra H. Then (f, A) is called a soft hyper K-algebra over H if f(x) is a hyper K-subalgebra of H for all $x \in A$.

*Example 3.2 :*

Consider the hyper K-algebra H={0, a, b} with the following table

| 'o' | 0 | a | b |
|---|---|---|---|
| 0 | {0} | {0} | {0} |
| a | {a} | {0, a} | {0,a} |
| b | {b} | {a, b} | {0, a, b} |

Let (f, A) be a soft set over H. where A=H Let f: $A \to P(H)$ be a set valued function f(0)={0}, f(a)={0, a}, f(b)={0, a, b} Then they are all hyper K-subalgebras of H. Therefore (f, A) is a soft hyper K-algebra over H.

*Definition 3.3 :*

Let (f, A) be a fuzzy soft set over H. Then (f, A) is said to be a fuzzy soft hyper K-subalgebra over H if f(x) is a fuzzy hyper K-subalgebra of H for all $x \in A$. (i.e.) A fuzzy set (f, A) over H is called a fuzzy soft hyper K-subalgebra of H if $\inf_{z \in x o y} f_\varepsilon(z) \geq \min\{f_\varepsilon(x), f_\varepsilon(y)\}$ for all $x, y, \in H$.

*Definition 3.4 :*

Let (f, A) & (g, B) be fuzzy soft hyper K-subalgebra over H. Then (f, A) is a fuzzy subalgebra of (g, B) if

i) $A \subset B$
iI) f(x) is a fuzzy subalgebra of g(x) for all $x \in A$.

*Example 3.5 :*

Consider the hyper K-algebra as in example 3.2

Let A={$e_1$, $e_2$, $e_3$}
B={$e_2$, $e_3$}
$f(e_1)$={(0, 0.7), (a, 0.7), (b, 0.6)}
$f(e_2)$={(0, 0.6), (a, 0.5), (b, 0.3)}
$f(e_3)$={(0, 0.4), (a, 0.2), (b, 0.1)}
$g(e_2)$={(0, 0.5), (a, 0.3), (b, 0.1)}
$g(e_3)$={(0, 0.3), (a, 0.2), (b, 0.1)}

clearly (f, A) and (g, B) are fuzzy soft sets over H. For all $x \in A$ & for all $x \in B$.f(x) and g(x) are fuzzy hyper K-subalgebra[5]

Hence (f, A) and (g, B) are fuzzy soft hyper K-subalgebras.

Also $B \subset A$ & g(x) is a fuzzy hyper K-subalgebra of f(x) for all $x \in$ Hence (g, B) is a fuzzy soft hyper K-subalgebra of (f, A).

*Remark.3.6 :*

Let (f, A) be a fuzzy soft hyper k-subalgebra over H. If B is the subset of A then ($f|_B$, B) is a fuzzy soft hyper K-algebra. The following example shows that there exists a fuzzy soft set (f, A) over H such that

i) (f, A) is not a fuzzy soft hyper K-subalgebra over H.
ii) there exists a subset B of A such that ($f|_B$,B) is a fuzzy soft hyper K-subalgebra over H.

*Example 3.7 :*

Let H be a hyper K-algebra as in *eg* 3.2 where H represents the set of houses.

Let A={small, medium, big}

Let (f, A) be a fuzzy soft set over H.

Then f(small), f(medium), f(big) are fuzzy sets defined as follows

| f | 0 | a | b |
|---|---|---|---|
| small | 0.7 | 0.8 | 0.6 |
| medium | 0.6 | 0.5 | 0.3 |
| big | 0.4 | 0.2 | 0.1 |

Then it is easy to check that (f, A) is not a fuzzy soft hyper K-subalgebra over H since f(small) is not a fuzzyhper K-subalgebra

Ler B={medium, big} Then ($f|_B$, B) is a fuzzy soft hyper K-subalgebra over H.

*Proposition 3.8 :*

Let (f, A) and (g, B) be fuzzy soft hyper K-subalgebra over H. Then (f, A) $\tilde{\cap}$ (g, B) is a fuzzy soft hyper K-subalgebra over H.

*Proof :*

By definition 2.10

$h(\varepsilon)=f_\varepsilon$ if $\varepsilon \in A-B$

since (f, A) is a fuzzy soft hyper K-subalgebra for all $\varepsilon \in A$, $f_\varepsilon$ is a fuzzy hyper K-subalgebra similarly if $\varepsilon \in B-A$ then $g_\varepsilon$ is a fuzzy hyper K-subalgebra.

Let $\varepsilon \in A \cap B$

Then $h(\varepsilon)=f_\varepsilon \cap g_\varepsilon$

Let $z \in xoy$ & $x, y \in H$.

$\inf_{z \in x o y} f_\varepsilon \cap g_\varepsilon (z)$

$= \inf_{z \in x o y} [\mu\iota\nu(f_\varepsilon(z), g_\varepsilon(z))]$

$=\min[\inf_{z \in x o y} f_\varepsilon(z), \inf_{z \in x o y} g_\varepsilon(z)]$

$\geq \min [\min(f_\varepsilon(x), f_\varepsilon(y), \min(g_\varepsilon(x), g_\varepsilon(y))]$

$=\min [\min\{f_\varepsilon(x), g_\varepsilon(x)\}, \min \{f_\varepsilon(y), g_\varepsilon(y)\}]$

$=\min [f_\varepsilon \cap g_\varepsilon(x), f_\varepsilon \cap g_\varepsilon(y)]$

Hence $f_\varepsilon \cap g_\varepsilon$ is a fuzzy hyper K-subalgebra over H for all $\varepsilon \in A \cap B$, $h(\varepsilon)$ is a fuzzy hyper K-subalgebra. Therefore (h, C) is a fuzzy soft Hyper K-subalgebra.

*Corollary 3.9 :*

Let (f, A) and (g, A) be two fuzzy soft hyper K-subalgebras over H. Then their extended intersection (f, A) $\tilde{\cap}$ (g, A) is a fuzzy soft hyper K-subalgebra over H.

*Corollary 3.10 :*

The restricted intersection of two fuzzy soft hyper K-subalgebras over H is a fuzzy soft hyper K-subalgebra.

*Proposotion 3.11 :*

Let (f, A) and (g, B) be two fuzzy soft hyper K-subalgebras over H. Then "(f, A) and (g, B)' is a fuzzy soft hyper K-subalgebra over H.

*Proof :*

By definition 2.11

(f, A) and (g, B)=(h, $A\times B$)

(i.e) (f, A) $\wedge$ (g, B)=(h, $A\times B$)

where $h(a, b)(z)=(f_a \cap g_b)(z)$, $z \in xoy$ $x, y \in H$

For all $(a, b) \in A\times B$ & $x, y \in H$

$\inf_{z \in x o y} h(a,b) (z)$

$= \inf_{z \in x o y} [\min(f_a(z), g_b(z))]$

$=\min[\inf_{z \in x o y} f_a(z), \inf_{z \in x o y} g_b(z)]$

$\geq \min[\min(f_a(x), f_a(y), \min(g_b(x), g_b(y))]$
$= \min[\min\{f_a(x), g_b(x)\}, \min\{f_a(y), g_b(y)\}]$
$= \min[f_a \cap g_b(x), f_a \cap g_b(y)]$

Therefore $\inf_{z \in x o y} (f_a \cap g_b)(z) \geq \min[(f_a \cap g_b(x), f_a \cap g_b(y)]$

(i.e) $\inf_{z \in x o y} h_{(a,b)}(z) \geq \min[h_{(a,b)}(x), h_{(a,b)}(y)]$

Thus $h_{(a,b)}$ is fuzzy soft hyper K-subalgebra over H.

*Proposition 3.12 :*

Let (f, A) and (g, B) be fuzzy soft hyper K-subalgebra over H. If $A \cap B = \phi$ then $(f, A) \tilde{\bigcup} (g,B)$ is a fuzzy soft hyper K-subalgebra of H.

*Proof :*

Straight forward from definition 2.12

*Proposition 3.13 :*

Consider the hyper K-algebra as given in the eg 3.2 where H denotes the set of Houses

Let A={beautiful, costly}

& B={costly, moderate}

be the parameter sets

$A \cap B \neq \phi$

Let (f, A) and (g, B) be the fuzzy soft sets over H Then f(beautiful), f(costly), g(costly), g(moderate) are fuzzy sets in H defined as follows.

| f | 0 | a | b |
|---|---|---|---|
| Beautiful | 0.7 | 0.7 | 0.6 |
| Costly | 0.6 | 0.5 | 0.3 |

| g | 0 | a | b |
|---|---|---|---|
| Costly | 0.5 | 0.3 | 0.1 |
| Moderate | 0.3 | 0.2 | 0.1 |

It is easy to check that (f, A) and (g, B) are fuzzy soft hyper K-subalgebra over H.

But $(f, A) \bigcup (g, B)$ is not a fuzzy soft hyper K-subalgebra

Let $z \in a \text{ o } a = \{0, b\}$

Then $\inf_{z \in a o a} [f(\text{costly}) \bigcup g(\text{costly})](z)] = 0.3$ is as seen below [

$f(\text{costly}) \bigcup g(\text{costly})](0) = \max\{f(\text{costly})(0), g(\text{costly}(0)\}$

$= \max\{0.6, 0.5\}$

$= 0.6$

$[f(\text{costly}) \bigcup g(\text{costly})](b) = \max\{f(\text{costly})(b), g(\text{costly})(b)\}$

$= \max\{0.3, 0.1\}$

$= 0.3$

Now, $\min\{[f(\text{costly}) \bigcup g(\text{costly})](a), [f(\text{costly}) \bigcup g(\text{costly})](a)\}$

$= \min\{\max[f(\text{costly}(a), g(\text{costly})(a)], \max[f(\text{costly})(a), g(\text{costly})(a)]\}$

$= \min\{\max[0.5, 0.5], \max[0.5, 0.5]\}$

$= \min\{0.5, 0.5\} = 0.5$

Hence

$\inf_{z \in a o a} [f(\text{costly}) \bigcup g(\text{costly})](z) \ngeq \min\{f(\text{costly}) \bigcup g(\text{costly})](a), [f(\text{costly}) \bigcup g(\text{costly})](a)\}$

*Theorem 3.14 :*

Let (f, A) and (g, B) be fuzzy soft sets over H and $(f, A) \subseteq (g, B)$ with $f_u(x)$. If (g, B) is a fuzzy soft hyper K- subalgebra over H then $g_u(0) \geq f_u(x)$ and $\inf_{z \in x o y} g_u(z) \geq \min\{f_u(x), f_u(y)\}$ for all x, y ∈ H and u ∈ A

*Proof :*

Let (g, B) be a fuzzy soft hyper K-subalgebra over H $\inf_{z \in x o y} g_u(z) \geq \min\{g_u(x), g_u(y)\}$ and (f, A) ⊆ (g, B) for all x, y ∈ H & u ∈ A

$\geq \min\{f_u(x), f_u(y)\}$

Put x = y

Then $\inf_{z \in x o x} g_u(z) \geq \min\{f_u(x), f_u(x)\} = f_u(x)$,

$g_u(0) \geq \inf_{z \in x o x} g_u(z) \geq f_u(x)$

In the following example we show that if (f, A) ⊆ (g, B) and (g, B) is a fuzzy soft hyper K-subalgebra over H but (f, A) is not a fuzzy soft hyper K-subalgebra over H.

*Example 3.15 :*

Let H={0, 1, 2, 3} be a hyper K-algebra with the following Table.

| o | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 0 | {0} | {0} | {0} | {0} |
| 1 | {1} | {0} | {0} | {0} |
| 2 | {2} | {2} | {0} | {2} |
| 3 | {3} | {1,2} | {0,1} | {0,2} |

Let H reperesent a set of houses & E={small, medium, big} is a parameter space[5,7].

Consider A={small}, B={small, big}

Clearly A⊆B

Let (f, A) be a fuzzy soft set over H.

Then f(small) is a fuzzy set defined as follows

| f | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| small | 0.6 | 0.6 | 0.3 | 0.6 |

Let (g, B) be a fuzzy soft over H.

| g | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| Small | 0.6 | 0.5 | 0.3 | 0.3 |
| Blg | 0.8 | 0.2 | 0.5 | 0.2 |

Then g(small), g(big) are fuzzy sets defined as follows

f(small ={0.6, 0.3}⊆{0.6, 0.5, 0.3}= {g(small), g(big)}

(g, B) is a fuzzy soft hyper K-subalgebra

Since g(small) & g(big)are fuzzy hyper K-subalgebra but f(small) is not a fuzzy hyper K-subalgebra.

Therefore (f, A) is not a fuzzy soft hyper K-subalgebra over H.

*Lemma 3.16 :*

Let S be a non-empty subset of H then S is a hyper K-subalgebra of H if and only if xoy ⊆ S for all x, y ∈ S.

*Theorem 3.17 :*

Let (f, A) be a fuzzy soft over H. Then (f, A) is a fuzzy soft hyper K-subalgebra over H if and only if $(f, A)^t$ is a soft hyper K-subalgebra over H for each t ∈ [0, 1]

*Proof :*

Suppose that (f, A) is a fuzzy soft hper K-subalgebra over H.

Then for ever $\varepsilon \in A$ $f_\varepsilon$ is a fuzzy hyper K-subalgebra over H and let x, y ∈ $(f, A)^t$ for all

$t \in [0, 1]$.

Let $z \in xoy$ Then $f_\varepsilon(z) \geq \inf_{w \in xoy} \geq \min \{f_\varepsilon(x), f_\varepsilon(y)\} \geq t$ & so $z \in (f, A)^t$. This shows that

$xoy \subseteq (f, A)^t$. It follows from lemma 3.15 that $(f, A)^t$ is a hyper K-subalgebra of H.

Conversely let $(f, A)^t \neq \phi$ be a hyper K-sub algebra over H. For every $t \in [0, 1]$ for any x, $y \in H$

Let $t=\min \{f_\varepsilon(x), f_\varepsilon(y)\}$. Then $x, y \in (f, A)^t$ and hence $xoy \subseteq (f, A)^t$.

It follows that $f_\varepsilon(z) \geq t$ for all $z \in xoy$ so that

$$\inf_{z \in xoy} f_\varepsilon(z) \geq t=\min \{f_\varepsilon(x), f_\varepsilon(y)\}.$$

This shows that $f_\varepsilon$ is a fuzzy hyper K-subalgebra of H. Since it is true for all $\varepsilon \in A$, (f, A) is a fuzzy soft hyper K-subalgebra over H.

## 4. Conclusion

Fuzzy soft sets are a new mathematical tool to deal with uncertanities. In this paper, we applied the theory to hyper K-algebras. We introduced the notions of fuzzy soft hyper K-subalgebras and many related properties of their union, intersection were surveyed. In our opinion the theory can be extended to intutionistic fuzzy soft hyper K-subalgebras.

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