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# Contra $\alpha g$-Closed and Almost Contra $\alpha g$-Closed Mappings

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## Abstract

The aim of this paper is to introduce and study the concepts of contra $\alpha g$-closed and almost contra $\alpha g$-closed mappings and the interrelationship between other contra-closed maps.

*Key words:* $\alpha g$-open set, $\alpha g$-open map, $\alpha g$-closed map, contra-closed map, contra $\alpha$-open map, contra $\alpha$-closed map, contra $\alpha g$-closed map and almost contra $\alpha g$-closed map.

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## 1. Introduction

Mappings plays an important role in the study of modern mathematics, especially in Topology and Functional analysis. Closed mappings are one such which are studied for different types of closed sets by various mathematicians for the past many years. N.Biswas[22], discussed about semiopen mappings in the year 1969, A.S.Mashhour[31], M.E. Abd El-Monsef and S.N. El-Deeb studied preopen mappings in the year 1982 and S.N. El-Deeb, and I.A.Hasanien[30] defind and studied about preclosed mappings in the year 1983. Further Asit kumar sen and P. Bhattacharya[2] discussed about pre-closed mappings in the year 1993. A.S.Mashhour, I.A. Hasanein and S.N. El-Deeb[31] introduced $\alpha$-open and $\alpha$-closed mappings in the year in 1982, F.Cammaroto and T. Noiri[24,25,34,35] discussed about semipre-open and semipreclosed mappings in the year 1989 and G.B. Navalagi[32,33] further verified few results about semipreclosed mappings. M.E. Abd El-Monsef[31], S.N. El-Deeb and R.A.Mahmoud[1] introduced $\beta$-open mappings in the year 1983. C.W. Baker, introduced Contra-open functions and contra-closed functions in the year 1997. M.Caldas and C.W.Baker[23] introduced contra pre-semiopen Maps in the year 2000. During the years 2010 to 2014, S. Balasubramanian[4-21] together with his research scholars defined and studied a variety of open, closed, almost open and almost closed mappings for *v*-open, rp-open *gpr*-closed and *vg*–closed sets as well contra-open and contra-closed mappings for semi-open, pre-open, rp-open, β-open and gpr-closed sets. Inspired with these concepts and its interesting properties the authors of this paper tried to study a new variety of closed maps called contra *αg*-closed and almost contra *αg*-closed maps. Throughout the paper *X*, *Y* means topological spaces (*X*, τ) and (*Y*,σ) on which no separation axioms are assured[26,29,36].

## 2. Preliminaries:

*Definition 2.1:* A$\subseteq$ X is said to be
a) regular open[α-open] if A = int(cl(A)) [A$\subseteq$ int(cl(int(A)))] and regular closed[α-closed] if A = cl(int(A))[cl(int(cl(A)))$\subseteq$ A]
b) g-closed[rg-closed, αg-closed] if cl(A)$\subset$U[rcl(A)$\subset$U, α-cl(A)$\subset$U] whenever A$\subset$U and U is open[r-open, α-open] in X and g-open[rg-open, αg-open] if its complement X- A is g-closed[rg-closed, αg-closed].

*Remark 1:* We have the following implication diagrams for closed sets.

$$\text{(i)}\quad \begin{array}{ccccccc} & & & & \text{r}\alpha\text{.closed} & & \text{g. closed} \\ & & & & \uparrow & & \downarrow \\ \text{r. closed} & \to & \text{closed} & \to & \alpha\text{.closed} & \to & \alpha\text{g. closed} \end{array}$$

None is reversible

The same relation is true for open sets also.

*Definition 2.2:* A function *f:X*→*Y* is said to be
a) continuous[resp: r-continuous, α-continuous] if the inverse image of every open set is open [resp: r-open, α-open].
b) r-irresolute [resp: α-irresolute] if the inverse image of every r-open [resp: α-open] set is r-open [resp: α-open].
c) closed[resp: r-closed, α-closed] if the image of every closed set is closed [resp: r-closed, α-closed].
d) g-continuous [resp: rg-continuous, αg-continuous] if the inverse image of every closed set is g-closed. [resp: rg-closed, αg-closed].

*Definition 2.3:* A function *f: X*→*Y* is said to be
a) contra closed[resp: contra *α*-closed; contra r*α*-closed; contra r-closed; contra *g*-closed] if the image of every closed set in *X* is open[resp: *α*-open; r*α*-open; r-open; g-open] in *Y*.
b) contra open[resp: contra *α*-open; contra r*α*-open; contra r-open; contra g-open] if the image of every open set in *X* is closed [resp: *α*-closed; r*α*-closed; r-closed; g-closed] in *Y*.
c) almost contra closed [resp: almost contra *α*-closed; almost contra r*α*-closed; almost contra r-closed; almost contra g-closed] if the image of every closed set in *X* is open [resp: *α*-open; r*α*-open; r-open; g-open] in *Y*.
d) almost contra open[resp: almost contra *α*-open; almost contra r*α*-open; almost contra r-open; almost contra *g*-open] if the image of every open set in *X* is closed[resp: *α*-closed; r*α*-closed; r-closed; *g*-closed] in *Y*.

*Definition 2.4:* *X* is said to be $T_{1/2}$[r-$T_{1/2}$] if every (regular) generalized closed set is (regular) closed.

### 3. *Contra $\alpha$G-Closed Mappings:*

*Definition 3.1:* A function $f: X \to Y$ is said to be contra $\alpha g$-closed if the image of every closed set in $X$ is $\alpha g$-open in $Y$.

*Theorem 3.1:* We have the following interrelation among the following contra closed mappings

(i)

$$\begin{array}{ccccccc} & & \text{c.r}\alpha\text{.c} & & & & \text{c.g.c} \\ & & \uparrow & & & & \downarrow \\ \text{c.r.c} & \to & \text{c.c} & \to & \text{c.}\alpha\text{.c} & \to & \text{c.}\alpha\text{g.c} \end{array}$$

None is reversible

(iii) If $\alpha$GO(Y) = RO(Y), then the reverse relations hold for all almost contra closed maps.

$$\begin{array}{ccccccc} & & \text{c.r}\alpha\text{.c} & & & & \text{c.g.c} \\ & & \uparrow\downarrow & & & & \uparrow\downarrow \\ \text{c.r.c} & \leftrightarrow & \text{c.c} & \leftrightarrow & \text{c.}\alpha\text{.c} & \leftrightarrow & \text{c.}\alpha\text{g.c} \end{array}$$

*Example 1:* Let $X = Y = \{a, b, c\}$; $\tau = \{\varphi, \{a\}, \{b, c\}, X\} = \sigma$. Let $f:X \to Y$ be defined $f(a) = c$, $f(b) = a$ and $f(c) = b$. Then $f$ is contra $\alpha g$–closed.

*Example 2:* Let $X = Y = \{a, b, c\}$; $\tau = \{\varphi, \{a\}, \{b, c\}, X\}$; $\sigma = \{\varphi, \{a\}, \{b\}, \{a, b\}, Y\}$. Let $f:X \to Y$ be defined $f(a) = c$, $f(b) = a$ and $f(c) = b$. Then $f$ is not contra $\alpha g$-closed, contra closed, contra $\alpha$-closed, contra r$\alpha$-closed and contra g-closed.

*Theorem 3.2:*

(i) If(Y, $\sigma$) is discrete, then $f$ is contra closed of all types.

(ii) If $f$ is contra closed and $g$ is $\alpha g$-open then $gof$ is contra $g$-closed.

(iii) If $f$ is closed and $g$ is contra $\alpha g$-closed then $gof$ is contra $\alpha g$-closed.

*Corollary 3.1:* If $f$ is contra closed and $g$ is [$r$-; $\alpha$-; $r\alpha$-] open then $gof$ is contra $\alpha g$-closed.

*Corollary 3.2:* If $f$ is closed[ r-closed] and $g$ is c-[$c$-$r$-; c-$\alpha$-; c-$r\alpha$-] closed then $gof$ is contra $\alpha g$-closed.

*Theorem 3.3:* If $f$: X$\to$ Y is contra $\alpha g$-closed, then $f(A^o) \subset \alpha g(f(A))^o$

*Proof:* Let A$\subseteq$ X be closed and $f$: X$\to$ Y is contra $\alpha g$-closed gives $f(A^o)$ is $\alpha g$-open in Y and $f(A^o) \subset f(A)$ which in turn gives $\alpha g(f(A^o))^o \subset \alpha g(f(A))^o$----(1)
Since $f(A^o)$ is $\alpha g$-open in Y, $\alpha g(f(A^o))^o = f(A^o)$----(2)
combining(1) and(2) we have $f(A^o) \subset \alpha g(f(A))^o$ for every subset A of X.

*Remark 2:* Converse is not true in general

*Corollary 3.3:* If $f$: X$\to$Y is c-[$c$-$r$-; c-$\alpha$-; c-$r\alpha$-] closed, then $f(A^o) \subset \alpha g(f(A))^o$

*Theorem 3.4:* If $f:X \to Y$ is contra $\alpha g$-closed and A$\subseteq X$ is closed, $f(A)$ is $\tau_{\alpha g}$-open in $Y$.

*Proof:* Let A$\subset$X be closed and $f$:X$\to$Y is contra $\alpha g$-closed $\Rightarrow f(A^o) \subset \alpha g(f(A))^o \Rightarrow f(A) \subset \alpha g(f(A))^o$, since $f(A) = f(A^o)$. But $\alpha g(f(A))^o \subset f(A)$. Combining we get $f(A) = \alpha g(f(A))^o$. Hence $f(A)$ is $\tau_{\alpha g}$-open in Y.

*Corollary 3.4:* If $f:X \to Y$ is c-[$c$-$r$-; c-$\alpha$-; c-$r\alpha$-]closed, then $f(A)$ is $\tau_{\alpha g}$ open in $Y$ if A is closed set in $X$.

*Theorem 3.5:* If $\alpha g(A)^o = r(A)^o$ for

every A$\subset$Y, then the following are equivalent:
a) $f$: X$\rightarrow$ Y is contra $\alpha g$-closed map
b) $f(A^o) \subset \alpha g(f(A))^o$

*Proof:* (a) $\Rightarrow$(b) follows from theorem 3.3.
(b) $\Rightarrow$(a) Let A be any closed set in X, then $f(A)=f(A^o) \subset \alpha g(f(A))^o$ by hypothesis. We have $f(A) \subset \alpha g(f(A))^o$, which implies $f(A)$ is $\alpha g$-open. Therefore $f$ is contra $\alpha g$-closed.

*Theorem 3.6:* If $\alpha(A)^o = r(A)^o$ for every A$\subset$Y, then the following are equivalent:

a) $f$: X$\rightarrow$ Y is contra $\alpha g$-closed map
b) $f(A^o) \subset \alpha g(f(A))^o$

*Proof:* (a) $\Rightarrow$(b) follows from theorem 3.3.
(b) $\Rightarrow$(a) Let A be any closed set in X, then $f(A)=f(A^o) \subset \alpha g(f(A))^o$ by hypothesis. We have $f(A) \subset \alpha g(f(A))^o$, which implies $f(A)$ is $\alpha g$-open. Therefore $f$ is contra $\alpha g$-closed.

*Theorem 3.7:* $f:X\rightarrow Y$ is contra $\alpha g$-closed iff for each subset S of $Y$ and each open set U containing $f^{-1}(S)$, there is an $\alpha g$-closed set V of $Y$ such that $S\subseteq V$ and $f^{-1}(V) \subseteq U$.

*Proof:* Assume $f:X\rightarrow Y$ is contra $\alpha g$-closed. Let $S \subseteq Y$ and $U\in RO(X, f^{-1}(S))$. Then $X$-U is closed in $X$ and $f(X\text{-}U)$ is $\alpha g$-open in $Y$ as $f$ is contra $\alpha g$-closed and $V=Y\text{-} f(X\text{-}U)$ is $\alpha g$-closed in $Y$. $f^{-1}(S) \subseteq U \Rightarrow S\subseteq f(U) \Rightarrow S \subseteq V$ and $f^{-1}(V) = f^{-1}(Y\text{-} f(X\text{-}U)) = f^{-1}(Y)\text{-} f^{-1}(f(X\text{-}U)) = f^{-1}(Y)\text{-}(X\text{-}U) = X\text{-}(X\text{-}U) = U$

Conversely Let F be open in $X \Rightarrow F^c$ is closed. Then $f^{-1}(f(F^c)) \subseteq F^c$. By hypothesis there exists an $\alpha g$-closed set V of $Y$, such that $f(F^c) \subseteq V$ and $f^{-1}(V)\supset F^c$ and so $F \subseteq [f^{-1}(V)]^c$. Hence $V^c \subseteq f(F) \subseteq f\,[f^{-1}(V)^c] \subseteq V^c \Rightarrow f(F) \subseteq V^c \Rightarrow f(F) = V^c$. Thus $f(F)$ is $\alpha g$-open in $Y$. Therefore $f$ is contra $\alpha g$-closed.

*Remark 3:* Composition of two contra $\alpha g$-closed maps is not contra $\alpha g$-closed in general

*Theorem 3.8:* Let $X$, $Y$, $Z$ be topological spaces and every $\alpha g$-open set is closed in $Y$. Then the composition of two contra $\alpha g$-closed maps is contra $\alpha g$-closed.

*Proof:* (a) Let $f$ and g be contra $\alpha g$-closed maps. Let A be any closed set in $X \Rightarrow f(A)$ is closed in $Y$(by assumption) $\Rightarrow g(f(A)) = g \; o \; f(A)$ is $\alpha g$-open in $Z$. Therefore $gof$ is contra $\alpha g$-closed.

*Corollary 3.5:* Let $X$, $Y$, $Z$ be topological spaces and every [$r$-; $\alpha$-; $r\alpha$-]open set is closed in $Y$. Then the composition of two c-[$c$-$r$-; c-$\alpha$-; c-$r\alpha$-]closed maps is contra $\alpha g$-closed.

*Example 3:* Let $X = Y = Z = \{a, b, c\}$; $\tau = \{\varphi, \{a\}, \{a, b\}, X\}$; $\sigma = \{\varphi, \{a, c\}, Y\}$ and $\eta = \{\varphi, \{a\}, \{b, c\}, Z\}$. $f:X\rightarrow Y$ be defined $f(a) = c$, $f(b) = b$ and $f(c) = a$ and $g:Y\rightarrow Z$ be defined $g(a) = b$, $g(b) = a$ and $g(c) = c$, then $g$, $f$ and $g \; o \; f$ are contra $\alpha g$-closed.

*Theorem 3.9:* If $f:X\rightarrow Y$ is contra g-closed[ contra rg-closed], g:$Y$'!$Z$ is $\alpha g$-open and $Y$ is $T_{½}$ [$r$-$T_{½}$] then $gof$ is contra $\alpha g$-closed.

*Proof:* (a) Let A be closed in $X$. Then $f(A)$ is $g$-open and so open in $Y$ as $Y$ is $T_{½} \Rightarrow g(f(A)) = gof(A)$ is $\alpha g$-open in $Z$. Hence $gof$ is contra $\alpha g$-closed.

*Corollary 3.6:* If $f:X\rightarrow Y$ is contra $g$-closed[contra $rg$-closed], $g:Y\rightarrow Z$ is [$r$-; $\alpha$-;

$r\alpha$-]open and $Y$ is $T_{1/2}$ [$r$-$T_{1/2}$] then $gof$ is contra $\alpha g$-open.

*Theorem 3.10:* If $f:X\to Y$ is g-closed[ rg-closed], g:$Y\to Z$ is contra $\alpha g$-closed and $Y$ is $T_{1/2}$ [$r$-$T_{1/2}$] then $gof$ is contra $\alpha g$-closed.

*Proof:* (a) Let A be closed in $X$. Then $f$(A) is $g$-closed and so closed in $Y$ as $Y$ is $T_{1/2}$ $\Rightarrow g(f(A)) = gof(A)$ is $\alpha g$-open in $Z$. Hence $gof$ is contra $\alpha g$-closed.

*Theorem 3.11:* If $f:X\to Y$ is g-closed[ rg-closed], g:$Y\to Z$ is c-[$c$-$r$-; c-$\alpha$-; c-$r\alpha$-] closed and $Y$ is $T_{1/2}$ [$r$-$T_{1/2}$] then $gof$ is contra $\alpha g$-closed.

*Theorem 3.12:* If $f:X\to Y$, $g:Y\to Z$ be two mappings such that $gof$ is contra $\alpha g$-closed [contra closed] then the following statements are true.
a) If $f$ is continuous [$r$-continuous] and surjective then $g$ is contra $\alpha g$-closed.
b) If $f$ is g-continuous[resp: rg-continuous], surjective and $X$ is $T_{1/2}$ [resp: $r$-$T_{1/2}$] then $g$ is contra $\alpha g$-closed.

*Proof:* A closed in $Y$, $f^{-1}$(A) closed in $X \Rightarrow (g\ o\ f)(f^{-1}(A)) = g(A)$ $\alpha g$-open in $Z$. Hence $g$ is contra $\alpha g$-closed.

Similarly one can prove the remaining parts and hence omitted.

*Corollary 3.7:* If $f:X\to Y$, g:$Y\to Z$ be two mappings such that $gof$ is c-[$c$-$r$-; c-$\alpha$-; c-$r\alpha$-]closed then the following statements are true.
a) If $f$ is continuous [$r$-continuous] and surjective then $g$ is contra $\alpha g$-closed.
b) If $f$ is g-continuous[rg-continuous], surjective and $X$ is $T_{1/2}$ [$r$-$T_{1/2}$] then $g$ is contra $\alpha g$-closed.

*Theorem 3.13:* If $f:X\to Y$, $g:Y\to Z$ be two mappings such that $gof$ is $\alpha g$-open then the following statements are true.
a) If $f$ is contra-continuous [contra-$r$-continuous] and surjective then $g$ is contra $g$-closed.
b) If $f$ is contra-g-continuous[contra-rg-continuous], surjective and $X$ is $T_{1/2}$ [resp: $r$-$T_{1/2}$] then $g$ is contra $\alpha g$-closed.

*Proof:* A closed in $Y$, $f^{-1}$(A) open in $X$ $\Rightarrow (g\ o\ f)(f^{-1}(A)) = g(A)$ $\alpha g$-open in $Z$. Hence $g$ is contra $\alpha g$-closed.

*Corollary 3.8:* If $f:X\to Y$, g:$Y\to Z$ be two mappings such that $gof$ is [$r$-; $\alpha$-; $r\alpha$-] open then the following statements are true.
a) If $f$ is contra-continuous [contra-$r$-continuous] and surjective then $g$ is contra $\alpha g$-closed.
b) If $f$ is contra-g-continuous[contra-rg-continuous], surjective and $X$ is $T_{1/2}$ [$r$-$T_{1/2}$] then $g$ is contra $\alpha g$-closed.

*Theorem 3.14:* If $X$ is $\alpha g$-regular, $f:X\to Y$ is $r$-open, $r$-continuous, contra $\alpha g$-closed surjective and $A^o = A$ for every $\alpha g$-open set in $Y$ then $Y$ is $\alpha g$-regular.

*Corollary 3.9:* If $X$ is $\alpha g$-regular, $f:X\to Y$ is $r$-open, $r$-continuous, contra $\alpha g$-closed, surjective and $A^o = A$ for every closed set in $Y$ then $Y$ is $\alpha g$-regular.

*Theorem 3.15:* If $f:X\to Y$ is contra $\alpha g$-closed and $A\in RC(X)$, then $f_A:(X,\tau(A))\to(Y,\sigma)$ is contra $\alpha g$-closed.

*Proof:* Let F be closed in $A$. Then F = A)$\cap$E for some closed set E of $X$ and so F is closed in $X \Rightarrow f$(A) is $\alpha g$-open in $Y$. But $f$(F) = $f_A$(F). Therefore $f_A$ is contra $\alpha g$-closed.

*Theorem 3.16:* If $f:X\to Y$ is contra

$\alpha g$-closed, $X$ is $rT_{½}$ and A is rg-closed set of $X$ then $f_A:(X,\tau(A))\rightarrow(Y,\sigma)$ is contra $\alpha g$-closed.

*Proof:* Let F be closed in $A$. Then F = A$\cap$E for some closed set E of $X$ and so F is closed in $X \Rightarrow f(A)$ is $\alpha g$-open in $Y$. But $f(F) = f_A(F)$. Therefore $f_A$ is contra $\alpha g$-closed.

*Corollary 3.10:* If $f:X\rightarrow Y$ is c-[$c$-$r$-; c-$\alpha$-; c-$r\alpha$-] open and $A\in RC(X)$, then $f_A:(X,\tau(A))$’!$(Y,\sigma)$ is contra $\alpha g$-open.

*Theorem 3.17:* If $f_i:X_i\rightarrow Y_i$ be contra $\alpha g$-closed for i =1, 2. Let $f: X_1 x X_2\rightarrow Y_1 x Y_2$ be defined as $f(x_1,x_2) =(f_1(x_1), f_2(x_2))$. Then $f:X_1 x X_2\rightarrow Y_1 x Y_2$ is contra $\alpha g$-closed.

*Proof:* Let $U_1 x U_2 \subseteq X_1 x X_2$ where $U_i$ is closed in $X_i$ for i = 1,2. Then $f(U_1 x U_2) = f_1(U_1) \text{ x } f_2(U_2)$ is $\alpha g$-open set in $Y_1 x Y_2$. Hence $f$ is contra $\alpha g$-closed.

*Corollary 3.11:* If $f_i:X_i\rightarrow Y_i$ be c-[$c$-$r$-; c-$\alpha$-; c-$r\alpha$-]closed for i =1, 2. Let $f:X_1 x X_2 \rightarrow Y_1 x Y_2$ be defined as $f(x_1,x_2) =(f_1(x_1), f_2(x_2))$, then $f:X_1 x X_2\rightarrow Y_1 x Y_2$ is contra $\alpha g$-closed.

## *4. Almost Contra $\alpha$G-Closed Mappings:*

*Definition 4.1:* A function $f: X\rightarrow Y$ is said to be almost contra $\alpha g$-closed if the image of every $r$-closed set in $X$ is $\alpha g$-open in $Y$.

*Theorem 4.1:* Every contra $\alpha g$-closed almost contra $\alpha g$ –closed map but not conversely.

*Theorem 4.2:* We have the following interrelation among the following almost contra closed mappings

(i)
$$\begin{tikzcd} & al.c.r\alpha.c & & al.c.g.c \arrow[d] \\ al.c.r.c \arrow[r] & al.c.c \arrow[u] \arrow[r] & al.c.\alpha.c \arrow[r] & al.c.\alpha g.c \end{tikzcd}$$
None is reversible

(ii)
$$\begin{tikzcd} & c.r\alpha.c & & c.g.c \arrow[d] \\ c.r.c \arrow[r] & c.c \arrow[u] \arrow[r] & c.\alpha.c \arrow[r] & al.c.\alpha g.c \end{tikzcd}$$
None is reversible

(iii) If $\alpha GO(Y) = RO(Y)$, then the reverse relations hold for all almost contra closed maps.
$$\begin{tikzcd} al.c.r\alpha.c \arrow[d, leftrightarrow] & & & al.c.g.c \arrow[d, leftrightarrow] \\ al.c.r.c \arrow[r, leftrightarrow] & al.c.c \arrow[r, leftrightarrow] & al.c.\alpha.c \arrow[r, leftrightarrow] & al.c.\alpha g.c \end{tikzcd}$$

*Example 4:* Let $X = Y = \{a, b, c\}$; $\tau = \{\varphi, \{a\}, \{b, c\}, X\} = \sigma$. Let $f:X\rightarrow Y$ be defined $f(a) = c$, $f(b) = a$ and $f(c) = b$. Then $f$ is contra almost $\alpha g$–closed.

*Example 5:* Let $X = Y = \{a, b, c\}$; $\tau = \{\varphi, \{a\}, \{b, c\}, X\}$; $\sigma = \{\varphi, \{a\}, \{b\}, \{a, b\}, Y\}$. Let $f:X\rightarrow Y$ be defined $f(a) = c$, $f(b) = a$ and $f(c) = b$. Then $f$ is not almost contra $\alpha g$-closed, almost contra closed, almost contra $\alpha$-closed, almost contra r$\alpha$-closed and almost contra g-closed.

*Theorem 4.3:*
(i) If $(Y, \sigma)$ is discrete, then $f$ is almost contra closed of all types.
(ii) If $f$ is almost contra closed and $g$ is $\alpha g$-open then $gof$ is almost contra $\alpha g$-closed.
(iii) If $f$ is almost closed and $g$ is contra $\alpha g$-closed then $gof$ is almost contra $\alpha g$-closed.

*Corollary 4.1:* If $f$ is almost contra closed and $g$ is [$r$-; $\alpha$-; $r\alpha$-] open then $gof$ is almost contra $\alpha g$-closed.

*Corollary 4.2:* If $f$ is almost closed [almost $r$-open] and $g$ is c-[$c$-$r$-; c-$\alpha$-; c-$r\alpha$-] closed then $gof$ is almost contra $\alpha g$-closed.

*Theorem 4.4:* If $f$: X→ Y is almost contra $\alpha g$-closed, then $f(A^o) \subset \alpha g\ (f(A))^o$

*Proof:* Let A⊆ X be $r$-closed and $f$: X→ Y is almost contra $\alpha g$-closed gives $f(A^o)$ is $\alpha g$-open in Y and $f(A^o) \subset f(A)$ which in turn gives $\alpha g\ (f(A^o))^o \subset \alpha g\ (f(A))^o$---- (1)
Since $f(A^o)$ is $\alpha g$-open in Y, $\alpha g(f(A^o))^o = f(A^o)$-----(2)
combining (1) and (2) we have $f(A^o) \subset \alpha g\ (f(A))^o$ for every subset A of X.

*Remark 4:* Converse is not true in general.

*Corollary 4.3:* If $f$: X→ Y is al-c-[al-$c$-$r$-; al-c-$\alpha$-; al-c-$r\alpha$-] closed, then $f(A^o) \subset \alpha g\ (f(A))^o$

*Theorem 4.5:* If $f$:$X$’!$Y$ is almost contra $\alpha g$-closed and A⊆$X$ is $r$-closed, $f$(A) is $\tau_{\alpha g}$-open in $Y$.

*Proof:* Let A⊂X be $r$-closed and $f$:X→Y is almost contra $\alpha g$-closed ⇒ $f(A^o) \subset \alpha g\ (f(A))^o \Rightarrow f(A) \subset \alpha g\ (f(A))^o$, since $f(A) = f(A^o)$. But $\alpha g\ (f(A))^o \subset f(A)$. Combining we get $f(A) = \alpha g\ (f(A))^o$. Hence $f$(A) is $\tau_{\alpha g}$-open in Y.

*Corollary 4.4:* If $f$:$X$→$Y$ is al-c-[al-$c$-$r$-; al-c-$\alpha$-; al-c-$r\alpha$-]closed, then $f$(A) is $\tau_{\alpha g}$ open in $Y$ if A is $r$-closed set in $X$.

*Theorem 4.6:* If $f$:$X$→$Y$ is c-[$c$-$r$-; c-$\alpha$-; c-$r\alpha$-] closed and A⊆$X$ is $r$-closed, $f$(A) is $\tau_{\alpha g}$-open in $Y$.

*Proof:* For A⊂X is $r$-closed and $f$:X→Y is c-$r$-closed, $f$(A) is $\tau_r$-open in Y and so $f$(A) is $\tau_{\alpha g}$-open in Y. [since $r$-open set is $\alpha g$-open]. Similarly we can prove the remaining results.

*Theorem 4.7:* If $\alpha g\ (A)^o = r(A)^o$ for every A⊂ Y, then the following are equivalent:
a) $f$: X→ Y is almost contra $\alpha g$-closed map
b) $f(A^o) \subset \alpha g\ (f(A))^o$

*Proof:* (a) ⇒(b) follows from theorem 4.4.
(b) ⇒ (a) Let A be any $r$-closed set in X, then $f(A) = f(A^o) \subset \alpha g\ (f(A))^o$ by hypothesis. We have $f(A) \subset \alpha g\ (f(A))^o$, which implies $f$(A) is $\alpha g$-open. Therefore $f$ is almost contra $\alpha g$-closed.

*Theorem 4.8:* If $\alpha(A)^o = r(A)^o$ for every A⊂ Y, then the following are equivalent:
a) $f$: X→ Y is contra $\alpha g$-closed map
b) $f(A^o) \subset \alpha g\ (f(A))^o$

*Proof:* (a) ⇒(b) follows from theorem 4.4.
(b) ⇒ (a) Let A be any $r$-closed set in X, then $f(A) = f(A^o) \subset \alpha g\ (f(A))^o$ by hypothesis. We have $f(A) \subset \alpha g\ (f(A))^o$, which implies $f$(A) is $\alpha g$-open. Therefore $f$ is almost contra $\alpha g$-closed.

*Theorem 4.9:* $f$:$X$→$Y$ is almost contra $\alpha g$-closed iff for each subset S of $Y$ and each U∈RO(X, $f^{-1}$(S)), there is an $\alpha g$-closed set V of $Y$ such that S⊆V and $f^{-1}(V) \subseteq$ U.

*Proof:* Assume $f$:$X$→$Y$ is almost contra $\alpha g$-closed. Let S⊆$Y$ and U∈RO(X, $f^{-1}$(S)). Then $X$-U is r-closed in $X$ and $f$($X$-U) is $\alpha g$-open in $Y$ as $f$ is almost contra $\alpha g$-closed and V=$Y$- $f$($X$-U) is $\alpha g$-closed in $Y$. $f^{-1}(S) \subseteq$ U

$\Rightarrow$ S$\subseteq$ $f$(U) $\Rightarrow$ S $\subseteq$ V and $f^{-1}$(V) = $f^{-1}$($Y$- $f$($X$-U)) = $f^{-1}$($Y$)- $f^{-1}$($f$ ($X$-U)) = $f^{-1}$($Y$)-($X$-U) = $X$-($X$-U) = U

Conversely Let F be r-open in $X$ $\Rightarrow$ $F^c$ is r-closed. Then $f^{-1}(f\,(F^c)) \subseteq F^c$. By hypothesis there exists an $\alpha g$-closed set V of $Y$, such that $f\,(F^c) \subseteq V$ and $f^{-1}(V) \supset F^c$ and so F $\subseteq [f^{-1}(V)]^c$. Hence $V^c \subseteq f\,(F) \subseteq f\,[f^{-1}(V)^c] \subseteq V^c \Rightarrow f\,(F) \subseteq V^c \Rightarrow f\,(F) = V^c$. Thus $f$ (F) is $\alpha g$-open in $Y$. Thus $f$ is almost contra $\alpha g$-closed.

*Remark 5:* Composition of two almost contra $\alpha g$-closed maps is not almost contra $\alpha g$-closed in general.

*Theorem 4.10:* Let $X$, $Y$, $Z$ be topological spaces and every $\alpha g$-open set is $r$-closed in $Y$. Then the composition of two almost contra $\alpha g$-closed maps is almost contra $\alpha g$-closed.

*Proof:* (a) Let $f$ and g be almost contra $\alpha g$-closed maps. Let A be any $r$-closed set in $X \Rightarrow f$(A) is $r$-closed in $Y$ (by assumption) $\Rightarrow$ $g(f\,(A)) = g\ o\ f(A)$ is $\alpha g$-open in $Z$. Therefore $gof$ is almost contra $\alpha g$-closed.

*Theorem 4.11:* Let $X$, $Y$, $Z$ be topological spaces and every [$r$-; $\alpha$-; $r\alpha$-]open set is $r$-closed in $Y$. Then the composition of two al-c-[al-$c$-$r$-; al-c-$\alpha$-; al-c-$r\alpha$-]open maps is almost contra $\alpha g$-closed.

*Proof:* Let A be $r$-closed set in X, then $f$(A) is $r$-open in Y and so $r$-closed in $Y$ (by assumption) $\Rightarrow g(f\,(A)) = gof(A)$ is $r$-open in $Z$. Hence $gof$ is almost contra $\alpha g$-closed

*Corollary 4.5:* Let $X$, $Y$, $Z$ be topological spaces and every [$r$-; $\alpha$-; $r\alpha$-]open set is closed [$r$-closed] in $Y$. Then the composition of two c-[$c$-$r$-; c-$\alpha$-; c-$r\alpha$-]closed maps is almost contra $\alpha g$-closed.

*Example 6:* Let $X = Y =$ Z = {a, b, c}; $\tau = \{\varphi, \{a\}, \{a, b\}, X\}$; $\sigma = \{\varphi, \{a, c\}, Y\}$ and $\eta = \{\varphi, \{a\}, \{b, c\}, Z\}$. $f{:}X \to Y$ be defined $f$(a) = c, $f$(b) = b and $f$(c) = a and $g{:}Y \to Z$ be defined $g$(a) = b, $g$(b) = a and $g$(c) = c, then $g$, $f$ and $g\ o\ f$ are almost contra $\alpha g$-closed.

*Theorem 4.12:* If $f{:}X \to Y$ is almost contra g-closed[almost contra rg-closed], g:$Y \to Z$ is $\alpha g$-open and $Y$ is $T_{½}$ [$r$-$T_{½}$] then $gof$ is almost contra $\alpha g$-closed.

*Proof:* (a) Let A be $r$-closed in $X$. Then $f$(A) is $g$-open and so open in $Y$ as $Y$ is $T_{½} \Rightarrow g(f\,(A)) = gof\,(A)$ is $\alpha g$-open in $Z$ (since $g$ is $\alpha g$-open). Hence $gof$ is almost contra $\alpha g$-closed.

*Corollary 4.6:* If $f{:}X$'!$Y$ is almost contra $g$-closed[almost contra $rg$-closed], $g{:}Y \to Z$ is [$r$-; $\alpha$-; $r\alpha$-]open and $Y$ is $T_{½}$ [$r$-$T_{½}$] then $gof$ is almost contra $\alpha g$-open.

*Theorem 4.13:* If $f{:}X \to Y$ is almost g-closed[almost rg-closed], g:$Y \to Z$ is contra $\alpha g$-closed and $Y$ is $T_{½}$ [$r$-$T_{½}$] then $gof$ is almost contra $\alpha g$-closed.

*Proof:* (a) Let A be $r$-closed in $X$. Then $f$(A) is $g$-closed and so closed in $Y$ as $Y$ is $T_{½} \Rightarrow g(f\,(A)) = gof\,(A)$ is $\alpha g$-open in $Z$ (since $g$ is contra $\alpha g$-closed). Hence $gof$ is almost contra $\alpha g$-closed.

*Theorem 4.14:* If $f{:}X \to Y$ is almost g-closed[almost rg-closed], g:$Y \to Z$ is c-[$c$-$r$-; c-$\alpha$-; c-$r\alpha$-]closed and $Y$ is $T_{½}$ [$r$-$T_{½}$] then $gof$ is almost contra $\alpha g$-closed.

*Theorem 4.15:* If $f:X\to Y$ is $g$-closed [$rg$-closed], $g:Y\to Z$ is c-[$c$-$r$-; c-$\alpha$-; c-$r\alpha$-]closed and $Y$ is $T_{½}$[$r$-$T_{½}$], then $gof$ is almost contra $\alpha g$-closed.

*Proof:* Let A be $r$-closed set in X, then $f$(A) is $g$-open in Y and so open in $Y$ (by assumption) $\Rightarrow g(f(A)) = gof(A)$ is $r$-open in $Z$. Hence $gof$ is almost contra $\alpha g$-closed.

*Theorem 4.16:* If $f:X\to Y$ is c-$g$-closed[c-$rg$-closed], $g:Y\to Z$ is [$r$-; $\alpha$-; $r\alpha$-]open and $Y$ is $T_{½}$[$r$-$T_{½}$], then $gof$ is almost contra $\alpha g$-closed.

*Proof:* Let A be $r$-closed set in X, then $f$(A) is $g$-open in Y and so open in $Y$ (by assumption) $\Rightarrow g(f(A)) = gof(A)$ is $r$-open in $Z$. Hence $gof$ is almost contra $\alpha g$-closed.

*Theorem 4.17:* If $f:X\to Y$, $g:Y\to Z$ be two mappings such that $gof$ is contra $\alpha g$-closed [almost contra $r$-closed] then the following statements are true.
a) If $f$ is continuous [$r$-continuous] and surjective then $g$ is almost contra $\alpha g$-closed.
b) If $f$ is g-continuous[resp: rg-continuous], surjective and $X$ is $T_{½}$ [resp: $r$-$T_{½}$] then $g$ is almost contra $\alpha g$-closed.

*Proof:* (a) For A $r$-closed in $Y$, $f^{-1}$(A) closed in $X \Rightarrow (g\ o\ f)(f^{-1}(A)) = g(A)$ $\alpha g$-open in $Z$. Hence $g$ is almost contra $\alpha g$-closed.

Similarly one can prove the remaining parts and hence omitted.

*Corollary 4.7:* If $f:X\to Y$, $g:Y\to Z$ be two mappings such that $gof$ is contra $\alpha g$-closed [almost contra $r$-closed] then the following statements are true.
a) If $f$ is almost continuous [almost $r$-continuous] and surjective then $g$ is almost contra $\alpha g$-closed.
b) If $f$ is almost g-continuous[resp: almost rg-continuous], surjective and $X$ is $T_{½}$ [resp: $r$-$T_{½}$] then $g$ is almost contra $\alpha g$-closed.

*Corollary 4.8:* If $f:X\to Y$, g:$Y\to Z$ be two mappings such that $gof$ is c-[$c$-$r$-; c-$\alpha$-; c-$r\alpha$-]closed then the following statements are true.
a) If $f$ is continuous [$r$-continuous] and surjective then $g$ is almost contra $\alpha g$-closed.
b) If $f$ is g-continuous[rg-continuous], surjective and $X$ is $T_{½}$ [$r$-$T_{½}$] then $g$ is almost contra $\alpha g$-closed.

*Corollary 4.9:* If $f:X\to Y$, g:$Y\to Z$ be two mappings such that $gof$ is c-[$c$-$r$-; c-$\alpha$-; c-$r\alpha$-]closed then the following statements are true.
a) If $f$ is almost continuous [almost $r$-continuous] and surjective then $g$ is almost contra $\alpha g$-closed.
b) If $f$ is almost g-continuous[almost rg-continuous], surjective and $X$ is $T_{½}$ [$r$-$T_{½}$] then $g$ is almost contra $\alpha g$-closed.

*Theorem 4.18:* If $f:X\to Y$, $g:Y\to Z$ be two mappings such that $gof$ is $\alpha g$-open then the following statements are true.
a) If $f$ is contra-continuous [contra-$r$-continuous] and surjective then $g$ is almost contra $\alpha g$-closed.
b) If $f$ is contra-g-continuous[contra-rg-continuous], surjective and $X$ is $T_{½}$ [resp: $r$-$T_{½}$] then $g$ is almost contra $\alpha g$-closed.

*Proof:* (a) For A $r$-closed in $Y$, $f^{-1}$(A) open in $X \Rightarrow (g\ o\ f)(f^{-1}(A)) = g(A)$ $\alpha g$-open in $Z$. Hence $g$ is almost contra $\alpha g$-closed.

*Corollary 4.10:* If $f:X\rightarrow Y$, $g:Y\rightarrow Z$ be two mappings such that $gof$ is $\alpha g$-open then the following statements are true.

a) If $f$ is almost contra-continuous [almost contra-$r$-continuous] and surjective then $g$ is almost contra $\alpha g$-closed.

b) If $f$ is almost contra-g-continuous[almost contra-rg-continuous], surjective and $X$ is $T_{½}$ [resp: $r$-$T_{½}$] then $g$ is almost contra $\alpha g$-closed.

*Corollary 4.11:* If $f:X\rightarrow Y$, $g:Y\rightarrow Z$ be two mappings such that $gof$ is [$r$-; $\alpha$-; $r\alpha$-] open then the following statements are true.

a) If $f$ is contra-continuous [contra-$r$-continuous] and surjective then $g$ is almost contra $\alpha g$-closed.

b) If $f$ is contra-g-continuous[contra-rg-continuous], surjective and $X$ is $T_{½}$ [$r$-$T_{½}$] then $g$ is almost contra $\alpha g$-closed.

*Corollary 4.12:* If $f:X\rightarrow Y$, $g:Y\rightarrow Z$ be two mappings such that $gof$ is [$r$-; $\alpha$-; $r\alpha$-] open then the following statements are true.

a) If $f$ is almost contra-continuous [almost contra-$r$-continuous] and surjective then $g$ is almost contra $\alpha g$-closed.

b) If $f$ is almost contra-g-continuous[almost contra-rg-continuous], surjective and $X$ is $T_{½}$ [$r$-$T_{½}$] then $g$ is almost contra $\alpha g$-closed.

*Theorem 4.19:* If $X$ is $\alpha g$-regular, $f:X\rightarrow Y$ is $r$-open, $r$-continuous, almost contra $\alpha g$-closed surjective and $A^o = A$ for every $\alpha g$-open set in $Y$ then $Y$ is $\alpha g$-regular.

*Proof:* Let $p\in U\in \alpha GO(Y)$, $\exists$ a point $x\in X \ni f(x) = p$ by surjection. Since X is $\alpha g$-regular and $f$ is nearly-continuous, $\exists$ $V\in RC(X) \ni x\in V^o\subset V\subset f^{-1}(U)$ which implies $p\in f(V^o)\subset f(V)\subset U$------ (1)
for $f$ is almost contra $\alpha g$-closed, $f(V^o)\subset U$ is $\alpha g$-open. By hypothesis $f(V^o)^o = f(V^o)$ and $f(V^o)^o = \{f(V)\}^o$-----(2)
combaining (1) and (2) $p\in f(V)^o\subset f(V) \subset U$ and $f(V)$ is $r$-closed. Hence Y is $\alpha g$-regular.

*Corollary 4.13:* If $X$ is $ag$-regular, $f:X\rightarrow Y$ is $r$-open, $r$-continuous, almost contra $ag$-closed, surjective and $A^o = A$ for every $r$-closed set in $Y$ then $Y$ is $ag$-regular.

*Theorem 4.20:* If $f:X\rightarrow Y$ is almost contra $\alpha g$-closed and $A\in RC(X)$, then $f_A:(X,\tau(A))\rightarrow(Y,\sigma)$ is almost contra $\alpha g$-closed.

*Proof:* Let F be an $r$-closed set in $A$. Then $F = A\cap E$ for some $r$-closed set E of $X$ and so F is $r$-closed in $X \Rightarrow f(A)$ is $\alpha g$-open in $Y$. But $f(F) = f_A(F)$. Therefore $f_A$ is almost contra $\alpha g$-closed.

*Theorem 4.21:* If $f:X\rightarrow Y$ is almost contra $\alpha g$-closed, $X$ is $rT_{½}$ and A is rg-closed set of $X$ then $f_A:(X,\tau(A))\rightarrow(Y,\sigma)$ is almost contra $\alpha g$-closed.

*Proof:* Let F be a $r$-closed set in $A$. Then $F = A\cap E$ for some $r$-closed set E of $X$ and so F is $r$-closed in $X \Rightarrow f(A)$ is $\alpha g$-open in $Y$. But $f(F) = f_A(F)$. Therefore $f_A$ is almost contra $\alpha g$-closed.

*Corollary 4.14:* If $f:X\rightarrow Y$ is c-[$c$-$r$-; c-$\alpha$-; c-$r\alpha$-] open and $A\in RC(X)$, then $f_A:(X,\tau(A))\rightarrow(Y,\sigma)$ is almost contra $\alpha g$-open.

*Corollary 4.15:* If $f:X$'!$Y$ is al-c-[al-$c$-$r$-; al-c-$\alpha$-; al-c-$r\alpha$-]closed and $A\in RC(X)$, then $f_A:(X,\tau(A))\rightarrow(Y,\sigma)$ is almost contra $\alpha g$-closed.

*Theorem 4.22:* If $f_i:X_i\rightarrow Y_i$ be almost contra $\alpha g$-closed for i =1, 2. Let $f: X_1 x X_2 \rightarrow Y_1 x Y_2$ be defined as $f(x_1,x_2) = (f_1(x_1), f_2(x_2))$.

Then $f:X_1 x X_2 \to Y_1 x Y_2$ is almost contra $\alpha g$-closed.

*Proof:* Let $U_1 x U_2 \subseteq X_1 x X_2$ where $U_i$ is $r$-closed in $X_i$ for i = 1,2. Then $f(U_1 x U_2) = f_1(U_1) \text{ x } f_2(U_2)$ is $\alpha g$-open set in $Y_1 x Y_2$. Hence $f$ is almost contra $\alpha g$-closed.

*Corollary 4.16:* If $f_i:X_i \to Y_i$ be al-c-[al-$c$-$r$-; al-c-$\alpha$-; al-c-$r\alpha$-] closed for i = 1, 2. Let $f:X_1 x X_2 \to Y_1 x Y_2$ be defined as $f(x_1,x_2) = (f_1(x_1), f_2(x_2))$, then $f:X_1 x X_2 \to Y_1 x Y_2$ is almost contra $\alpha g$-closed.

*Corollary 4.17:* If $f_i:X_i \to Y_i$ be c-[$c$-$r$-; c-$\alpha$-; c-$r\alpha$-]closed for i =1, 2. Let $f:X_1 x X_2 \to Y_1 x Y_2$ be defined as $f(x_1,x_2) = (f_1(x_1), f_2(x_2))$, then $f:X_1 x X_2 \to Y_1 x Y_2$ is almost contra $\alpha g$-closed.

## Conclusion

In this paper the authors introduced the concepts of contra $\alpha g$-closed mappings, almost contra $\alpha g$-closed mappings, studied their basic properties and interrelationship between other such contra closed maps.

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