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Steady MHD flow of a Visco-Elastic fluid Through Horizontal Channel Bounded by Horizontal Plates in Porous Medium in Presence of Transverse Magnetic field

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Abstract

The steady laminar flow of an electrically conducting visco-elastic fluid through horizontal channel, bounded by two horizontal plates, lower being a stretching sheet and the upper being permeable plate bounded by porous medium in presence of transverse magnetic field, has been discussed analytically. The expressions for skin-friction coefficient at both the surfaces have been derived and discussed numerically, their values being presented through table.

Key words: Visco-elastic, MHD, stretching sheet, stretching Reynolds number, Hartmann number, permeable parameter, skin-friction.

1. Introduction

The problem of laminar flow control is gaining considerable importance for engineers in the design of machinery, particularly in the design of turbo machinery e. g. rotary compressor and turbines. Flow between porous and non-porous boundaries is of practical as well as theoretical interest. The steady laminar flow of a visco-elastic fluid between two parallel porous discs when the fluid is injected at uniform rate through both the discs has been discussed by Das and Haque[4]. Unsteady symmetric viscous incompressible fluid flow between two rotating porous disc was discussed by Sharma and Singh[7]. Sharma and Singh[6] also discussed steady laminar flow and heat transfer of viscous incompressible fluid between two porous discs. The flow of a heat transfer in second order fluid between two naturally permeable, coaxial, stationary parallel discs in presence of magnetic field has been discussed by Chouhan and Ghiya[3]. Alves, Molchanov and Tagilva[1] studied the flow of non-Newtonian fluid between parallel planes and showed the possibility of calculation of technological parameters in filling process by non-Newtonian fluids of narrow gaps. The steady flow of an electrically conducting viscous fluid through horizontal channel bounded by porous medium in presence of transverse magnetic field has been discussed by Sharma and Mishra[5]. The flow of second order fluid through porous medium in between two parallel plates has been discussed Charyulu and Ram[2]. The object of the present paper is to investigate the trend of non-Newtonian behaviour by considering the flow of a visco-elastic fluid characterized by Walters liquid (Model B$^{/}$) in a highly porous medium the presence of transverse magnetic field which is bounded by two horizontal non-conductive plates[7].

The constitutive equation for the Walters liquid B$^{/}$ is given by

$$\sigma_{ij} = -pg_{ik} + \tau_{ij},$$

$$\tau^{ij}(x,t) = 2\int_0^\infty \varphi(t-t^{/})\frac{dx^j}{dx^{im}}\cdot\frac{dx^j}{dx^{ir}}e^{(1)mr}(x^{/},t^{/})dt^{/} \quad (1.1)$$

where σ_{ij} is the stress tensor, p is an arbitrary isotropic pressure, g_{ij} is the metric tensor of fixed coordinate system, $x^i, x^{/i}$ the position at the time $t^{/}$ of the element which is instantaneously at the point x^i at time t, $e_{ij}^{(1)}$ the rate of strain tensor and

$$\varphi(t-t^{/}) = \int_0^\infty \frac{N(\tau)}{\tau}e^{-(t-t^{/})/\tau}d\tau \quad (1.2)$$

$N(\tau)$ being the distributive function of the relaxation time τ.

It has been shown by Walters[8,9] that in case of liquids with short memories (i.e. short relaxation times), the equation of state can be simplified to

$$\tau^{ij} = 2\eta_0 e^{(1)ij} - 2K_0\frac{\delta}{\delta t}e^{(1)ij} \quad (1.3)$$

$$\frac{\delta}{\delta t}b^{ij} = \frac{\partial b^{ij}}{\partial t} + v^m\frac{\partial b^{ij}}{\partial x^m} - \frac{\partial v^j}{\partial x^m}b^{im} - \frac{\partial v^i}{\partial x^m}b^{mj} \quad (1.4)$$

v_i is the velocity vector.

1. Equation:

Consider the steady flow of a Walters liquid (ModelB$^{/}$) in a highly porous medium in the presence of transverse magnetic field which is bounded by two horizontal non-conductive plates. The lower plate is taken as stretching sheet and the upper one is solid porous plate. The fluid is electrically conducting. The fluid motion is due to stretching of the lower plate with injection applied in the upper plate. All the physical quantities are referred to the Cartesian coordinate system, the x-axis being taken along the lower plate whereas y-axis is normal to the plates. The positions of the pates are at y = 0 and y = h, h being the distance between the two parallel plates. Two equal and opposite forces are introduced to stretch the lower plate so that its position remains unchanged. The fluid is injected through the upper solid plate with constant velocity v_0. The induced magnetic field is neglected which is valid for small magnetic Reynolds number. The external electric field is zero and the electric field due to polarization of charges is negligible[1-9].

The governing equations of motion are

$$\frac{\partial\bar{u}}{\partial\bar{x}} + \frac{\partial\bar{v}}{\partial\bar{y}} = 0 \quad (2.1)$$

$$\sigma a_x = \frac{\partial\sigma_{xx}}{\partial\bar{x}} + \frac{\partial\sigma_{xy}}{\partial\bar{y}} + \sigma\frac{B_0^2}{\rho}\bar{u} - \frac{\vartheta}{\bar{k}}\bar{u} \quad (2.2)$$

$$\sigma a_y = \frac{\partial\sigma_{xy}}{\partial\bar{x}} + \frac{\partial\sigma_{yy}}{\partial\bar{y}} - \frac{\vartheta}{\bar{k}}\bar{v} \quad (2.3)$$

where $a_x = \bar{u}\,\frac{\partial\bar{u}}{\partial\bar{x}} + \bar{v}\frac{\partial\bar{u}}{\partial\bar{y}}$

$$a_y = \bar{u}\,\frac{\partial\bar{v}}{\partial\bar{x}} + \bar{v}\frac{\partial\bar{v}}{\partial\bar{y}}$$

$$\sigma_{xx} = -\bar{p} + 2\eta_0\frac{\partial\bar{u}}{\partial\bar{x}} - 2\overline{K_0}\left[\left(\bar{u}\frac{\partial}{\partial\bar{x}} + \bar{v}\frac{\partial}{\partial\bar{y}}\right)\frac{\partial\bar{u}}{\partial\bar{x}} - \left\{2\left(\frac{\partial\bar{u}}{\partial\bar{x}}\right)^2 + \left(\frac{\partial\bar{v}}{\partial\bar{x}} + \frac{\partial\bar{u}}{\partial\bar{y}}\right)\frac{\partial\bar{u}}{\partial\bar{y}}\right\}\right]$$

$$\sigma_{yy} = -\bar{p} + 2\eta_0\frac{\partial\bar{v}}{\partial\bar{y}} - 2\overline{K_0}\left[\left(\bar{u}\frac{\partial}{\partial\bar{x}} + \bar{v}\frac{\partial}{\partial\bar{y}}\right)\frac{\partial\bar{v}}{\partial\bar{y}} - \left\{2\left(\frac{\partial\bar{v}}{\partial\bar{y}}\right)^2 + \left(\frac{\partial\bar{v}}{\partial\bar{x}} + \frac{\partial\bar{u}}{\partial\bar{y}}\right)\frac{\partial\bar{v}}{\partial\bar{x}}\right\}\right]$$

$$\sigma_{xy} = \eta_0\left(\frac{\partial\bar{u}}{\partial\bar{y}} + \frac{\partial\bar{v}}{\partial\bar{x}}\right) - 2\overline{K_0}\left[\frac{1}{2}\left(\bar{u}\frac{\partial}{\partial\bar{x}} + \bar{v}\frac{\partial}{\partial\bar{y}}\right)\left(\frac{\partial\bar{v}}{\partial\bar{x}} + \frac{\partial\bar{u}}{\partial\bar{y}}\right) - \left\{\frac{\partial\bar{u}}{\partial\bar{x}}\frac{\partial\bar{v}}{\partial\bar{x}} + \frac{1}{2}\left(\frac{\partial\bar{v}}{\partial\bar{x}} + \frac{\partial\bar{u}}{\partial\bar{y}}\right)\frac{\partial\bar{v}}{\partial\bar{y}}\right\} - \left\{\frac{1}{2}\left(\frac{\partial\bar{v}}{\partial\bar{x}} + \frac{\partial\bar{u}}{\partial\bar{y}}\right)\frac{\partial\bar{u}}{\partial\bar{x}} + \frac{\partial\bar{u}}{\partial\bar{y}}\frac{\partial\bar{v}}{\partial\bar{y}}\right\}\right],$$

$\bar{u}, \bar{v}$ are velocity components of the fluid along the axes of $\bar{x}$ and $\bar{y}$, ρ is the density, ϑ is the kinematic viscosity, σ is the magnetic conductivity and B_0 is the magnetic intensity.

The boundary conditions are:

$$\bar{y} = 0;\ \bar{u} = c\bar{x}\ ;\ \bar{v} = 0$$
$$\bar{y} = h;\ \bar{u} = 0\ ;\ \bar{v} = -v_0 \qquad (2.4)$$

where c is the stretching parameter.

1. Solution:

Substituting

$$\bar{u} = c\bar{x}f^{/}(y);\quad \bar{v} = -chf(y) \qquad (3.1)$$

in equations (2.2) and (2.3), we get

$$-\frac{\vartheta^2}{h^2}\frac{\partial p}{\partial x} = c^2xh\left[\left(f^{/2} - ff^{//} - \frac{f^{///}}{Re} + \frac{M^2}{Re}f^{/} + \frac{1}{\kappa Re}f^{/}\right) + K_0\left(ff^{IV} - 2f^{/}f^{///} + f^{//2}\right)\right] \quad (3.2)$$

$$-\frac{\vartheta^2}{h^2}\frac{\partial p}{\partial y} = c^2h\left[\left(ff^{/} + \frac{f^{//}}{Re} + \frac{1}{\kappa Re}f\right) + K_0\left(3f^{/}f^{//} - ff^{///}\right)\right] \qquad (3.3)$$

where $Re = \frac{ch^2}{\vartheta}$ is the stretching Reynolds number, $M = \left(\frac{\sigma}{\rho\vartheta}\right)^{\frac{1}{2}}B_0h$, the Hartmann number, $\kappa = \frac{\bar{\kappa}}{h^2}$ the permeability parameter, $x = \frac{\bar{x}}{h}, y = \frac{\bar{y}}{h}, K_0 = \frac{\overline{K_0}}{\rho h^2}$.

Eliminating p between (3.2) and (3.3), we get

$$\left(f^{/2} - ff^{//} - \frac{f^{///}}{Re} + \frac{M^2}{Re}f^{/} + \frac{1}{\kappa Re}f^{/}\right) + K_0\left(ff^{IV} - 2f^{/}f^{///} + f^{//2}\right) = -A \qquad (3.4)$$

A is a constant to be determined.

The Boundary Conditions become

$$y = 0;\ f^{/} = 1, f = 0$$
$$y = 1;\ f^{/} = 0, f = R_c \qquad (3.5)$$

where $R_c = \frac{v_0}{ch}$ the cross-flow Reynolds number.

Using the regular perturbation technique f and A can be expanded in the form:

$$f = \sum_{n=0} Re^n f_n\ ,\quad A = \sum_{n=0} Re^n A_n \qquad (3.6)$$

Substituting (3.6) in (3.4) and equating like powers of *Re*, we have

$$f_0^{///} - \left(M^2 + \frac{1}{\kappa}\right) f_0^{/} = A_0 \tag{3.7}$$

$$f_1^{///} - \left(M^2 + \frac{1}{\kappa}\right) f_1^{/} = A_1 + \left(f_0^{/2} - f_0 f_0^{//}\right) +$$

$$K_0 \left(f_0 f_0^{IV} - 2 f_0^{/} f_0^{///} + f_0^{//2}\right) \tag{3.8}$$

The corresponding boundary conditions become

$$y = 0;\ f_0 = 0;\ f_0^{/} = 1;\ f_n = f_n^{/} 0\ \ \forall\, n > 0$$

$$y = 1; f_0 = R_c; f_0^{/} = 0, f_n = f_n^{/} 0\ , \forall\, n > 0 \tag{3.9}$$

The solutions of f_0 and f_1 are given by

$$f_0 = \beta_1 + \beta_2 e^{\alpha y} + \beta_3 e^{-\alpha y} - \frac{A_0}{\alpha^2} y \tag{3.10}$$

$$f_1 = \beta_4 + \left(\beta_5 - \frac{N_5}{\alpha^2}\right) y + \left(\beta_5 + \frac{7}{8}\frac{N_3}{\alpha^4}\right) e^{\alpha y} +$$

$$\left(\beta_6 + \frac{7}{8}\frac{N_4}{\alpha^4}\right) e^{-\alpha y} - \left(\frac{N_1}{2\alpha^2} + \frac{3}{4}\frac{N_3}{\alpha^3}\right) y e^{\alpha y} +$$

$$\left(\frac{N_2}{2\alpha^2} + \frac{3}{4}\frac{N_3}{\alpha^3}\right) y e^{-\alpha y} + \frac{N_3}{2\alpha^2} y^2 e^{\alpha y} + \frac{N_4}{2\alpha^2} y^2 e^{-\alpha y} \tag{3.11}$$

where $\alpha = \sqrt{M^2 + \frac{1}{\kappa}}$ and β_1 to β_6 , N_1 to N_5 are constants given bellow.

$$A_0 = \frac{1}{2}\left(\frac{1 + e^{-\alpha}}{1 - e^{-\alpha}}\alpha^3 - \alpha^2\right)$$

$$\beta_1 = \frac{\propto - \alpha e^{\alpha} - 2}{2(1 - e^{\alpha})} - \frac{1}{2\alpha}$$

$$\beta_2 = \frac{1 - \propto}{2(1 - e^{\alpha})}$$

$$\beta_3 = -\frac{\propto + e^{-\alpha}}{2(1 - e^{-\alpha})} - \frac{1}{2\alpha}$$

$$\beta_4 = -\beta_5 - \beta_6 - \frac{7}{8\alpha^4} N_3 - \frac{7}{8\alpha^4} N_4$$

$$\beta_5 = \frac{1}{2\alpha^2} N_1 + \frac{2\alpha - 1}{2\alpha^2(e^{\alpha} + \alpha e^{2\alpha} - \alpha)} N_2 + \frac{e^{-\alpha} - (4\alpha^2 + 2\alpha + 1)e^{\alpha}}{8\alpha^3(1 + \alpha e^{\alpha} - \alpha e^{-\alpha})} N_3 + \frac{2\alpha - 1}{4\alpha^2(1 + \alpha e^{\alpha} - \alpha e^{-\alpha})} N_4 + \frac{1}{\alpha^2(1 + \alpha e^{\alpha} - \alpha e^{-\alpha})} N_5$$

$$\beta_6 = \beta_5 - \frac{1}{2\alpha^2}(N_1 - N_2) + \frac{1}{8\alpha^4}(N_3 - N_4)$$

$$N_1 = \left\{\frac{2A_0}{p} + p^2\beta_1 - Kp(p^3\beta_1 + 2A_0)\right\}\beta_2$$

$$N_2 = \left\{\frac{2A_0}{p} - p^2\beta_1 + Kp(p^3\beta_1 - 2A_0)\right\}\beta_3$$

$$N_3 = A_0\beta_2(1 - Kp^2)$$

$$N_4 = A_0\beta_3(1 - Kp^2)$$

$$N_5 = \frac{A_0^{\,2}}{p^4} - 4p^2\beta_2\beta_3 + 8Kp^4\beta_2\beta_3$$

The skin friction coefficient is given by

$$c_f = \frac{\tau_{xy}/_{\bar{y}=0,\ h}}{\rho c^2 h^2} = \frac{x}{Re} f^{//}/_{y=0,\ 1}$$

1. Results and Discussion

The results have been numerically worked out for various combinations of the parameters. It has been observed from table-1 that the skin friction coefficient at the stretching sheet increases with the increase in the values of visco-elastic parameter. Similarly the skin friction coefficient at the permeable plate also increases due to increase in the value of the visco-elastic parameter. The fluid velocity distribution and the normal velocity distribution have been presented graphically as seen in figure 1 and 2. The purpose of the present paper is to bring out effects of visco-elastic parameter on the flow characteristics as the effects of the other parameters have been discussed in details by Sharma and Mishra[5]. It is observed from figure 1 that fluid velocity increases with the increase in the value of visco-elastic parameter. From figure 2, it has been seen that normal velocity also increases with the increase in the value of visco-elastic parameter.

Table-1. The values of skin friction coefficient at the stretching sheet and porous plate for different values of visco-elastic parameter (K). Values of other parameters being taken as $x = 2, M = 1.0, \kappa = .25, Re = 0.2, Rc = 1.0$

K	$(c_f)_0$	$(c_f)_1$
0.01	-84.28583024	4.624491562
0.1	-77.83898337	12.05400424
0.25	-72.7355031	23.83359873

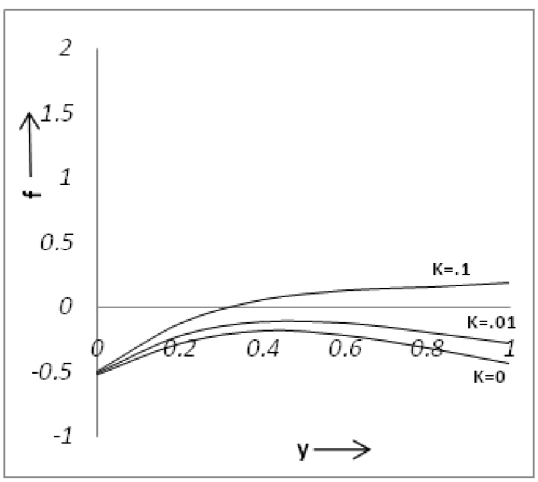

Figure-1: Normal velocity distribute
$(M = 1.0, \ \kappa = .25, Re = 0.2, Rc = 1.0)$

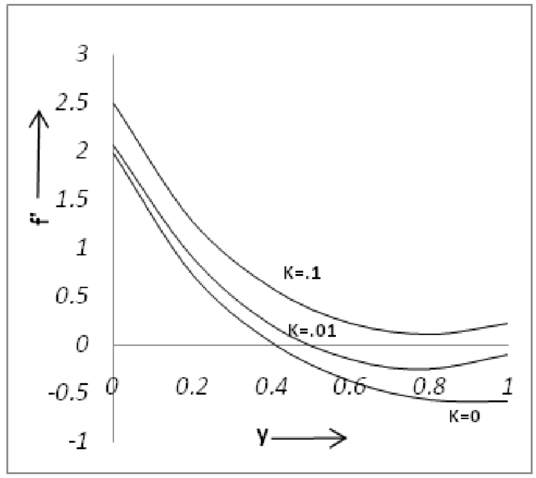

Figure-2: Fluid velocity distribution
$(M = 1.0, \kappa = .25, Re = 0.2, Rc = 1.0)$

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