Ultra Scientist Vol. 27(2)A, 135-144 (2015).

Modified Multivariable H-Function and its Fractional Integration via Pathway Operator

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(Acceptance Date 24th June, 2015)

Abstract

In recent paper Saxena, Ram and Daiya[1] developed fractional integration of Multivariable H-function via pathway operator. The object of this paper is to derive fractional integration of Modified Multivariable H-function via pathway operator.

1 Introduction

The modified multi-variable H-function employed as kernel of multi-dimensional transform defined by Prasad and Singh[5] on the lines of Srivastava and Panda[7], Prasad and Maurya[4] is as follows:

$$H^{m,n:|R':m_1,n_1;\dots;m_r,n_r}_{p,q:|\,R:p_1,q_1;\dots;\,p_r,q_r}\left[\begin{matrix} z_1 \\ \vdots \\ z_r \end{matrix}\middle| \begin{matrix} (a_j;\alpha'_j,\dots,\alpha_j^{(r)})_{1,p}:(e_j;u'_j g'_j,\dots,u_j^{(r)} g_j^{(r)})_{1,R'}:(c'_j,\gamma'_j)_{1,p_1};\dots;(c_j^{(r)},\gamma_j^{(r)})_{1,p_r} \\ (b_j;\beta'_j,\dots,\beta_j^{(r)})_{1,q}:(L_j;U'_j f'_j,\dots,U_j^{(r)} f_j^{(r)})_{1,R}:(d'_j,\delta'_j)_{1,q_1};\dots;(d_j^{(r)},\delta_j^{(r)})_{1,q_r} \end{matrix}\right]$$

$$= \frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r} \Phi_1(\xi_1)\dots\Phi_r(\xi_r)\psi(\xi_1,\dots,\xi_r) z_1^{\xi_1}\dots z_r^{\xi_r} d\xi_1\dots d\xi_r \qquad \dots\dots (1.1)$$

where

$$\Phi_i(\xi_i) = \frac{\prod_{j=1}^{m_i}\Gamma\left(d_j^{(i)}-\delta_j^{(i)}\xi_i\right)\prod_{j=1}^{n_i}\Gamma\left(1-c_j^{(i)}-\gamma_j^{(i)}\xi_i\right)}{\prod_{j=m_i+1}^{q_i}\Gamma\left(1-\alpha_j^{(i)}+\delta_j^{(i)}\xi_i\right)\prod_{j=n_i+1}^{p_i}\Gamma\left(c_j^{(i)}-\gamma_j^{(i)}\xi_i\right)} (i = 1,2,\dots,r) \qquad \dots\dots\dots\dots (1.2)$$

$$\psi(\xi_i,\ldots,\xi_r)=\frac{\prod_{j=1}^{m_i}\Gamma\left(b_j-\sum_{i=1}^{r}\beta_j^{(i)}\xi_i\right)\prod_{j=1}^{n}\Gamma\left(1-a_j+\sum_{i=1}^{r}\alpha_j^{(i)}\xi_i\right)\prod_{j=1}^{R'}\Gamma\left(e_j+\sum_{i=1}^{r}u_j^{(i)}g_j^{(i)}\xi_i\right)}{\prod_{j=m+1}^{p}\Gamma\left(a_j-\sum_{i=1}^{r}\alpha_j^{(i)}\xi_i\right)\prod_{j=n+1}^{q}\Gamma\left(1-b_j+\sum_{i=1}^{r}\beta_j^{(i)}\xi_i\right)\prod_{j=1}^{R}\Gamma\left(L_j+\sum_{i=1}^{r}U_j^{(i)}f_j^{(i)}\xi_i\right)} \tag{1.3}$$

The multiple integral (1.1) converges absolutely if

$$|\arg z_i|<\frac{1}{2}U_i\pi,\qquad (i=1,2,\ldots,r).$$

where

$$U_i=\sum_{j=1}^{m}\beta_j^{(i)}-\sum_{j=m+1}^{q}\beta_j^{(i)}+\sum_{j=1}^{n}\alpha_j^{(i)}-\sum_{j=n+1}^{p}\alpha_j^{(i)}\sum_{j=1}^{m}\delta_j^{(i)}-\sum_{j=m_1+1}^{q_i}\delta_j^{(i)}\sum_{j=1}^{n_1}\gamma_j^{(i)}-\sum_{j=n_i+1}^{p_i}\gamma_j^{(i)}+\sum_{j=1}^{R'}g_j^{(i)}-\sum_{j=1}^{R}f_j^{(i)}>0(i=1,2\ldots,r) \tag{1.3a}$$

from Srivastava and Panda [29, p. 131], we have

$$H[z_1\ldots z_r]=0\left(|z_1|^{e_1}\ldots|z_r|^{e_r}\right)\left(\max_{1\le j\le r}\ \|z_j\|\to 0\right) \tag{1.4}$$

where

$$e_i=\min_{1\le j\le r}\ \left[\frac{\mathrm{Re}(d_i{}^{(i)})}{\delta_j{}^{(i)}}\right](i=1,\ldots r)$$

for n=p=q=0 the modified multivariable H-function breaks up into product of ‘r’ H-functions and consequently there holds the following result (see Mathai *et al.*[16]).

$$H^{0,0:|R':m_1,n_1;\ldots;m_r,n_r}_{0,0:|R:p_1,q_1;\ldots;p_r,q_r}\left[\begin{matrix}z_1\\ \vdots\\ z_r\end{matrix}\left|\begin{matrix}(e_j;u_j{}'_jg'_j,\ldots,u_j^{(r)}g_j^{(r)})_{1,\backslash R'}:(c'_j,\gamma'_j)_{1,p_1};\ldots;(c_j{}^{(r)},\gamma_j{}^{(r)})_{1,p_r}\\ (l_j;U'_jf'_j,\ldots,U_j^{(r)}f_j^{(r)})_{1,\backslash R}:(d'_j,\delta'_j)_{1,q_1};\ldots;(d_j{}^{(r)},\delta_j{}^{(r)})_{1,q_r}\end{matrix}\right.\right]$$

$$=\prod_{i=1}^{r}H^{|R':m_i,n_i}_{|R:p_i,q_i}\left[z_i\left|\begin{matrix}(e_j;u_j{}'_jg'_j,\ldots,u_j^{(r)}g_j^{(r)})_{1,\backslash R'}:(c_j{}^{(i)},\gamma_j{}^{(i)})_{1,p_i}\\ (l_j;U'_jf'_j,\ldots,U_j^{(r)}f_j^{(r)})_{1,\backslash R}:(d_j{}^{(i)},\delta_j{}^{(i)})_{1,q_i}\end{matrix}\right.\right] \tag{1.5}$$

A general class of multivariable polynomials of real or complex variables $x_1,...x_s$ is defined and studied by Srivastava and Garg[26] in the following form[8-12]

$$S_L^{h_1,\dots,h_s}(x_1,\dots,x_s) = \sum_{k_1,\dots,k_s=0}^{h_1+k_1+\dots+h_sk_s\le L} (-L)h_1k_1 + \dots + h_sk_s$$

$$A(L;k_1,\dots,k_s)\frac{x_1^{k_1}}{k_i!}---\frac{x_s^{k_s}}{k_s!} \qquad (1.6)$$

where L, $(h_1,\dots h_s \in \mathbb{N}_0 = \{0,1,\dots\}$ and the coefficients $A(L;k_1,\dots,k_s)$ $(k_j \in \mathbb{N}_0; j = 1,\dots,S)$are arbitrary constants real or complex. In the case S=1 of the above polynomials would correspond to the polynomials (See Srivastava[24])

$$S_L^h(x) = \sum_{k=0}^{[^1/h]} \frac{(-1)_{hk}}{k!} A_{1k}x^k \qquad (1.7)$$

The coefficients $A_{1k} = (1, k \in \mathbb{N}_0)$ are arbitrary constants real or complex.

It is well known fact that fractional integration operators play an important role in the solution of several problems of science and engineering. There are many fractional integral operators like Riemann-liouville, Wegle, Kober, Erdily-Kober and Soigo operators. These operators are studied by various Mathematicians due to their applications in the solution of integral equations in several problem of many areas of physical, engineering and technological sciences.[2,3,15]

A detailed description of these operators can be found in the survey paper by Srivastava and Saxena[30].

Let $f(x) \in L(a,b), \alpha \in C, R(\alpha) > 0$, then left sided Riemann-Liouville fractional integral operator is defined as[6]

$$(I_{0+}^{\alpha}F)(x) = \frac{1}{\Gamma(\alpha)}\int_0^x (x-t)^{\alpha-1} f(t)dt.$$

where $R(\alpha) > 0$.

Definition

Let $f(x) \in L(a,b), \eta \in C, R(\eta) > 0, a > 0$ and let us take a "pathway parameter" $\alpha < 1$. Then the Pathway fractional integration operator is defined by Nair[17] as

$$\left(P_{0+}^{(\eta,\alpha)}f\right)(x) = x^{\eta}\int_0^{\left[\frac{x}{a(1-\alpha)}\right]}\left(1-\frac{a(1-\alpha)t}{x}\right)^{\frac{\eta}{1-\alpha}} f(t)dt \qquad (1.9)$$

For further details of Pathway model and its applications one can refer to paper by Mathai and Haubold[13, 14].

For $R(\alpha) > 0$, the pathway model for scalar random variables is represented by the following probability density function.

$$f(x) = c|x|^{\gamma-1}\left[1-a(1-\alpha)|x|^{\delta}\right]^{\frac{\beta}{1-\alpha}} \qquad (1.10)$$

$$-\infty < x < \infty, \delta > 0, \beta \ge 0,$$

$$\left[1-a(1-\alpha]|x|^{\delta}\right] > 0, \gamma > 0,$$

where c is the normalizing constant and α is the pathway parameter. For real α, the normalizing constant is as follows:

$$C = \frac{1}{2}\frac{\delta[a(1-\alpha)]^{\frac{\gamma}{\delta}}\ \Gamma\left[\frac{\gamma}{\delta}+\frac{\beta}{1-\alpha}+1\right]}{\Gamma\left(\frac{\gamma}{\delta}\right)\ \Gamma\left(\frac{\beta}{1-\alpha}+1\right)}, \text{For } \alpha < 1 \quad (1.11)$$

$$= \frac{1}{2}\frac{\delta[a(1-\alpha)]^{\frac{\gamma}{\delta}}\ \Gamma\left[\frac{\beta}{\alpha-1}\right]}{\Gamma\left(\frac{\gamma}{\delta}\right)\Gamma\left(\frac{\beta}{\alpha-1}-\frac{\gamma}{\delta}\right)}, \text{For } \frac{1}{\alpha-1}-\frac{\gamma}{\delta} > 0, \alpha > 1 \quad (1.12)$$

$$= \frac{1}{2}\frac{\delta(a\beta)^{\frac{\gamma}{\delta}}}{\Gamma\left(\frac{\gamma}{\delta}\right)}, \text{For } \alpha \to 1 \quad (1.13)$$

For $\alpha < 1$, it is a finite range density with $1 - a(1-\alpha)|x|^{\delta} > 0$ and Eq[n] (1.10) remains in the extended generalized type-I beta family. The Pathway density in Eq[n](1.10), for α <1, includes the extended type –I beta density, the triangular density, the uniform density and many other probability density function.

When α > 1, we write 1-α = -(α -1), then

$$\left(P_{0+}^{(\eta,\alpha)}\right)(x) = x^{\eta}\int_0^{\left[\frac{x}{-a(\alpha-1)}\right]}\left[1+\frac{a(\alpha-1)t}{x}\right]^{\frac{\eta}{-(\alpha-1)}} f(t)dt \quad (1.14)$$

$$f(x) = c|x|^{\gamma-1}\left[1+a(\alpha-1)|x|^{\delta}\right]^{-\frac{\beta}{\alpha-1}} \quad (1.14)$$

$-\infty < x < \infty, \delta > 0, \beta \geq 0, \alpha > 1$ which is the extended generalized type-2 beta model for real x. It includes the type-2 beta density, the F density, the student –t density, the cauchy density.

Here we consider the case of Pathway parameter for á < 1. For á →1 both Eq[n](1.10) and Eq[n](1.15) take the exponential form, Since

$$\lim_{\alpha\to 1} c|x|^{\gamma-1}\left[1-a(1-\alpha)|x|^{\delta}\right]^{\frac{n}{1-\alpha}}$$

$$= \lim_{\alpha\to 1} c|x|^{\gamma-1}\left[1-a(\alpha-1)|x|^{\delta}\right]^{\frac{-n}{\alpha-1}}$$

$$= c|x|^{\gamma-1}e^{-a\eta}|x|^{\delta} \quad (1.16)$$

When

$$\alpha \to 1, \left(1-\frac{a(1-\alpha)t}{x}\right)^{\frac{\eta}{1-\alpha}} \to e^{-\frac{a\eta}{x}t},$$

then operator (1.9) reduce to

$$\left(P_{0+}^{(\eta,1)} f\right)(x) = x^{\eta}\int_0^{\infty} e^{-\frac{a\eta}{x}t}\ f(t)dt$$

$$= x^{\eta}L_f\left(\frac{a\eta}{x}\right) \quad (1.17)$$

it reduces to the Laplace integral transform of f with parameter, $\frac{a\eta}{x}$:

$$L_f(x) = \int_0^{\infty} e^{-xt}\ f(t)dt$$

When α = 0, a = 1 and η is replaced by η = -1 in (1.9)

$$\left(I_{0+}^{\eta}\ f\right)(x) = \frac{1}{\Gamma(\eta)}\int_0^{\infty}(x-t)^{\eta-1} f(t)dt \quad (1.18)$$

Which is the left– sided Riemann – Liouville fractional integral discussed by Samko (*et al.* [22])

In 2012 integration of Aleph function by means of Pathway model is denonstrated by Saxena *et al.*[23].

Main Result

Theorem 2.1

Let $\eta, p \in C, R(\beta) > 0, R\left(1+\frac{\eta}{a-\alpha}\right) > 0, R(p+\mu_i e_i) > 0$

(i=1, ..., r) and α <1, b $\in$ R. Then for the pathway fractional operator $P_{O+}^{(\eta,\alpha)}$ the following formula holds:

$$(P_{O+}^{(\eta,\alpha)} t^{p-1} H[z_1 t^{\mu_1}, \ldots, z_r t^{\mu_r}] = \frac{x^{\eta+p\Gamma\left(1+\frac{\eta}{1-\alpha}\right)}}{[a(1-\alpha)]^p} \text{ x}$$

$$H^{o,n+1:|R':m_1,n_1,\ldots,m_r,n_r}_{p+1,q+1:|R:p_1,q_1;\ldots;p_r,q_r}$$

$$\left[\begin{matrix} z_1\left[\frac{x}{a(1-\alpha)}\right]^{\mu_1} \\ \vdots \\ z_r\left[\frac{x}{a(1-\alpha)}\right]^{\mu_r} \end{matrix}\left|\begin{matrix} (1-\rho;\mu_1,\ldots,\mu_r),(a_j;\alpha_j^{(1)},\ldots,\alpha_j^{(r)})_{1,p}; \\ \\ (1-\rho-\frac{\eta}{1-\alpha};\mu_1,\ldots,\mu_r),(b_j;\beta_j^{(1)},\ldots,\beta_j^{(r)})_{1,q}; \end{matrix}\right.\right.$$

$$\left.\begin{matrix} (e_j;U_j',\ldots,U_j^{(r)}g_j^{(r)})_{1,|R};(c_j';\gamma_j')_{1,q_1};\ldots;(c_j^{(r)},\gamma_j^{(r)})_{1,p_r} \\ (l_j;U_j'f_j',\ldots,U_j^{(r)}f_j^{(r)})_{1,|R'};(d_j';\delta_j')_{1,q_1};\ldots;(d_j^{(r)};\delta_j^{(r)})_{1,q_r} \end{matrix}\right] \qquad (2.1)$$

where e_i is defined in (1.4)

Proof : Using equation (1.1) and (1.9), it follows that

$$I = x^\eta \int_0^{\frac{x}{a(1-\alpha)}} t^{\rho-1}\left[1-\frac{a(1-\alpha)t}{x}\right]^{\frac{\eta}{1-\alpha}} dt \frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\Psi(\xi_1,\ldots,\xi_r)$$

$$\left\{\prod_{i=1}^{r}\Phi_i(\xi_i)[z_i t^{\mu_i}]^{\xi_i}\right\} d\xi_1,\ldots,d\xi_r$$

$$= x^\eta \int_0^{\frac{x}{a(1-\alpha)}} t^{\rho-1}\left[1-\frac{a(1-\alpha)}{x}\right]^{\frac{\eta}{1-\alpha}} \frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\Psi(\xi_1,\ldots,\xi_r)$$

$$\left\{\prod_{i=1}^{r} \Phi_i\,(\xi_i) {z_i}^{\xi_i}\right\} t^k d\xi_i, \ldots, d\xi_r$$

$$\text{where } k = \sum_{i=1}^{r} \mu_i \xi_i$$

$$= x^\eta \frac{1}{(2\pi\omega)^r} \int_{L_1}, \ldots, \int_{L_r} \Psi(\xi_1, \ldots, \xi_r) \left\{\prod_{i=1}^{r} \Phi_i\,(\xi_i) {z_i}^{\xi_i}\right\} d\xi_i, \ldots, d\xi_r$$

$$\int_0^{\frac{x}{a(1-\alpha)}} t^{\rho+K-1} \left[1 - \frac{a(1-\alpha)t}{x}\right]^{\frac{\eta}{1-\alpha}} dt$$

put $\frac{a(1-\alpha)t}{x} = v$ interchange the order of integration and evaluate the inner integral by means of beta function formula, it gives

$$= \frac{x^{\eta+\rho}\Gamma\left(1-\frac{\eta}{1-\alpha}\right)}{[a(1-\alpha)]^\rho} \frac{1}{(2\pi\omega)^r} \int_{L_1}, \ldots, \int_{L_r} \Psi(\xi_1, \ldots, \xi_r) \left\{\prod_{i=1}^{r} \Phi_i\,(\xi_i) {z_i}^{\xi_i}\right\}$$

$$d\xi_1, \ldots, d\xi_r \left[\frac{x}{a(1-\alpha)}\right]^k \frac{\Gamma(\rho+k)}{\Gamma\left(1+\frac{\eta}{1-\alpha}+\rho+k\right)} \qquad (2.2)$$

$$= \frac{x^{\eta+\rho}\Gamma\left(1-\frac{\eta}{1-\alpha}\right)}{[a(1-\alpha)]^p} H^{o,n+1:|R':m_1,n_1,\ldots,m_r,n_r}_{p+1,q+1:|R:p_1,q_1;\ldots;\,p_r,q_r} \left[\begin{matrix} z_1\left[\frac{x}{a(1-\alpha)}\right]^{\mu_1} \\ \vdots \\ z_r\left[\frac{x}{a(1-\alpha)}\right]^{\mu_r} \end{matrix}\right.$$

$$\left|\begin{matrix} (1-\rho;\mu_1,\ldots,\mu_r), & (a_j;\alpha_j^{(1)},\ldots,\alpha_j^{(r)})_{1,p};\ (e_j;U_j',\ldots,U_j^{(r)}g_j^{(r)})_{1,|R'}; \\ \left(-\rho-\frac{\eta}{1-\alpha};\mu_1,\ldots,\mu_r\right), & (b_j;\beta_j',\ldots,\beta_j^{(r)})_{1,q};\ (l_j;U_j'f_j',\ldots,U_j^{(r)}f_j^{(r)})_{1,|R}; \end{matrix}\right.$$

$$\left.\begin{matrix}(c'_j;\gamma'_j)_{1,q_1};\dots;(c_j^{(r)},\gamma_j^{(r)})_{1,p_r}\\[2ex](d'_j;\delta'_j)_{1,q_1};\dots;(d_j^{(r)};\delta_j^{(r)})_{1,q_r}\end{matrix}\right]$$

If we set n = p = q = 0 then by eqn. (1.16), we obtain

Corollary 2.1

If η , $\rho \in$ C, $R(\beta) > 0$, $R\ (1+\frac{\eta}{1-\alpha}) > 0$, $(i = 1,\dots,r)$, $\alpha < 1$, $b \in$ R, then

$$\left(P_{0+}^{(\eta,\alpha)} t^{\rho-1}\prod_{i=1}^{r} H_{|R:p_i,q_i}^{|R':m_i,n_i}\left[z_i t^{\mu_i}\left|\begin{matrix}(e_j;U'_j,\dots,U_j^{(r)}g_j^{(r)})_{1,1R'} & (c'_j,\gamma'_j)_{1,p_i}\\[2ex](l_j;U'_jf'_j,\dots,U_j^{(r)}f_j^{(r)})_{1,1R}; & (d'_j,\delta'_j)_{1,q_i}\end{matrix}\right.\right]\right)$$

$$=\frac{x^{\rho+\eta}\Gamma\left(1+\frac{\eta}{1-\alpha}\right)}{[a(1-\alpha)]^{p}} H_{1,1:|R:p_1,q_1;\dots;\,p_r,q_r}^{0,1:|R':m_1,n_1,\dots,m_r,n_r}\left[z_i\left[\frac{x}{a(1-\alpha)}\right]^{\mu_i}\right|$$

$$\left.\begin{matrix}\left(1-\rho;\mu_1,\dots,\mu_r\right):(e_j;U'_j,\dots,U_j^{(r)}g_j^{(r)})_{1,|R'}:(c'_j,\gamma'_j)_{1,p_i}\\[2ex]\left(-\rho-\frac{\eta}{1-\alpha};\mu_1,\dots,\mu_r\right):(l_j;U'_jf'_j,\dots,U_j^{(r)}f_j^{(r)})_{1,|R};(d'_j,\delta'_j)_{1,q_1}\end{matrix}\right]. \qquad (2.3)$$

In order to obtain the Laplace transform of Modified Multivariable H-function, we evaluate the expression.

$$\lim_{\infty\to 1}\frac{x^{\rho+\eta}\Gamma(\rho+k)\Gamma\left(1+\frac{\eta}{1-\alpha}\right)}{[a(1-\alpha)]^{\rho+k}\,\Gamma\left(\rho+k+1+\frac{\eta}{1-\alpha}\right)}$$

which can be seen from (2.2).Expanding the gamma function by Stirling formula for gamma function.

$$\lceil(1-z)\sim (2\pi)^{\frac{1}{2}}(z)^{1-z-\frac{1}{2}}e^{-z}$$

the above expression becomes

$$= \underset{\alpha\to 1}{\mathrm{Lim}}\frac{x^{\rho+\eta}\Gamma(\rho+k)\left(\frac{\eta}{1-\alpha}\right)^{1+\frac{\eta}{1-\alpha}-\frac{1}{2}}}{[a(1-\alpha)]^{\rho+k}\left(\frac{\eta}{1-\alpha}\right)^{\frac{\eta}{1-\alpha}+\rho+k+1-\frac{1}{2}}}$$

$$= \underset{\alpha\to 1}{\mathrm{Lim}}\frac{x^{\rho+\eta}\Gamma(\rho+k)}{[a(1-\alpha)]^{\rho+k}(\frac{\eta}{1-\alpha})^{\rho+k}}$$

$$= \frac{x^{\rho+\eta}\Gamma(\rho+k)}{[a\eta]^{\rho+k}}$$

Now using the limit formula (1.16) we arrive at

Corollary 2.2

$$\text{If } R(\rho+\mu_i e_i) > 0, R\left(\frac{a\eta}{x}\right) > 0, (i = 1, \dots, r)\text{then}$$

$$\int_0^\infty e^{-\frac{a\eta}{x}t} + t^{\rho-1}H[z_1 t^{\mu_1}, \dots, z_r t^{\mu_r}]\,dt = \frac{x^{\rho+\eta}}{(a\eta)^{\rho}} H^{o,n+1:|R':m_1,n_1,\dots,m_r,n_r}_{p+1,q+1:|R:p_1,q_1;\dots;\,p_r,q_r}$$

$$\left[\begin{matrix} z_1\left[\frac{x}{a\eta}\right]^{\mu_1} \\ \vdots \\ z_r\left[\frac{x}{a\eta}\right]^{\mu_r}\end{matrix}\left|\begin{matrix}(1-\rho;\mu_1,\dots,\mu_r),(a_j;\alpha_j^{(1)},\dots,\alpha_j^{(r)})_{1,p}: \\ \\ \left(-\rho-\frac{\eta}{1-\alpha};\mu_1,\dots,\mu_r\right),\left(b_j;\beta_j',\dots,\beta_j^{(r)}\right)_{1,q}:\end{matrix}\right.\right.$$

$$\left.\begin{matrix}(e_j;U_j',\dots,U_j^{(r)}g_j^{(r)})_{1,|R'};(c_j^{(1)};\gamma_j^{(1)})_{1,p_1,};\dots;(c_j^{(r)}\gamma_j^{(r)})_{1,p_r}; \\ \\ (l_j;U_j'f_j',\dots,U_j^{(r)}f_j^{(r)})_{1,|R};(d_j^{(1)};\delta_j^{(1)})_{1,q_1,};\dots;d_j^{(r)}\delta_j^{(r)})_{1,q_r};\end{matrix}\right]$$

The fractional integration of multivariable H-function has been discussed earlier, among others, by Saigo and Saxena[18-20], Saigo *et al*.[21], Srivastava *et al*.[27], Srivastava and Panda[28, 29], Gupta *et al*.[10] and others.

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