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Free Convection MHD flow between two vertical walls filled with a Porous matrix

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Abstract

The proposed study analyzes the free convection flow between two vertical walls filled with a porous matrix in presence of a transverse magnetic field considering the effect of both viscous and Darcy dissipation. The effect of pertinent parameters on the flow field is analyzed with help of graphs.

1. Introduction

The free convection flow between two vertical walls have been a subject of interest of researcher in metal production, underground bed flow and agricultural engineering. Mishra *et al.*[1] investigated mixed convection in a porous medium bounded by two vertical walls. Das *et al.*[2] analyzed Mass transfer effects on MHD flow and heat transfer past a vertical porous plate through a porous medium under oscillatory suction and heat source. Panda *et al.*[3] investigated Heat and mass transfer in MHD flow of viscous fluid past a vertical plate under oscillatory suction velocity with heat source. Sahoo *et al.*[4] analyzed Unsteady two dimensional MHD flow and heat transfer of an elastico-viscous liquid past an infinite hot vertical porous surface bounded by porous medium with source/sink.. Panda *et al.*[5] investigated Heat and mass transfer on MFD flow through porous media over an accelerating surface in presence of suction and blowing. Sahoo *et al.*[6] analyzed the MHD mixed convection stagnation point flow and heat transfer in a porous medium.

The proposed study analyzes the free convection flow between two vertical walls filled with a porous matrix in presence of a transverse magnetic field considering the effect of both viscous and Darcy dissipation.

2. *Formulation of the problem :*

A steady natural convection flow of a

viscous incompressible fluid in a porous region bounded by two vertical parallel walls is considered, one of the walls is heated and other is cooled. The x′-axis is taken along one of the walls while y′-axis is taken normal to it. The temperature of the wall at $y' = 0$ is taken T'_h whereas for the other wall $y' = h$ it is taken T'_0 $(T'_h > T'_0)$ so that free convection flow can occur between the walls.Under the usual Boussinesq's approximation the natural convection flow is governed by the following system of equations

$$\frac{1}{\rho}\mu_{eff}\frac{d^2u'}{dy'^2}-\frac{\upsilon_f u'}{K}+g\beta\left(T'-T'_0\right)-\frac{\sigma B_0^2 u'}{\rho}=0 \quad (1)$$

$$\frac{d^2T'}{dy'^2}+\frac{1}{k}\mu_{eff}\left(\frac{du'}{dy'}\right)^2+\frac{\rho}{kK}u'^2=0 \quad (2)$$

along with the boundary condition

$$u'=0, \qquad T'=T'_H \qquad \text{at } y'=0$$
$$u'=0,\ T'=T_0' \qquad \text{at } y'=H \quad (3)$$

Introducing the non-dimensional quantities

$$y=\frac{y'}{H}, \qquad D_a=\frac{K}{H^2}, \qquad B_v=\frac{\mu_{eff}}{\mu_f},$$

$$u=\frac{u'\nu_f}{g\beta H^2(T'_h-T'_0)}, \qquad \theta=\frac{(T'-T'_0)}{(T'_h-T'_0)},$$

$$N=\frac{\rho g^2\beta^2H^4(T'_h-T'_0)}{k\nu_f},$$

$$M=\sqrt{\frac{\sigma B_0^2H^2}{\mu_f}} \quad (4)$$

equations (1) and (2) become

$$B_v\frac{d^2u}{dy^2}-\frac{u}{D_a}+\theta-M^2u=0 \quad (5)$$

$$\frac{d^2\theta}{dy^2}+B_vN\left(\frac{du}{dy}\right)^2+N\frac{u^2}{D_a}=0 \quad (6)$$

Where H is the distance between the vertical walls; K, the permeability of the porous medium; μ_{eff}, effective viscosity of the porous region; μ_f dynamic viscosity of the fluid; ν_0 kinematics viscosity of the fluid; u′, the fluid velocity; g, acceleration due to gravity; β, coefficient of thermal expansion; T ′, the temperature of the fluid; ρ, the density, B_v , the Bsrinkman number and k, the thermal conductivity of the fluid.

The boundary conditions concerned with the problem in non-dimensional form are obtained as follows :

$$u=0, \quad \theta=1 \quad \text{at } y=0;$$
$$u=0, \quad \theta=0 \quad \text{at } y=1; \quad (7)$$

3. *Solution of the zeroth order :*

The problem discussed in the previous section, due to viscous and Darcy dissipation terms, can be solved by using the perturbation technique because of the fact that N is small (N<1) in most of the practical problems. As a result of this assumption, we consider the following expansions:

$$u = u_0 + N u_1 + O(N^2),$$
$$\theta = \theta_0 + N\theta_1 + O(N^2), \quad (8)$$

The quantities u_0 and θ_0 are the solutions for N= 0 *i.e.*, when viscous and Darcy dissipations are neglected whereas u_1 and θ_1 are perturbed quantities related to u_0 and θ_0 respectively when viscous and Darcy dissipations are taken into account. The solution for u_0 and θ_0 are

$$u_0 = \frac{1-y}{m^2 B_v} + A\left[e^{-m(2-y)} - e^{-my}\right] \tag{9}$$

$$\theta_0 = 1 - y, \tag{10}$$

which are solutions of the differential equations obtained by setting N = 0 in equation (5) and (6) under the boundary conditions given in (7). In equation (9), m and A are given by

$$m = \sqrt{\frac{1}{B_v}\left(M^2 + \frac{1}{D_a}\right)},\; A = \frac{1}{m^2 B_v (1 - e^{-2m})} \tag{11}$$

4. Solution of the zeroth order :

We have to find the solution of the differential equations corresponding to first order to see the effect of viscous as well as Darcy dissipations. The governing equations are

$$B_v \frac{d^2 u_1}{dy^2} - \frac{u_1}{D_a} + \theta_1 - M^2 u_1 = 0 \tag{12}$$

$$\frac{d^2\theta_1}{dy^2} + B_v\left(\frac{du_0}{dy}\right)^2 + \frac{u_0^2}{D_a} = 0 \tag{13}$$

the boundary conditions now reduced to

$u_1 = 0$, $\theta_1 = 0$ at $y = 0$;

$u_1 = 0$, $\theta_1 = 0$ at $y = 1$; (14)

Solving equations (12) and (13) using boundary conditions (14) we get,

$$u_1 = A_1 e^{my} + \left[A_2 + A_7 + \frac{1}{2m^2 D_a}\left(\frac{y^2}{2} - y + \frac{y}{2m}\right)\right] - A_3\left(y^2 + \frac{2}{m^2}\right) + \frac{A_6 y + A_5}{B_v m^2}$$
$$+ A_4\left[(1-y)^4 + \frac{12}{m^2}(1-y)^2 + \frac{24}{m^4}\right]$$
$$+ \left[A_7 + \frac{1}{2m^2 D_a}\left(\frac{y^2}{2} - y - \frac{y}{2m}\right)\right]e^{m(y-2)}$$
$$+ A_8\left[e^{2m(y-2)} + e^{-2my}\right] \tag{15}$$

$$\theta_1 = A_6 y + A_5 - \left(B_v - \frac{1}{D_a}\right)y^2 e^{-2m}$$
$$- \frac{y^2}{2m^4 B_v} - \frac{(1-y)^4}{12 m^4 B_v^2 D_a} +$$
$$\frac{2A}{m^3 B_v}\left[B_v - \frac{y-1}{mD_a} - \frac{2}{m^2 D_a}\right]\left(e^{-my} + e^{2m(y-2)}\right) + A_{10} e^{-2my} +$$
$$\frac{2A}{m^3 B_v}\left[B_v + \frac{y-1}{mD_a} - \frac{2}{m^2 D_a}\right]e^{m(y-2)} \tag{16}$$

5. Results and Discussion

Numerical values of analytical solutions pertaining to the combined effects of Darcy and viscous resistances on the fully developed free convection flow of a viscous fluid in a porous media confined between two vertical walls and saturated with the same fluid are presented through graphs to bring out the effects of various important parameters governing the flow field.

The discussions of the problem modeled in the previous sections are divided into two parts. Case I presents the discussion on u_0, θ_0 for N =0 representing the model when viscous and Darcy dissipations are neglected.

From fig. 1 it is observed that the magnetic parameter M reduces the velocity at all points of the flow field. An increase in the value of magnetic parameter M reduces the velocity further. Magnetic parameter contributes to the velocity profiles by pushing the crests towards cooler plate. The decrease in Darcy parameter results in linearity of the profiles.An increase in Brinkman number reduces the velocity significantly for higher Darcy number.

Temperature distribution in N = 0 is given by $\theta_0 = 1 - y$ which shows a linear variation from $\theta_0 = 1$ at the heated wall to $\theta_0 = 0$ at the other one.

6. Conclusion

The analysis of above results brings following conclusion.

1. The decrease in viscosity of the fluid under study *i.e.*, for the fluid with low viscosity, velocity is greatly reduced for higher Darcy number.
2. Combined effect of Darcy diffusion and transverse magnetic field opposes the motion of the fluid.
3. The uniform magnetic field which has been applied to the flow opposes the motion of the fluid.

Curve	B_v	D_a	M
1	1	0.1	0
2	2	0.1	0
3	1	0.01	0
4	2	0.01	0
5	1	0.001	0
6	2	0.001	0
7	1	0.1	1
8	1	0.1	3

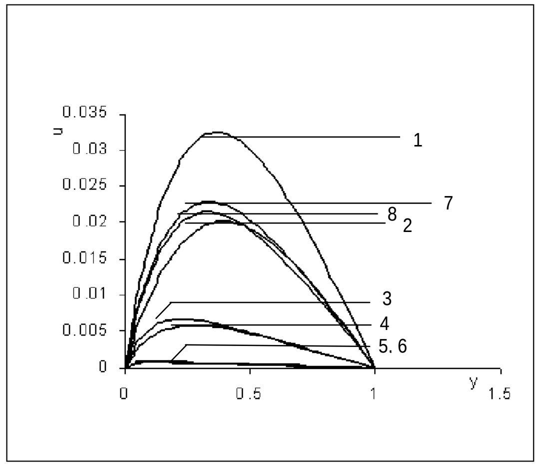

Fig . 1. Velocity profiles when dissipations are neglected

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