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Algorithm for realization of Type-I Discrete Sine Transform

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Abstract

In this paper, an algorithm for realization of type -I discrete sine transform (DST-I) of length $N-1$, where $N=2^m (m=2,3,4,....)$, is presented. The DST-I is realized by decomposing it into DST-I of length $\frac{N}{2}-1$ and type-II discrete sine transform (DST-II) of length $\frac{N}{2}$.

Key words: Discrete sine transform, Discrete cosine transform.

1. Introduction

Discrete transforms play a significant role in digital signal processing. Discrete sine transform (DST) and discrete cosine transform (DCT) are used as key functions in many signal and image processing applications. There are eight types of DST. The DST transform of types I, II, III and IV, form a group of so-called “even” sinusoidal transforms. Much less known is group of so-called “odd” sinusoidal transforms: DST of types V, VI, VII and VIII. The DSTs of “even” type have found wide applications.

The DST was first introduced to the signal processing by Jain[1], and several versions of this original DST were later developed by Kekre *et al.*[2], Jain[3] and Wang *et al.*[4]. Ever since the introduction of the first version of the DST, the different DST’s have found wide applications in several areas in Digital signal processing (DSP), such as image processing[1,5,6], adaptive digital filtering[7] and interpolation[8]. The performance of DST can be compared to that of the DCT and it may therefore be considered as a viable alternative to the DCT. For images with high correlation, the DCT yields better results; however, for images with a low correlation of coefficients, the DST yields lower bit rates[9]. Yip and Rao[10] have proven that for large sequence length ($N \geq 32$) and low correlation coefficient ($\rho < 0.6$), the DST performs even better than the DCT.

In this paper, a new algorithm for computation of DST-I of length $N-1$, where

$N=2^m (m=2,3,4,....)$ is presented. The DST-I is realized from two DST sequences, one is DST-I of length $\frac{N}{2}-1$ and the other is DST-II of length $\frac{N}{2}$.

The rest of the paper is organized as follows. The proposed algorithm for DST-I is presented in Section-2. An example for computation of DST-I for N=8 is given in Section-3. Conclusion is given in Section-4.

2. Proposed algorithm for DST-I :

The DST-I for input sequence $x(n)$ is defined as

$$Y_1(k)=\sqrt{\frac{2}{N}}\sum_{n=1}^{N-1} x(n)\sin\left[kn\frac{\pi}{N}\right] \quad (1)$$

for k = 1,2,...,N-1

The DST-II for input data array $x(n), 1 \le n \le N,$ is defined as

$$Y_{II}(k)=\sqrt{\frac{2}{N}}C_k\sum_{n=0}^{N-1} x(n)\ \sin\left[\frac{(2n+1)k\pi}{2N}\right] \quad (2)$$

for k = 1,2,..., N

where,

$$C_k=\begin{cases}\frac{1}{\sqrt{2}}\ if\ k=N\\ 1 if\ k=1,2,...,N-1\end{cases}$$

The $Y_1(k)$ and $Y_{II}(k)$ values in (1) and (2) respectively represent the transformed data. Without loss of generality, the scale factors in (1) and (2) are ignored in the rest of the paper. After ignoring the scale factor, (1) can be written as

$$Y_1(k)=\sum_{n=1}^{N-1} x(n)\sin\left[kn\frac{\pi}{N}\right] \quad (3)$$

for k = 1,2,..., N-1

Taking $N=2^m (m=2,3,....)$, (3) can be written as

$$Y_1(k)=\sum_{n=1}^{\frac{N}{2}-1} x(2n)\sin\left[\frac{2nk\pi}{N}\right]$$

$$+\sum_{n=0}^{\frac{N}{2}-1} x(2n+1)\sin\left[(2n+1)\frac{k\pi}{N}\right] \quad (4)$$

$$\Rightarrow Y_1(k)=P(k)+Q(k) \quad (5)$$

where,

$$P(k)=\sum_{n=1}^{\frac{N}{2}-1} x(2n)\sin\left[\frac{2nk\pi}{N}\right] \text{ for}$$

$$k=1,2,.....,\frac{N}{2}-1 \quad (6)$$

and

$$Q(k)=\sum_{n=0}^{\frac{N}{2}-1} x(2n+1)\sin\left[(2n+1)\frac{k\pi}{N}\right]$$

$$\text{for } k=1,2,.....,\frac{N}{2} \quad (7)$$

According to the definitions (1) and (2) of DST-I and DST-II respectively, $P(k)$ represents DST-I of length $\frac{N}{2}-1$ for even input data sequence $x(2n)$ and $Q(k)$ represents DST-II of length $\frac{N}{2}$ for odd input data sequence $x(2n+1)$. Therefore, DST-I of length N-1 is decomposed into DST-I of length $\frac{N}{2}-1$ and DST-II of length $\frac{N}{2}$.

Replacing k by $N-k$ in (5), we have

$$Y_1(N-k) = P(N-k) + Q(N-k) \qquad (8)$$

From (6), we obtain

$$P(N-k) = \sum_{n=1}^{\frac{N}{2}-1} x(2n)\sin\left[(N-k)\frac{2n\pi}{N}\right]$$

$$= \sum_{n=1}^{\frac{N}{2}-1} x(2n)\sin\left[2n\pi - \frac{2nk\pi}{N}\right]$$

$$= -\sum_{n=1}^{\frac{N}{2}-1} x(2n)\sin\left[\frac{2nk\pi}{N}\right]$$

$$\Rightarrow P(N-k) = -P(k) \qquad (9)$$

Using (7), we get

$$Q(N-k) = \sum_{n=0}^{\frac{N}{2}-1} x(2n+1)\sin\left[(2n+1)(N-k)\frac{\pi}{N}\right]$$

$$= \sum_{n=0}^{\frac{N}{2}-1} x(2n+1)\sin\left[(2n+1)\pi - (2n+1)\frac{k\pi}{N}\right]$$

$$= \sum_{n=0}^{\frac{N}{2}-1} x(2n+1)\sin\left[(2n+1)\frac{k\pi}{N}\right]$$

$$\Rightarrow Q(N-k) = Q(k) \qquad (10)$$

Using (9) and (10) in RHS of (8), we have

$$Y_1(N-k) = Q(k) - P(k) \qquad (11)$$

For $k = \frac{N}{2}$, we have from (6)

$$P\left(\frac{N}{2}\right) = \sum_{n=1}^{\frac{N}{2}-1} x(2n)\sin(n\pi) = 0 \qquad (12)$$

Using (12) in (11), we obtain for $k = \frac{N}{2}$,

$$Y_1\left(\frac{N}{2}\right) = Q\left(\frac{N}{2}\right) \qquad (13)$$

The block diagram for implementation of DST-I using (5), (11) and (13) is shown in Fig.1.

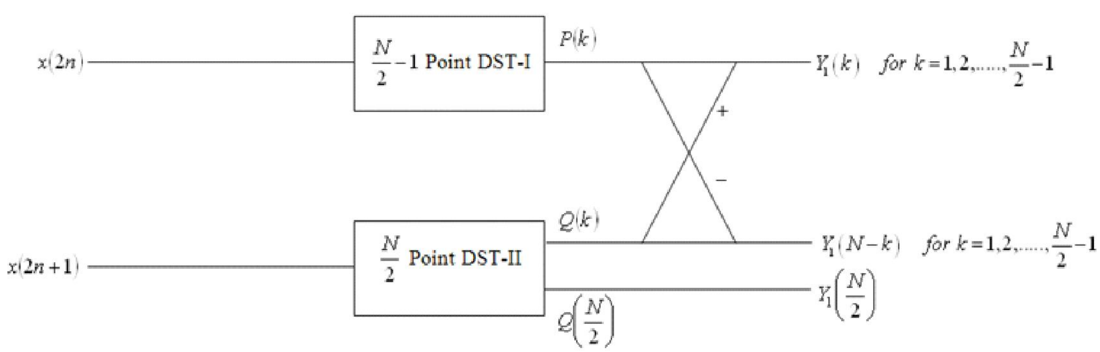

Figure 1. Block Diagram for implementation of DST-I of length N-1

3. Example for realization of DST-I for N=8

An example to clarify the proposal for $N = 8$ is considered.

Using (5), we have for $k = 1,2\&3$

$$Y_1(1) = P(1) + Q(1)$$

$$Y_1(2) = P(2) + Q(2) \qquad (14)$$

$$Y_1(3) = P(3) + Q(3)$$

From (11), we obtain for $N = 8$ and $k = 1,2\&3$

$$Y_1(7) = Q(1) - P(1)$$

$$Y_1(6) = Q(2) - P(2) \qquad (15)$$

$$Y_1(5) = Q(3) - P(3)$$

For $N = 8$, we have from (13)

$$Y_1(4) = Q(4) \qquad (16)$$

The output data components $Y_1(1), Y_1(2), \ldots\ldots, Y_1(7)$ of DST-I are computed from the input data sequence $x(1), x(2), \ldots.., x(7)$ using (14), (15) and (16) as shown in the Fig. 2.

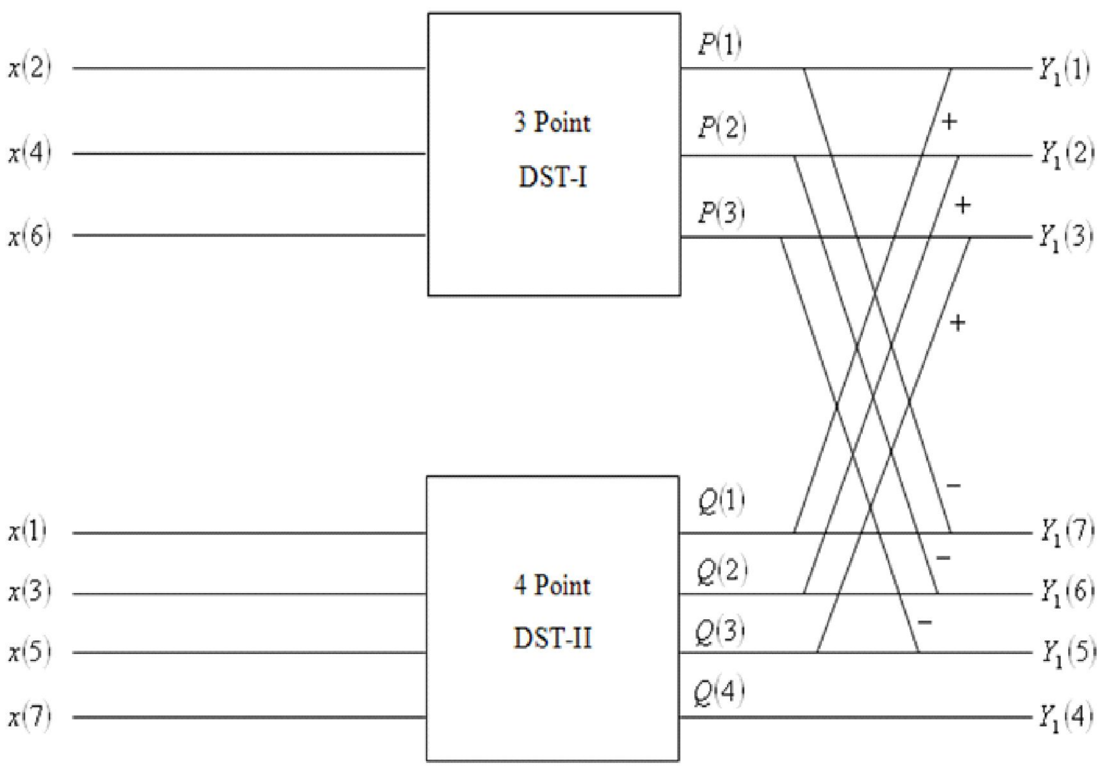

Figure 2. Block Diagram for implementation of DST-I for *N*=8

4. Conclusion

A novel algorithm for realization of DST-I of length $N-1$, where $N=2^m (m = 2, 3, 4, ...)$, has been presented. The DST-I is realized from two DST sequences, one is DST-I of length $\frac{N}{2}-1$ and the other is DST-II of length $\frac{N}{2}$. Block diagram for implementation of DST-1 using this algorithm is shown.

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