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Some Results on PC –Compactness in Fuzzy Bitopological Spaces

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Abstract

In this paper we have introduced the concept of PC –Compactness in Fuzzy Bitopological Spaces, as a generalization of P compactness **in Bitopological Spaces.**

Key words: Fuzzy topological spaces, Fuzzy bitopological spaces, Compact Fuzzy topological spaces, PC- Compact Fuzzy bitopological spaces.

Introduction

The concept of Fuzzy Set was introduced by Zadeh[7] in 1965 to describe those phenomena which are imprecise, vague or fuzzy in nature. Fuzzy set theory is similar to ordinary set theory, but there are some remarkable differences between these two. Compactness in fuzzy bitopological space has been studied earlier by Abd El-Monsef and Ramadan[2] and Safiya[1] *et. al.* as generalizations of P compactness or S- Compactness in bitopological space. In this paper we have obtained some interesting results related to PC –Compactness introduced by Safiya[1] *et. al.* as a concept paralleling that of P compactness of bitopological spaces.

A fuzzy Bitopological Space (in short fbts) is a triple(X, τ_1, τ_2) where X is a set and τ_1, τ_2 two fuzzy topology on X. In this paper we have considered fuzzy topology and compactness in the sense of Lowen[4].

Now we mention some notations/ definitions which will be used in this paper. Abd El-Monsef and Ramadan[2] have defined λ – adjoined fuzzy topology, but this may not be a fuzzy topology in Lowen sense. Here we give a modified definition of λ – adjoined fuzzy topology[3,5-8].

Definition 1: If τ is a fuzzy topology on X and λ is a non empty fuzzy set in X then the λ adjoint fuzzy topology denoted by $\tau(\lambda)$ is the fuzzy topology on X generated by

$S = \{\lambda \cup \mu : \mu \in \tau\} \cup \{\underline{\alpha} : \alpha \in [0,1]\}$

The following definitions are from Abu Safiya[1] *et al.*.

A pairwise open cover(in short, P-open cover) of a fbts (X, τ_1, τ_2) is a collection $U=\{u_s: u_s\in \tau_1 \cup \tau_2 : s\in S\}$ where U contains at least one non zero member of τ_1 and at least one non zero member of τ_2.

A fuzzy set A is called PC – compact if $\forall$ P-open cover U of A and $\varepsilon>0$ $\exists$ a finite sub family U^{ε} of U which covers A-ε. An fbts (X, τ_1, τ_2) is called PC –compact if each constant fuzzy set $\underline{\alpha}$ ($\alpha \in [0,1]$) is PC-compact.

Theorem 1: Let(X, τ_1, τ_2) this be a fbts. Then the following statements are equivalent:

(i) $\forall \lambda \in \tau_1$, the adjoint fuzzy topology $\tau_2(\lambda)$ is compact and $\forall\ \mu\in\tau_1$, the adjoint fuzzy topology $\tau_1(\mu)$ is compact

(ii) (X, τ_1, τ_2) is PC compact

Proof: (i) $\Rightarrow$(ii)

Let $U = \{u_s : s \in S\}$ be a P-open cover of $\underline{\alpha}$. Let $U=U_1 \cup U_2$ where $U_1=\{\lambda_s : s \in S_1\}\subseteq \tau_1$ and $U_2 = \{\mu_s : s \in S_2\} \subseteq \tau_2$. Choose ε such that $0< \varepsilon<\alpha$. Let $\lambda=\cup \lambda_s$ then $\lambda \in \tau_1$ and now consider the family $T_1=\{\lambda\cup \mu_s : s \in S_2\}$ then T_1 is a $\tau_2(\lambda)$ open cover of $\underline{\alpha}$ and therefore

$s\in S1$

since $\tau_2(\lambda)$ is compact, for $0<\varepsilon_1<\varepsilon$, $\exists$ a finite subfamily of T_1, say, $T_2=\{\lambda \cup \mu_s : s \in M_2\subseteq S_2$, M_2 is finite$\}$ which covers $\underline{\alpha} - \varepsilon_1$. Further, let $\mu = \sup \mu_s$. Then $\mu\in \tau_2$ and since $\underline{\alpha} - \varepsilon_1 \subseteq \sup (\lambda \cup \mu_s)=\lambda \cup (\sup \mu_s) =(\sup \lambda_s)\cup \mu= \sup \{\lambda_s$

$s\in M2$

$\cup\ \mu: s\in S_1\}$, thus the family $T_3 = \{\lambda_s \cup \mu: s \in S_1\}$ is a $\tau_1(\mu)$ open cover of $\underline{\alpha} - \varepsilon_1$ and

$s\in M2$ $s\in S1$ $s\in S1$

since $0<\alpha-\varepsilon<\alpha-\varepsilon_1$, using compactness of $\tau_1(\mu)$, $\exists$ a finite subfamily of T_3, say $T_4=\{\lambda_s \cup \mu: s \in M_1\subseteq S_1$, M_1 is finite$\}$ which covers $\underline{\alpha} - \varepsilon$. Hence, $\{\lambda^s : s \in M_1\} \cup \{\mu_s: s \in M_2\}$ is a finite subfamily of U which covers $\underline{\alpha} - \varepsilon$. Thus (X, τ_1, τ_2) is PC- compact.

(ii) $\Rightarrow$ (i)

Let us assume that (X, τ_1, τ_2) is PC-compact. Let $\lambda\in \tau_1$, $\lambda\neq\phi$ then $\tau_2(\lambda)$ is generated by

$S=\{\lambda\cup\mu:\mu\in\tau_2\}\cup\{\underline{\alpha}:\alpha\in[0,1]\}=S_1\cup S_2$, (say).

Now to show that $\tau_2(\lambda)$ is compact, using theorem 4.6 of Lowen[4], which is fuzzy form of Alexander's subbase lemma, it is sufficient to prove that any open cover U of $\underline{\alpha}$ such that $U\subseteq S$ and $\forall\ \varepsilon$ such that $0<\varepsilon<\alpha$, $\exists$ a finite subfamily say U^{ε} of U which covers $\underline{\alpha}-\varepsilon$. Now, let $U=\{u_s : s \in S\}\subseteq S$ be an open cover of $\underline{\alpha}$. Then $U=U_1 \cup U_2$ where $U_1\subseteq S_1$ and $U_2 \subseteq S_2$. Let $U_1 =\{u_s:s\in S_1\subseteq S\}$ and $U_2=\{u_s:s\in S_2\subseteq S\}$ where $S=S_1 \cup S_2$. Then $\underline{\alpha} \subseteq\{u_s: s \in S\}=\sup\{\{u_s: s \in S_1\} \cup \{u_s: s \in S_2\}\}= \sup\{\{\lambda \cup\mu_s: s \in S_1\} \cup \{\underline{\beta_s}: s \in S_2, \underline{\beta_s} \in S_2=\sup\{\{\lambda\} \cup\{\mu_s: s \in S_1\} \cup\{\underline{\beta_s}: s \in S_2\}$ and hence, the family $U_1 =\{\lambda\} \cup\{\mu_s: s \in S_1\} \cup\{\underline{\beta_s}: s \in S_2\}$ is a P open cover of $\underline{\alpha}$. Now using PC- compactness, $\exists$ a finite subfamily of U_1 say U^{ε}_1 which covers $\underline{\alpha} - \varepsilon$. Now there are two possibilities U^{ε}_1 may or may not contain λ. If it contains λ then U^{ε}_1 will be of the form $\{\lambda\} \cup \{\mu_s: s \in S_0' \subseteq S_1\} \cup\{\underline{\beta_s}: s \in S_0''\subseteq S_2\}$.

In this case, $\{\lambda \cup \mu_s : s \in S_0'\} \cup \{\underline{\beta_s} : s \in S_0''\}$ is a finite subfamily of U which covers $\underline{\alpha} - \varepsilon$. In the other case, if U^{ε}_1 does not contain λ then $\{\lambda\} \cup U^{\varepsilon}_1 = \{\lambda\} \cup \{\mu_s : s \in S_0'\} \cup \{\underline{\beta_s} : s \in S_0''\}$ is again a finite subfamily of U_1 which covers $\underline{\alpha} - \varepsilon$ which further implies that $\{\lambda \cup \mu_s : s \in S_0'\} \cup \{\underline{\beta_s} : s \in S_0''\}$ is a finite subfamily of U which covers $\underline{\alpha} - \varepsilon$. Thus, $\tau_2(\lambda)$ is compact[4-8].

Similarly, it can be shown that $\tau_1(\mu)$ is compact for every non empty $\mu \in \tau_2$.

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